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CONTENTS

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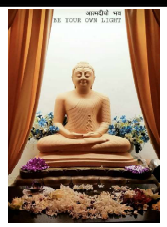

We feel extremely happy to place before you this issue **(NSV 17, December 2021, No. 4). This completes 17 years for our work with SVJ.** We express our very sincere thanks to all our contributors, evaluators, readers and well wishers for their continuous and consistent support, without which we would not have achieved our goal.

This issue contains for the section of Management and Statistics, one review article and two articles. Other portion of the journal contains three Research Artciles, one Biography, one Book Review and other items like SV News Letter and Readers Forum as usual. You will find the details in the contents.

We are very highly obliged to **our following referees** who have helped us for evaluation of articles / papers submitted for this issue. **(Their names are given one by one in the order of their apperance in the journal.)**

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We express our sincere thanks to our **Research Team** for this work. In particular we thank **Shree Dinesh Darji for DTP work and Shree Ashish Bhatt for website.**

All our contributors will get digital copy and official certificates.

For the coming year 2022, we express our best wishes for your good health, progress and prosperity. **WISH YOU HAPPY NEW YEAR 2022.**

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SANKHYA VIGNAN

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MANAGEMENT AND STATISTICS
(REVIEW ARTICLE)

CONTROL AND TRACK INVENTORY WITH DIGITAL AUTOMATION

Jayesh R. Purohit*

ABSTRACT

This paper deals with the difficulties as observed in traditional Inventory Management. The modern concept to deal with them is the use of digital automation with its benefits over the earlier approaches.

KEY WORDS

Siloed SCM, Automation, FRID-RED, Productivity, Customer Satisfaction.

* * *

If your business sells, handles or stores products, its success depends largely on the quality of its inventory management system. It's not a suggestive topic. But it sure is important. Unfortunately, when it comes time to spend capital on innovations that will boost business results, automated inventory management systems are often overlooked.

After all, if you don't have products in stock to send to customers, any efforts to create viral marketing campaigns will have been wasted. And, if you're making mistakes with inventory management, your customers will be unhappy and your results will suffer. Since inventory management plays such a large role in your business strategy and results, it makes sense to give it the attention it deserves. Here's what you need to know about controlling and tracking inventory with digital automation solutions.

Challenges for Traditional Inventory Management

With the surge in e-commerce sales and the increase in on-demand logistics,

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traditional inventory methods are no longer sufficient to guarantee an efficient supply chain, particularly now that many brands choose to use multiple fulfilment canters and outsource their logistics functions. If you're on the fence about upgrading to digital automation for inventory management, here are some of the challenges that traditional approaches are still facing.

Insufficient Demand Visibility

For an inventory system to be fully functional, it must have total visibility. As a business expands its supply chain, it's vital to have a system in place that allows for viewing and tracking inventory in real-time.

Traditional inventory approaches don't provide this type of transparency. Instead, businesses are left guessing how much inventory is available or moving across multiple channels and locations.

Siloed Supply Chain Communication

Traditional inventory management methods are often incapable of tracking inventory between outbound and inbound logistics operations and sometimes even within a single warehouse. This stifles the communication between third-party logistics providers and internal teams.

With traditional inventory systems, scaling operations becomes challenging if you want to maintain control over the inventory in your network. This can lead to higher logistics costs, backorders and stock outs, inaccuracies and lower customer satisfaction levels.

Lack of Demand Forecasting

One of the most critical elements of inventory management is the ability to forecast demand accurately. This can give just about any business a competitive advantage and improve results. But traditional inventory management is incapable of providing logistics companies, suppliers and retailers with the information they need to make smart business decisions.

Automated Inventory Management

As an online business or third-party logistics provider, you should always be concerned about inventory levels. Specifically, you may wish to ask yourself:

- How much inventory do I have on hand?
- What are my best-selling items?
- When should I re-order, and how much?

Since inventory is a moving target, keeping track of it through traditional approaches, like manually counting items and updating spreadsheets, isn't practical. The rise of automated inventory management solutions has made it easier for companies to control inventory based on demand, save on costs, and improve overall results.

RFID-READ

The solution for many businesses is to usher in automated inventory control and tracking solutions. These are modern processes that use advanced technology to manage inventory within the supply chain. With software, mobile solutions and other integrated hardware, like barcodes, RFID tags and scanners, warehouse management can gain the visibility it needs.

Benefits of Digital Automation in Inventory Control and Tracking

Whether you've been neglecting inventory management or just been reluctant to invest in it, this isn't something you want to ignore. Here are some of the benefits of digital automation in this area of your business.

1. Digital automation saves you time.

Manual inventory management processes are quickly becoming outdated due to inefficiencies. It takes too much time to extract information from various systems, figure out stock levels, record changes and generate reports. Automated inventory control makes updating and tracking inventory levels a seamless process. When used with other integrated services, such as point-of-sale systems, a business can save a ton of time and trench the spreadsheets for good.

2. Inventory management helps you predict demand.

What's in your warehouse now may not be the perfect predictor of demand, but the movement of products is certainly one of them. Unexpected shortages are never something you want to have because it impacts the customer experience. When you automate your inventory management system, you get real-time data on changes in your stock and can set up alerts for automatic re-ordering.

3. Automation minimizes human error.

Many Companies are still using manual methods for inventory control and tracking find that they also have more issues with human errors. Employees can miscalculate, miscount or make other mistakes that cause a warehouse to understock, overstock or simply misplace items that significantly impact their bottom line. Since automated inventory systems work with minimum human intervention, these types of errors are substantially reduced. For example, when an inventory count is completed using scanners, the totals are automatically stored in the system and ready for use by management or other stakeholders.

4. Digitization makes reporting simple.

If a warehouse has to pull figures from a spreadsheet or clipboard for reporting, how timely or accurate will those figures be? To understand what's going on in your business at any given moment, you need to know what's going on with your inventory. When you automate your inventory management and tracking, you can create reports on the fly that tell you what you have and where it is located. You can also pull up trends that help you make better decisions about the business.

5. Automated inventory management is scalable.

Graphical chart analysis; A business that relies heavily on an outdated inventory management system will be at its mercy when it comes to scaling operations. For example, a warehouse that still uses manual lists on clipboards is going to find it challenging to expand to multiple locations where collaboration is necessary. But an automated inventory management system invites scalability. It gives the business real-time data that can be integrated with other locations. This makes it much easier for

a business to scale its operations quickly and as needed.

6. It can make your business more money.

Of course, you'll need to invest in digital automation for your warehouse system. But this list illustrates that the rewards will be worth the effort. You will certainly be able to reduce some of the money spent on the manual maintenance of inventory. You can achieve other cost savings and make more money in the form of improved productivity and efficiency and opportunities to maximize sales opportunities through better customer service.

Digital inventory control and tracking solutions are transforming the modern industry. These are behind-the-scenes solutions that make businesses more efficient, resulting in higher levels of customer satisfaction and better bottom-line results. Contrary to popular belief, a business doesn't have to get rid of its entire workforce or completely reconfigure a warehouse to implement many of these solutions. They can be integrated seamlessly to work with existing staff and will only create improvements in outdated warehouse systems.

Selected References :

[1] **BUFFA (1990)** Modern Production operations Management (7/e, Wiley Eastern Ltd., New Delhi)

[2] **Naddor (2003)**, Inventory Systems (Wiley Pub.)

[3] **Tossi Sheffi,** Logistic CLusters (MIT Press, cambridge, Massachusetts, London, UK)

MANAGEMENT AND STATISTICS
(ARTICLE)

IMPORTANCE OF PSYCHOGRAPHIC DATA IN MARKETING AND ITS MEASUREMENT.

A. C. BRAHMBHATT*

ABSTRACT

This article is a special one representing customer preferences and choices pertaining to the psychographic data as obtained from the questionnaires designed for that purpose. It also signifies how it differes from the usual type questionnaires only representing demographic data that are available from them.

This has immense impact for the marketers which in turn can be much more beneficial to the custmers looking to their choices and preferences. Such an approach can also be benefitial and important in Market research.

KEYWORDS.

Psychography, STP, SEM, Triggers

* * *

Mosty in the descriptive research design, where data are collected through survey mode, the powerful research instrument is questionnaire. The questionnaire should not be a questionbank but is should be designed scientifically.

In such questionnaires, generally we find the questions related to the demographics of the customers e.g. their age, geneder, income, occupation, education, marital status, domicile etc. Such informations are found to be useful in calculating the measures of central tendency, measures of dispersion etc. (e.g. Mean, Range, Mean Deviation, S.D., C.V., Measures of Skewness and Kurtosis etc.)

They can also be useful in testing the correlational or causal hypothesis. But collecting only demographical data is not enough. The same age group people or the same income level people or the same educational level people may have different attitude towards the product or the brand.

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Consumers of the same age group of certain product or service may have diverse buying intention for the product or service - they may have different satisfcational level, they may have different affection level. Customers of the same educational level may have different life styles, different beleifs and systems. The candidates of the same educational level and a work experience background may have different motives for joining the Ph.D. program of the University.

The Psychographic profile helps, therefore, the marketes to describe fully his/ her customer segment and accordingly cater to their needs.

The Psychographic profile also helps to judicialy design their segmenting targetting and positionary (STP) strategy, the Pscyhographics help in designing a promotional programme for the company; it dictates as to which tagline, which logo, which format would be more appropriate.

The marketers on the basis of Psychographics could create diverse customer segments such as the customers having a specific political belief, those having specifia preference for specific brands, those believing in body fitness, the segment having a feminial approach etc.

The marketers on the basis of the Psychographics can think of generating the triggers activating the feeling of hedonism, hopeg, excitement, fear, love etc. Using such triggers, one can enhance the brand value.

Psychographics help the marketers to comprehend the entire buying process of their present and potential customers, that helps them to be a step ahead of their competitors. The Pschographics data are measured in certain scales like Thurston Scale, Likers Scale, Cauhert Scale etc. Such input data serve as a basis for the advanced statistical analysis like Factor Analysis, Multiple Regreeion Analysis, Structural Equation Modelling (SEM) etc.

The Psychographics helps to create more emotionally appealing advertisements, reinforce the broad values, use aspirational images and messaging, update the buyer preferenes etc.

SELECTED REFERENCES.

[1] **William D. Weils (1975**), Psychographics - A critical review journal of marketing research. (Vol. 17, No.2, PP 196-213, Sare Publications)

[2] **Lyhn R. Kahle, Larry Chiagouris (2016)** Values, Life Styles and Psychographics (Psychology press)

MANAGEMENT AND STATISTICS
(ARTICLE)

SUPPLY CHAIN BLOCKAGE AT THE U.S. WEST COAST : A GENERAL DISCUSSION

Ajay K. Aggarwal[1], Dinesh S. Dave[2] and Varinder M. Sharma[3]

ABSTRACT

A recent blockage of the materials at the West Coast of the United States has generated both shortages and higher prices of products regularly sourced from foreign locations. This study discusses the reasons of this unique ocean logistics disruption. The analysis presented in the study reveals that this specific West coast supply chain disturbance occurred from the combination of the lasting effects of COVID-19 pandemic and structural limitations of U.S. ports.

KEYWORDS

Global SCM, Covid pendemic, Logistics

INTRODUCTION

Supply chain management (SCM) deals with the coordination all activities and processes beginning with raw materials acquisition to manufacturing to distribution till the final product reaches to the end-consumers. It is no doubt that the, supply chain management (SCM) is regarded as the most critical intra- and inter-organizational function of firms. Globalization of production and acquisition of the raw material, work-in-process components, and finished goods have made supply chain networks complex because of the worldwide location of supply chain member

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organizations boosted by enhanced technologies, trade agreements, raw material availability, environmental, economic, political, and cultural conditions. Additionally, logistics or transportation play a vital role in a global supply chain. The global supply chain uses the combination of air, water, road, rail, and pipeline media of transportation. However, ocean transportation is quite common in the transportation of relatively heavy goods.

Logistics in a global supply chain arena has currently become an even more crucial business function because of the pandemic. Most goods moving through the supply chain network arrive at their consumption destination through the ocean. Therefore, any disturbance at the sourcing locations and/or at the consumption locations is likely to create the blockage at those ends. Recently, the United States and other countries have experienced prolonged disruption in goods arriving from other countries due to logjams at their seaports due to the lasting effects of the pandemic. In particular, the Western United States ports experienced severe disruption due to the effect of the pandemic which provided significant challenges to organizations in the United States to satisfy consumer demand in a timely manner.

Several social media, magazines, experts' opinions, and other business sources gleaned interesting causes that influence the supply chain disturbance at the Western U. S. ports (Lindsay, 2021; Ng, 2021; Velshi, 2021; Swanson, Smialek, and Tankersley, 2021; Sharan, 2021; Clark, Patt, Gamble, Bisceglie, and Hodgkins, 2021; Boak, 2021; and Young, 2021). These causes include: Limitations in Port Operations, Structural Design of the ports, Implementation of Technology, Status of Current Road Infrastructure, Increased Consumer Demand, Current Fleet and Operators, Effect of Pandemic, and External Issues.

GENERAL DISCUSSION

Ports in the US normally operate from 8 am to 5 pm with an hour break for lunch. The ship congestion at the ports has renewed calls for a 24/7 operation. Given that a lot of port workers are unionized, and the unemployment rate is 4.2%, it may be hard to implement the idea expeditiously. Also, there are federal regulations in effect at the ports pertaining to worker and vessel safety, container organization,

environment protection, and goods transfer protocols that significantly impact operations.

The infrastructure for most US ports is old. For instance, the Los Angeles port, regarded as the top container port in the western hemisphere for the 2000-2020 duration, was constructed in 1907 has only seven containers and twenty-five cargo terminals and, six rail yards. It's equipped with 82 cranes. Unlike ships that pack in containers by stacking them high, the stacking height of containers on land is kept significantly down by regulations. In addition, several old ports are not deeper than 30 feet to allow the anchoring of large shipping vessels. To alleviate shipping congestion some have floated the idea of constructing inland ports. However, the idea has not gained much traction since it is regarded as a long-term venture with dubious chances of success.

There is a surprising lack of state-of-the-art technology at work at the US Ports. Most operations are largely manually driven. The worker pool, while competent, is apparently hesitant to adopt innovative technologies to help expedite operations.

The deteriorated state of US highways and bridges impedes efficient truck flowing traffic. While the US Congress has passed the trillion-plus dollar infrastructure bill, it will be a while before the monies percolate down to the local projects level and even more time before they are successfully updated.

Retailers in the US conduct 50-60% of their yearly business during the holiday season, between Thanksgiving and New Year. To ensure great customer service, major retail chains plan their inventory build-up and sales strategy earlier in the year. Easy credit terms and special layaway plans help boost holiday sales further. The pandemic gutted out face-to-face retail, moving virtually all the traditional retail online. The nearly two-trillion dollars in stimulus spending by the US government sent directly into consumer pockets, also boosted demand for consumer discretionary and technology products. Taken together, this resulted in a perfect storm that overwhelmed retailers, who in turn engaged in chaotic ordering from suppliers. With most goods coming from overseas, the log-jam at US ports was to be expected!

The trucking industry has been mostly run by a small sliver of the US population

with very specific demographics (e.g., race, age, and gender), that has largely remained consistent over the last couple of decades. The pandemic has disrupted it by creating significant demand for additional trucks and truck drivers. The sheer scale of the problem has opened the flood doors to change (manufacturing faster and technology-fitted trucks, workforce composition, a truck-stops redesign to make them appealing to a broader demographic, etc., driver pay, regulations regarding driver's age, gender, etc.)

The impact of the COVID pandemic has been felt worldwide. Millions who have been infected faced flu-like symptoms followed by significant respiratory distress and many lost their lives within 2-3 weeks of initial infection. Worst outcomes were felt by those with already weak immune systems, with children escaping the wrath for the most part. People in the US were clinging close to their families, with a significant percentage of women taking time off work or leaving it altogether to take care of loved ones. Some male members followed the same pattern as well. Since most US schools were closed to in-person instruction, a lot of parents assisted their kids with schoolwork or home-schooled them.

Social media has played a detrimental role in the US throughout the pandemic. Since a lot of people get their news from it (especially Facebook), its ability to impact consumer behavior by its displayed content cannot be underestimated. With its penchant for housing and actively recommending misinformation content, it may have helped perpetuate the pandemic despite the availability of multiple vaccines (e.g., Moderna, Pfizer, J&J). Even in December 2021, almost 40% of the US population is unvaccinated.

CONCLUSION

Supply chain management is unequivocally regarded as the most critical function in any global organization. The world depends on the efficient working of global supply chains. Any disruption in the orderly processing will have a detrimental effect on organizations and consumers. The present study provides a glimpse into the type of issues grappling with most global supply chains, with specific reference to the United States. Significant future research will be needed to develop a

comprehensive understanding of the interrelatedness of various factors and their associated issues influencing the disruption. In addition, cost effective strategies for achieving the pre-pandemic global supply chain status will need to be identified.

ACKNOLWEDGEMENTS

Authors thank the referee for reviewing our paper.

REFERENCES

[1] **Boak, Josh** "L.A. port to operate 24/7 as U.S. aims to ease supply chain bottlenecks," The Associated Press, https://globalnews.ca/news/8262636/port-of-los-angeles-going-24-7-supply-chain-bottlenecks/, October 13, 2021

[2] **Clark, Bryan, Patt, Dan, Gamble, Duane A.,** Bisceglie, Jennifer, and Hodgkins, Dennis,"Preventing the Next Supply Chain Disruption," https://www.youtube.com/watch?v=kYE2QjoiYlw, November 15, 2021.

[3] **Davis, Lindsay,** "ABC News Prime: Supply Chain Nightmare; Deadly Bow and Arrow attack; Conversation with David Grohl," https://theglobalherald.com/news/abc-news-prime-supply-chain-nightmare-deadly-bow-and-arrow-attack-conversation-with-david-grohl/, October 14, 2021

[4] **Ng, Abigail** "Supply chain disruptions may ease in the second half of 2022, insurer says," https://www.cnbc.com/2021/12/09/supply-chain-crunch-may-ease-in-the-second-half-of-2022-euler-hermes.html?&qsearchterm=supply%20chain, December 9, 2021.

[5] **Sharan, Anjali** " Supply Chain Disruption: The Impact on Los Angeles and Singapore," https://www.youtube.com/watch?v=8L99WkN141M, December 2, 2021.

[6] **Swanson, Ana, Smialek, Jeanna, and Tankersley,** Jim, "Biden Announces Measures at Major Ports to Battle Supply Chain Woes," New York Times, https://www.nytimes.com/2021/10/13/us/politics/biden-port-los-angeles-supply-chain.html, October 13, 2021.

[7] **Velshi, Ali** "Oh, Ship!: How the supply chain crisis impacts one's everyday life," MSNBC.comhttps://www.msnbc.com/ali-velshi/watch/oh-ship-how-the-supply-

chain-crisis-impacts-one-s-everyday-life-123783749906, October 17, 2021.

[8] Young, Jessica, "US ecommerce sales increase 6.8% in Q3 2021," Digital Commerce 360, https://www.digitalcommerce360.com/article/quarterly-online-sales, November 18, 2021.

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RESEARCH ARTICLE

THE CONVOLUTIONS OF DEBTS RECOVERY LAWS IN INDIA : ISSUES AND POSSIBLE SOLUTIONS

Hitesh N. Dave[1] and Pradeep Prajapati[2]

ABSTRACT

Health of Banking Sector depends on the health of economy. Banks accept the money and lend in the form of loan to the needy person. If loans are defaulted and turn the Non Performing Assets, the Banks recover the same through the recovery mechanism. Interestingly, there are three different laws for the recovery of the Bad Debts. There is no statutory restriction for the recovery to be made by the Banks and as such it is sole discretion of the Banks to elect the remedy of recovery. In a given situation the Banks may adopt three methods of recovery parallel or it may be avail one after another depending upon the circumstances. This research attempts to highlight the confusion being created by the existent three statutes and it attempts to find the problems faced by the creditors and borrowers both, besides the issue of backlog of cases. It also provides impetuous and discusses the remedial aspects and possible solution. It parses the solution that by harmonizing the three different statutes the problem could be redressed.

KEY WORD

Banks, Debt, Harmonization of Statutes, Non Performing Assets.

RESEARCH METHODOLOGY:

Scientific and descriptive method for analysis is used. Secondary data has been used in the present Research Article.

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1. INTRODUCTION:

Prior to Nationalization of Banks in India, the banking sector was mainly regulated by the private banks. Government of India nationalised 14 major private banks on 19th July, 1969. Thereafter, six more private sector banks were nationalised on 15th July, 1980. Thereafter, there was massive branch expansion of the Public Sector Banks (PSBs); the Private Sector Banks (PVBs) kept on running their shows. However, due to government supports and infusion of capital, PSBs increased their focuses on the agriculture sector and thereby the banking sector grew rapidly in India.

The reformative phase of Indian Banking Sector was begun in the year 1991, as part of the financial sector reforms and with a view to economic liberalisation, Reserve Bank of India (RBI) granted licenses to open new private banks and foreign banks were also allowed the entry in India. There were sea changes in functioning of PSBs but they were still not able to perform well compare to PVBs.

As a matter of policy decision, Government of India (GoI) started reducing share holdings in all PSBs and State Bank of India and it associates, with an object to improve the management of PSBs and thereby the government retained the control of PSBs with it by diluting stake. However, due to the control of GoI in PSBs, yet the Banks continued to face the problem of recovery of money lent to the borrowers in the form of loans.

To strengthen the recovery, it was felt by the government to open the Spate Court in the form of Tribunal for expeditious recovery of money and as such the separate Tribunal exclusively mean for the recovery of Bank's money got birth in the year 1993. There was the first statute in the form of specialised tribunal known as the Recovery of Debts Due to Banks and Financial Institutions Act, 1993 (RDB Act, 1993), (later on it was changed to The Recovery and Debts and Bankruptcy Act, 1993). However, as time limit for finalisation of the cases within 180 days was not adhered by the Debts Recovery Tribunals (DRT) over the passage of time, hence, the government had introduced the another statues for recovery empowering the Banks

and Financial Institutions (FIs) to recover the money without approaching the Courts or Tribunals by promulgating the Securitisation and Reconstruction of Financial Assets and Enforcement of Securities Interest Act, 2002 (SARFAESI Act, 2002). But, in that case also, as the Banks were insufficiently secured with the Secured Assets, for the balance recovery of money approach to be made to DRTs, which was already loggerhead with cases.

Thereafter, in the year 2015, Government formed the Bankruptcy Law Reforms Committee and on the report given by the Committee, another statute was introduced namely the Insolvency and Bankruptcy Code, 2016 (IBC, 2016). The IBC, 2016 was introduced to improve the recovery percentage and to minimise the losses to the creditors under the IBC, 2016 also the Bank and creditors are empowered to initiate the process for the resolution of debt, as the idea of introducing the IBC was not for the recovery but for resolution of sick units as well.

As such, now there is three major recovery laws, under which, the Banks and FIs may go for recovery of their debts and there is no statutory embargo that having availing the remedy under one statute, the creditor may not avail the remedy under another law. As such, the Banks usually, if not in all cases, avails the remedy under either two or three statutes together, resultantly, the Courts or Tribunals flooded with the multiple litigations, it may also happen that against one particular borrower different forums are elected for the recovery of debt.

2. THREE STATUTORY PROVISIONS FOR RECOVERY OF BAD DEBTS IN NUTSHELL:

For recovery of dues of the Banks and FIs, there are different statues as stated below:

(A) The Recovery of Debts and Bankruptcy Act, 1993 (RDB Act, 1993) :

Expeditious adjudication and recovery of debts due to banks and financial institutions was the motto in enacting The Recovery of Debts Due to Banks and Financial Institutions Act, 1993. The Tribunal and the Appellate Tribunal follow

provisions and rules of The RDB Act, 1993 read with The Debt Recovery Tribunal (Procedure) Rules, 1993, The Debts Recovery Appellate Tribunal (Procedure) Rules, 1994 and The Debts Recovery Tribunals (Refund of Court Fee) Rules, 2013. They shall have powers to regulate their own procedure including the places at which they shall have their sittings. Recovery Officers of the Tribunal follow procedure and proceedings as per the Second Schedule of Income Tax Act i.e. ITCP Rules, 1962 for recovery.

Proceedings before the Tribunal are summary in nature and the need for the witnessing the documents was dispensed with. The Tribunal and the Appellate Tribunal, as stated in the Act, for purposes of procedures and powers, shall not be bound by the procedure laid down by The Code of Civil Procedure, 1908 (5 of 1908), but shall be regulated and guided by the principles of natural justice subject to the other provisions of the Act and Rules.

Moreover, the proceedings before the Tribunal shall be deemed to be the judicial proceedings within the meaning of Section 193 and 228 and for the purpose of Section 196 of Indian Penal Code (45 of 1860) and Tribunal and the Appellate Tribunal shall be deemed to be a Civil Court for all the purposes of Section 195 and Chapter XXVI of Code of Criminal Procedure, 1973 (2 of 1974). Apart from it provisions of the Bankers' Books Evidence Act 1891 in respect of proof of entry in the 'banker's books' shall apply to all the proceedings before the Tribunal and Appellate Tribunal.

❖ **Establishment of the DRT:**

The Central Government shall, by notification, establish one or more Tribunals to be known as the Debts Recovery Tribunal, to exercise the jurisdiction, powers and authority conferred on such Tribunal by or under this Act. The Central Government shall, by notification, establish such number of Debts Recovery Tribunals and its benches as it may consider necessary, to exercise the jurisdiction, powers and authority of the Adjudicating Authority conferred on such Tribunal by or under the IBC Code, 2016.

❖ **Pecuniary Jurisdiction of the DRT :**

The pecuniary jurisdiction of the Tribunal, after the amendment carried out on 06/09/2018, is increased from Rs.10.00 Lakhs to Rs.20.00 Lakhs.

(B) The Securitisation and Reconstruction of Financial Assets and Enforcement of Security Interest Act, 2002.

The Recovery of Debts Due to Banks and Financial Institutions Act, 1993 was promulgated in the year 1993 to set up an independent Tribunal for the recovery of Non-Performing Assets or bad debts speedily. After passage of time, it was felt by the Government that the provisions as contained in the RDB Act have not yielded positive result. Therefore, on the recommendations of the Committee appointed earlier – Narashimham Committee - II, promulgated The Securitisation and Reconstruction of Financial Assets and Enforcement of Security Interest Act, 2002 (SARFAESI Act) with effect from December 17, 2002. Under the SARFAESI Act, secured creditors are given wide powers to sell secured hypothecated and mortgaged assets to recovery their dues without any interference of the court or Tribunal and without approaching them. Thus, the SARFAESI Act was enacted with the object that it would enable banks and financial institutions to realise long-term assets, manage problem of liquidity, asset liability mismatches and improve recovery by exercising powers to take possession of securities, sell them and reduce non-performing assets by adopting measures for recovery or reconstruction.

❖ **Scheme of the SARFAESI Act, 2002 :**

SARFAESI Act, 2002, enables the Banks and FIs to take the actions or recovery measures by issuing the Demand Notice of 60 days and in failure to follow, measures for recovery of assets and sale of assets including taking management and appointment of receiver can be exercised. As far as the reasons for enacting the SARFAESI Act, which was referred to as NPA Act, 2002 also, was to regulate securitization and reconstruction of financial assets. With that motto the NPA Act was introduced with further creation of Asset Reconstruction Companies (ARCs) to take over the management of the business of the borrower.

(C) Insolvency and Bankruptcy Code :

With an intention to remove difficulties experienced under the different jurisdictions and courts under the provisions of The Sick Industrial Companies (Special Provisions) Act, The Companies Act, The Presidency Towns Insolvency Act, 1909 and The Provincial Insolvency Act, 1920, as well as with an intent to provide common platform, having noticed leeway in different statutes, the IBC 2016, was introduced.

❖ **Scheme of The IBC, 2016:**

Bankruptcy Law Reform Committee (BLRC) appointed under the Chairmanship of Dr. T.K. Vishwanathan submitted his report to the Ministry of Finance on 04/ 11/ 2015. The objective of the BLRC was to resolve insolvency with lesser time, lesser loss in recovery and higher level of debt financing across instruments. The report of the BLRC was in two parts. The Insolvency and Bankruptcy Code, 2016 (hereinafter to be referred to as the IBC or Code) was passed in Lok Sabha on 05/ 05/2016 and Rajya Sabha on 11/05/2016. The IBC was enacted on 28/05/ 2016. Simultaneously, various required rules and regulations under The IBC 2016 are being brought into force at regular intervals. At the same time, as and when required, Sections of The IBC 2016 as well as Rules and Regulations are being amended.

The Insolvency and Bankruptcy Code, 2016 (Code) introduced in India is a combination of consolidating and bringing all the matters relating to insolvency, liquidation, voluntary liquidation or bankruptcy of companies, limited liability partnerships, partnership firms and individuals under a single umbrella. The Official Gazette was published on 28/05/2016 and on the effective notification of the Code, provisions of Presidency Towns Insolvency Act 1909 and Sick Industrial Companies (Special Provisions) Act 1985 were repealed whereas the amendments to The Indian Partnership Act 1932, The Central Excise Act 1944, The Customs Act 1962, The Finance Act 1994, The Companies Act 2013, The Limited Liability Partnership Act 2008, The Recovery of Debts Due to Banks and Financial Institutions Act 1993 (RDDBFI), The Securitization and Reconstruction of Financial Assets and Enforcement of Security Interest Act, 2002 (SARFAESI Act), The Income Tax Act 1961 and Payment, and

The Settlement System Act 2007 were made effective.

❖ **Purposes of The IBC, 2016:**

The Code aims to consolidate and amend the laws relating to the re-organization and insolvency resolution of various entities in India with time being essence and to promote entrepreneurship, maximization of value of assets of such entities, availability of credit and balance the interest of all stakeholders. It further intended to encourage entrepreneurship and innovation. Entrepreneurs and lenders be able to move on instead of being bogged down for wrong business decision. The Code is a landmark in the Indian history because insolvency and bankruptcy were introduced by consolidating various acts under the single umbrella. The purpose is to achieve insolvency resolution in a time bound manner of 180 days (now 330 days) and extendable by another 90 days under certain circumstances. Bankruptcy regimes vary across countries. France and Italy are debtors friendly whereas U.K., Sweden and Germany are creditors friendly.

(D) Civil Courts including Commercial Courts, SFC Act etc. :

Apart from above three main statutes of the recovery, for the recovery of amount below Rs. 20 Lakhs, the powers are vested with the Civil Courts and Commercial Courts. In addition, one more statute empowers the recovery unlike the SARFAESI Act i.e. under the respective states, State Finance Corporations being empowered to recovery the dues under the State Financial Corporation Act, 1991. The gist of the said allied statutes is as under:

❖ **Commercial Courts and Civil Court :**

The Commercial Courts Act is a by-product of 'Justice delayed Justice denied' system of Indian courts for adjudication of commercial disputes of specified value. The Commercial Courts Act introduced with effect from 23/10/2015. Initially the Commercial Court Act was introduced as "The Commercial Courts, Commercial Division and Commercial Appellate Division High Court Act, 2015" and considering the long title of the Act and after certain modifications and amendments carried out with effect from 03/05/2018, the name of the Act was changed to "The Commercial

Courts Act, 2015". The purposes for bringing the commercial courts are stated in the objects and reasons.

The Civil Procedure Code deals with ordinary civil suits and also the money suits other than the suits of banks and financial institutions and disposal of the suits takes longer time. The Commercial Court Act was aimed to introduce the fast track procedure for resolution of the commercial dispute as defined under the Act. Case Management Hearing System is the unique feature of the Commercial Courts Act, 2015 to expedite the commercial court proceedings. Commercial dispute covers and means a dispute arising out of the list enumerated under Section 2(1)(c)(i to xxii).

Under The Commercial Courts Act the definition of the specified value is also stated for which and for the said subject matter the commercial dispute in a suit, appeal or application be determined and it covers the money sought to be recovered. Specified value is defined under the Act which is shall mean the value of subject matter in respect of subject matter of a suit determined in accordance with Section 12 of which shall not be less than Rs.3.00 Lakhs or such higher value as notified by the Central Government. Interestingly, initially before the amendment the specified value was Rs.1.00 Crore. In a way, it was also argued that the designation of the Commercial Courts lose its partake by lowering the amount from Rs. 1 Crore to Rs. 3 lakhs.

❖ **Civil Procedure Code, 1908 :**

It used to take years to recover dues by banks and financial institutions through civil suits under Civil Procedure Code (CPC). Besides, there were number of suits dealing with the cases other than those filed by banks or financial institutions for recovery. There was no provision under the CPC to accord any special treatment and/or rights to hear and dispose of the civil suits filed by banks and financial institutions. Civil suits normally take decades for decrees considering the backlogs and variety of cases taken up by civil courts under the CPC. Parties to civil suits are required to remain present personally before the civil court as witnesses and subjected to cross- examination and thereafter the process for the witnesses of the

cases used to take several long months and thereby the trial and final disposal was a mirage. Satirically, the concept for introduction of DRT in the year 1993 was lost by the Government and minimum limit of filing of case in specialized Tribunal was enhanced from Rs. 10 lakhs to Rs. 20 lakhs, by relegating nearly 38% cases to the docket of Civil Courts which were already flooded with dearth of other issues.

Apart from it, there are several other legislations which can be exercised by the Banks and FIs are Prevention of Money laundering Act 2002, The Fugitive Economic Offenders Act 2018 etc.

❖ **State Financial Corporation Act, 1951 :**

The State Financial Corporation Act (SFC) was enacted in the year 1951. State Financial Corporations used to play a major role in industrial development by providing finance to set up new units. SFCs had the refinance arrangement with All India Financial Institutions. With a view to recover loans the special powers were conferred to the SFCs when there were no laws like The RDB Act and The SARFAESI Act. Special powers under Sections 29 of the SFC Act for recovery from the defaulting industries are given under the said act. SFCs, exercising special powers under The SFC Act against the defaulting industrial concerns and guarantors, need to file the civil suit before the civil court for enforcement of their residual claim.

3. ISSUES WITH THE DIFFERENT STATUTES OF RECOVERY OF DEBTS:

After discussing above statutes of recovery, les us, discuss the issues being faced by the DRT and NCLT due to different legislation of recovery.

1) Vacancy in the Tribunals:

The issue of vacancies were considered by different High Courts and Supreme Court of India, from time to time, Union of India was severely reprimanded too. In the case of the DRTs, issue of vacancy were noticed several times and until and unless the issues are taken up before the Court of Law, there is lackadaisical approach from the government about timely filling in the post. In the case of NCLT also, when it is still passing from the teething problems, as after competition of five years of

introduction of IBC, NCLTs are working under-staff. In the recent past also Chief Justice of India, Justice V. N. Ramanna (as he then was) has sternly observed that fact of vacancies in the Tribunals went unnoticed by the Government and it is a very sorry state of affairs. In a given case despite the observation of Apex Court, the government went ahead with introduction of Tribunals Reforms Bill, 2021 and it was passed without any debate. The Apex Court struck down the rules, were introduced by the Government on previous two occasions, however the Tribunals Reforms Bill was also challenged now and pending once again before the Court of Law. While a rationalisation of the functioning of Tribunals may be necessary, the solution was handy but the government lacks the interest and as such the Tribunalisaiton which was aimed for the speedy disposal of litigation, but, it could not attained its object. Inadequate specialised and expertise amongst DRT and NCLT Judges besides the problem of massive staff shortage at DRT, NCLT and NCLAT are still the unsolved issue.

2) A Stark Contrast in DRT vis-à-vis NCLT:

Despite, the fact that the three statutes are more or less concerned with the recovery, however, two Tribunals are regulated under different statutes i.e. DRT under the RDB Act as well as SARFAESI Act, whereas the NCLT under the Companies Act. The nature of relief being provided by the two statutes are also different that the former concerned with the recovery, appointment of receiver etc., later concerned with liquidation, insolvency or winding up due to bankruptcy. The DRT provides recovery mechanism for debts strictly for the Banks and FIs whereas the creditors other than banks and FIs are also allowed to take action with the aid of NCLT. NCLT provides remedy in case of default of payment of debt, be it by the operational or financial creditor. Piling of cases because of merger of the provisions of the Companies Act and recovery are being faced by the NCLT.

Table 1: Distribution of NPAs Recovered by Banks, Various Channels

Year	Lok Adalat			DRTs			ARCs			IBC			Total	
	Percentage of total amt involved	Percentage of total amt recovered	Amt recovered as per cent of amt involved	Percentage of total amt involved	Percentage of total amt recovered	Amt recovered as per cent of amt involved	Percentage of total amt involved	Percentage of total amt recovered	Amt recovered as per cent of amt involved	Percentage of total amt involved	Percentage of total amt recovered	Amt recovered as per cent of amt involved	Total amount involved (₹ crore)	Total amount recovered (₹ crore)
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2003-04	5.1	3.3	13.5	53.3	79.1	30.9	41.5	17.6	8.9	-	-	-	47535	9921
2004-05	2.8	2.2	14.1	50.5	51.8	18.8	46.7	46.1	18.1	-	-	-	28342	5192
2005-06	6.5	2.7	20.3	35.9	56.4	76.9	57.6	41.0	34.8	-	-	-	17055	8356
2006-07	4.0	1.4	14.0	48.3	47.3	37.8	47.7	51.2	41.4	-	-	-	18972	7318
2007-08	14.1	2.3	8.2	38.2	39.6	51.9	47.7	58.1	61.0	-	-	-	15224	7625
2008-09	19.9	1.3	2.4	20.4	45.1	81.1	59.7	53.6	33.0	-	-	-	20220	7426
2009-10	23.1	1.5	1.5	31.3	41.7	32.0	45.6	56.8	30.0	-	-	-	31281	7514
2010-11	10.5	1.0	2.9	28.2	25.1	27.9	61.3	73.9	37.8	-	-	-	49950	15642
2011-12	2.8	1.4	11.8	39.4	28.5	17.0	57.8	70.1	28.6	-	-	-	61100	14400
2012-13	6.2	1.7	6.1	29.3	18.9	14.2	64.4	79.4	27.2	-	-	-	105700	23300
2013-14	13.3	4.4	6.0	31.8	16.6	9.6	54.8	79.1	26.5				173800	32000
2014-15	12.5	3.2	3.2	24.3	13.6	7.0	63.2	83.1	16.3	-	-	-	248200	30800
2015-16	32.5	14.0	4.4	31.3	28.1	9.2	36.2	57.9	16.5	-	-	-	221400	22800
2016-17	13.0	6.0	6.4	36.2	26.8	10.2	50.8	67.3	18.3	-	-	-	278300	38500
2017-18	16.9	4.5	4.0	49.2	17.9	5.4	30.3	65.4	32.2	3.7	12.2	49.6	270631	40352
2018-19	7.4	2.3	5.1	37.0	8.9	3.9	35.6	32.8	15.0	20.4	56.2	45.7	725996	118647
2019-20	9.1	2.4	6.2	33.1	5.8	4.1	26.5	30.5	26.7	31.3	61.3	45.5	742431	172565

* Amount recovered refers to amount recovered during a given year, which could be with reference to cases referred during the given year as well as earlier years.
Source: Report on Trend and Progress of Banking in India.

[1] Lok Adalat were developed as part of the Legal Services Authorities Act, 1987 for ensuring a civil settlement between banks and non-performing borrowers. Lok Adalat have been useful for settlement of dues in respect of smaller loans. In 2019-20, 59,86,790 cases involving ₹67,801 crore were pending before Lok Adalat, reflecting the small-size orientation of these institutions.

3) IBC vis-à-vis SARFAESI Act:

The IBC aid for the recovery and also for resolution, under the SARFAESI Act it is for the recovery of money. Under the SARFAESI Act and IBC powers of recovery can be exercised, however, the preferred mode of recovery is SARFAESI Act. It was also argued that Section 14 of IBC puts embargo to the secured creditor to take action under any other law due to the moratorium period. SARFAESI Act protects financial creditors who are mostly Banks and FIs whereas the IBC guarantees the interest of all creditors without any distinction. Lenders preferred to take action under SARFAESI Act as it was strengthen by amending the Act from time to time and recovery is faster than the IBC.

Table-2: Pendency of cases under IBC and its Tenure for Resolution

(Number)

Year / Quarter	CIRPs at beginning of the period	Admitted	Closure by				CIRPs at the end of the Period
			Appeal/ Review/ Settled	Withdrawal under Section 12A	Approval of Resolution Plan	Commencement of Liquidation	
2016-17	0	37	1	0	0	0	36
2017-18	36	706	94	0	20	91	537
2018-19	537	1,156	149	97	79	305	1,063
Apr-Jun, 2019	1,063	301	53	32	26	96	1,157
Jul-Sep, 2019	1,157	596	57	51	34	156	1,455
Oct-Dec, 2019	1,455	637	114	60	42	153	1,723
Jan-Mar, 2020	1,723	444	95	58	39	137	1,838
Apr-Jun, 2020	1,838	84	13	27	20	26	1,836
Jul-Sep, 2020	1,836	96	25	35	35	81	1,756
Oct-Dec, 2020	1,756	107	8	30	24	83	1,718
Jan-Mar, 2021	1,718	212	8	21	29	149	1,723
Total	**NA**	**4376**	**617**	**411**	**348**	**1,277**	**1,723**

Note: 1) These CIRPs are in respect of 4289 CDs.
2) This excludes 1 CD which has moved directly from BIFR to resolution.
Source: Compilation from website of the NCLT and filing by Insolvency Professionals.

4) Legislative intent of three different statues:

Legislative intents of three recovery laws are distinct and separate. However, one underlying object is for the recovery of money. That, we the objects and reasons are studied carefully, it would answer that right away from 1993 the first statute, the Government's wanted to recover the bad debts speedily, so the introduction of the special Tribunal, however, as it was not did well, hence, in the year 2002, the SARFAESI Act, thereafter, rationalisation of few statutes, the addition of power of liquidation and recovery is given under IBC. The IBC in a way enjoys the supremacy of two previous statues, reason being it has come at later point of time and it can be said that it has overriding effect and as such able to suspend the provisions of DRT and SARFAESI Act. The existence of several companies – related legislations with respect to recovery of debt mandates interpretation. A delay in insolvency or liquidation proceedings due to overlapping proceedings is worrisome and it remained unresolved. Permitting the creditors to take parallel or simultaneous proceedings for recovery of debt is contributing two and in sufficient insolvency regime as the recovery proceedings can be resolved by DRTs as well compare to NCLTs.

Table - 3: Recovery of Amount from different statures

(Amount in ₹ crore)

Recovery Channel	2018-19				2019-20			
	No. of cases referred	Amount involved	Amount recovered*	Col. (4) as per cent of Col. (3)	No. of cases referred	Amount involved	Amount recovered*	Col. (8) as per cent of Col. (7)
1	2	3	4	5	6	7	8	9
Lok Adalats	40,87,555	53,484	2,750	5.1	59,86,790	67,801	4,211	6.2
DRTs	51,679	2,68,413	10,552	3.9	40,818	2,45,570	10,018	4.1
SARFAESI Act	2,35,437	2,58,642	38,905	15.0	1,05,523	1,96,582	52,563	26.7
IBC	1,152@	1,45,457	66,440	45.7	1,953@	2,32,478	1,05,773	45.5
Total	**43,75,823**	**7,25,996**	**1,18,647**	**16.3**	**61,35,084**	**7,42,431**	**1,72,565**	**23.2**

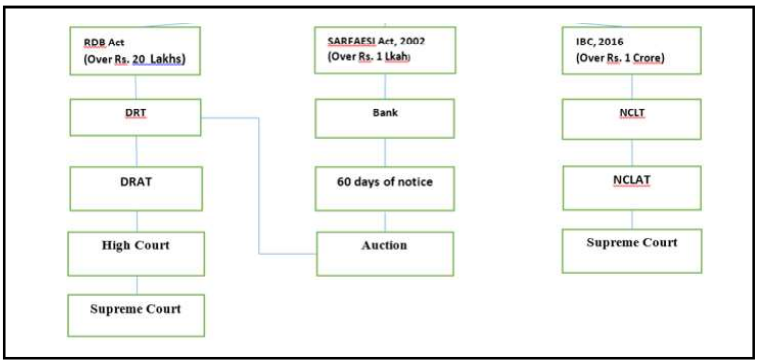
Notes: 1. Data are provisional.
2. DRTs: Debt Recovery Tribunals
3. *: Refers to the amount recovered during the given year, which could be with reference to the cases referred during the given year as well as during the earlier years. In the case of IBC, the realisation does not include amount realisable for operational creditors, from guarantors of corporate debtors and disposal of avoidance transactions.
4. @: Cases admitted by National Company Law Tribunals (NCLTs) under IBC. However, figures appearing for amount involved and amount recovered are for cases whose resolution plan was approved during the given financial year *i.e.* 81 cases for 2018-19 and 135 cases in 2019-20. Also, the amount recovered refers to realisables by all financial creditors, not just SCBs.
5. The resolution plan of Essar Steel India Ltd. was approved in 2018-19. However, as apportionment among creditors was settled in 2019-20, the recovery is reflected in the latter year data.

Source: Off-site returns, RBI and Insolvency and Bankruptcy Board of India (IBBI).

5) Massive haircut and role of Resolution Professional faulted with:

The issue of massive haircut is a point of worry for the functioning of the IBC apart from the issue of delay in the closure of the cases. Low recovery were observed between the FY 2017 and FY 2019 as it was put average around 43-50 percent that too by extending the time period. Out of 277 cases resolved by NCLT as of September, 2020, the average time for resolution was 440 days. IBC has come in for a lot of dissension and its success ratio is yet to be compared but it failed in its core purpose about the speedy disposal and in improving the recovery percentage. The creditors use forum shopping to harass the borrowers and thereby increasing the backlog of cases. Apart from it, frequent changes in the minimum threshold limit also creates the clog in functioning of the IBC, due to enhancement of amount of default from Rs. 1 Lakh to Rs. 1 Crore by amendment in 2020. So, the hierarchy and mechanism of the Tribunals can be very well adjudged from the flow chart below.

Flow Chart-1: Flow Chart Showing Hierarchy of Tribunals (DRT & NCLT) :



4. CONCLUSION:

Existing regulatory regimes for the recovery of debts poses serious issues with respect to backlog of cases, shortage of staff and delay in recovery of bank's dues. Apart from it, due to availability of different statutes, the creditors uses the different modes of recovery and in a given case elect remedy one after another. Harmonisation of the recovery laws, would certainly reduce the backlog of cases, forum shopping want be there and the creditors would recovery their money speedily. Yet apart, the different limits of recovery also a cause of concern, enhancement of minimum limit of DRT from Rs. 10 lakhs to Rs. 20 lakhs, created additional burden on Civil Courts and the Bank's dues within the said bracket would take long years in recovery, parallel, the default limit under IBC was enhanced from Rs. 1 lakh to Rs. 1 Crore, thereby the default amount below Rs. 1 Crore would be dealt with by the DRT or Civil Court and thereby leaving the creditors at lurch for recovery. Moreover, vesting additional Appellate Jurisdiction in the the case of DRT compare to NCLT also creates discretion. The minimum amount of default required to be rationalised and there should be one jurisdiction amongst all, so as to enable the banks to recovery optimum with no loss of time.

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6. SELECTED REFERENCES:

[1]. **Anil Kumar Basal (2020)**; Management of Legal Risk in Banks, 3 (6) The Indian Banker 45, 45-49 (2020).

[2]. **Arijeet Ghosh et al. (2018);** *Reforming the Tribunals Framework In India : An Interim Report,* Vidhi Centre for Legal Policy, 2018.

[3]. **Ashish Pandey (2019);** High Bids and Low Recovery : A Possible Case for Non-Performing Loan Auctions in India, 31(3) IIMB Management Review 250, 250-257 (2019).

[4]. BANKING AND FINANCIAL INSTITUTIONS LAWS, PROFESSIONAL BOOK PUBLISHERS (2015).

[5]. **Bawa Jaslene Kaur et al (2019);** An analysis of NPAs of Indian banks : Using a comprehensive framework of 31 financial ratios, 31 (1) IIMB Management Review 51, 51-62 (2019).

[6]. **Bhandari Meenu (2019);** A Study of Non-Performing Assets (NPAs) of Public and private Sector Banks – Comparative Analysis, 9 (6) International Journal for Basic and Applied Research 958, 958-964 (2019).

[7]. COMPTROLLER AND AUDITOR GENERAL OF INDIA, EXECUTIVE SUMMARY, REPORT No. 28/2017 (2017).

[8]. **Datta Pratik et al. (2017);** *Understanding Judicial Delay at the Income Tax Appellate Tribunal in India*, W.P. No. 208, National Institute of Public Finance and Policy (2017).

[9]. **Dave Hitesh and Rathore (2020);** Regulating Collateral Management in Public Sector Banks: Need of the Hour Vidhya Publication, Vidya (2020), A Journal of Gujarat University, Vol. 15(1), 13-26

[10]. **Dave Hitesh et. al (2020);** Debts Recovery Laws Guide for Bakers, Legal Practitioners and Businessmen, 2020.

[11]. **Dave Hitesh (2018);** Elimination of Debt Recovery Appellate Tribunals, 7 (1) The Indian Banker 48, 48-55 (2018).

[12]. Dave Hitesh (2019); Service of Sale Notice Under SARFAESI: Dilemma Over Precedent, 18 Debts Recovery Tribunal Cases A.5, A.5-A.8 (2019).

[13]. Durgaprasad M. V. (2016); Commentary on The Commercial Courts Act, 2015, Asia Law House (3rd ed. 2016).

[14]. European Banking Authority, Guidelines on Management of Non Performing and Forborne Exposures Final Report, EBA/GL/2018/06, (Oct. 31 2018).

[15]. Insolvency and Bankruptcy Code as amended by The Insolvency and Bankruptcy Code (Amendment) Act, 2018 alongwith NCLT Rules (Bare Act), Bharat Law House Pvt. Ltd., (2nd ed. 2018).

[16]. Kohli R. C., Practical Guide To NPA Resolution, Taxmann (4th ed. 2017).

[17]. Rajan Dr. Raghuram G. (2016); *Interesting, Profitable, and Challenging : Banking in India Today,* Speech Delivered at FICCI – IBA Annual Banking Conference, Aug. 16, 2016.

[18]. Rajan Dr. Raghuram G. (2016); *Resolving Stress in Banking System,* Speech Delivered at ASSOCHAM – Interactive Meet with Industry & Trade at Bengaluru, Jun. 22, 2016.

[19]. Rajan, Raghuram G. (2018); Why Bank Credit Policies Fluctuate : A Theory and Some Evidence, 109 (2) The Quarterly Journal of Economics 339, (May, 1994), pp. 399-441, Oxford University Press, http: www.jstor.org/stable/2118468 accessed on Jan. 16, 2018.

[20]. RBI Discussion Paper, *Early Recognition of Financial Distress, Prompt Steps for Resolution and Fair Recovery for Lenders : Framework for Revitalising Distressed Assets in Economy,* RBI

[21]. Regy Prasanth V & Roy Shubho (2017); *Understanding Judicial Delays in Debt Tribunals,* WPS No. 195 National Institute of Public Finance and Policy (2017).

[22]. Reinout De Bock & Alexander Demyanets, *Bank Asset Quality in Emerging Markets: Determinants and Spillovers*, IMF Working Paper, WP/12/71, International Monetary Fund.

[23]. Repko Maria (2019); *Banking Fragility rooted in justice failures Evidence from Ukraine,* W.P. No. 2019-08 CEPS Paper in Liberty and Security in Europe (2019).

[24]. Shaktikant Das (2019); *Indian Banking at Crossroads: Some Reflections,* Speech Delivered at the First Economics Conference, Nov. 16, 2019.

[25]. Srivastava's Recovery of Debts & Bankruptcy Act, 1993, Law Publishers (India) Pvt. Ltd. (1st ed. 2020).

[26]. The Code of Civil Procedure, 1908 (Bare Act) Commercial Law Publishers (India) Pvt. Ltd. (2018).

[27]. The Commercial Courts Act, 2015 (Bare Act), Commercial Law Publishers (India) Pvt. Ltd. (2018).

[28]. The Companies Act, 2013 with Rules and Forms, LexisNexis (4th ed. 2016).

[29]. The Recovery of Debts and Bankruptcy Act, 1993 (Bare Act), Universal Law Publishing (2019).

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RESEARCH ARTICLE

GLOBAL ECONOMIC CRISIS AND INDIAN ECONOMY

Himanshu Desai[1] and Manish B. Thaker[2]

ABSTRACT

World has witness global economic crisis in the recent years in terms of recession on the financial front and increasing rate of unemployment at the individual firm level.

The purpose of this paper is to focus on two issues [1]. Global economic crisis and their impact on key economic indicator for the period Q_1 2006 to Q_3 2008.

[2].The impact of global crisis on the Indian economy with respect to the similar variables and the additional variables will be added to have the overall macro economic impact.

KEYWORDS

GDP, CPI, MACRO ECONOMIC, INDICATORS, GLOBAL RECESSION

* * *

Section I — Word Economy

The global crisis may be due to wrong expectations of the market share and/ or wrong expectations of the consumer demand by the producer in the product market.

The oil price related inflation was highest till May 2008 but then it has shown downward trend. The important aspect is to examine crisis on key macro-economic indicators.

The indicators that we have included are

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1. Real GDP
2. Unemployment rate
3. Industrial production index
4. Consumer Price Index
5. Broad money growth
6. Short term interest rate.

The economies are divided in three parts:

1. European Union, USA, and Japan
2. Asia
3. Latin America and others

I. First Group

Any kind of global economic crisis must affect at two levels: Real GDP and Unemployment. At the macro economic level there will be the slow down of the real GDP and on the other hand the unemployment accelerates.

The impact of global crisis is almost consistent for first group of economies. Here year to year growth of real GDP shows decline in some cases. The base year growth rate itself was very low and then it continued to remain low. These economies are Germany, France, Italy, Euro zone, UK and Canada.

Unemployment rate reduced only in Denmark but in other economies it has moderately declined.

The index of industrial production has declined steadily in these economies followed by a moderate rise in the prices. In the Q3 2008, in USA the production index was countries in the first group are

1. Germany	2.France	3. Italy	4.Denmark	5. Sweden
6.EuroZone	7. UK	8.Canada	9. US	10. Japan

In the second group are

1. India	2. China	3. HongKong	4.Indonesia	5.Malaysia
6.Philippines	7. S.Korea	8.Singapore	9.Thailand	

In the third group

1. Argentina	2. Brazil	3. Chile	4. Mexico	5. Czech Repub
6. Russia	7. Hungary	8. Poland		

Unemployment data are available on yearly basis for India and China and no data for Indonesia. The decline in unemployment in both the economy is not very sharp. In other economies it has remained fairly consistent. This reflects that the impact of global economic crisis is relatively less serious on the Asian economies.

The industrial production index on the other hand showed much better situation as compared to the other group of economies. In India, it declined remarkably but in China, it remained almost steady. In Philippines it has increased Singapore showed very wide fluctuations. Hong Kong remains at the lowest level with negative rate of industrial production.

The money growth and CPI data reveals that the money growth is fairly high in India accompanied by other economies like China, Hong Kong, and Singapore. -2.03 and price shows rise by 5.03% which is the highest among the group.

The growth of money supply continued to remain high in the reported economies with fairly low short term interest rates.

This reflects that even with the high growth of money supply it failed to enhance the real GDP and due to that the industrial production remains at the low level associated with high rate of unemployment. This reflects the intensity of the global crisis in the developed economies.

II. Second Group

In the second group, India and China has shown considerable consistency in their performance with reference to real GDP. Indonesia showed slow rise, only Singapore showed quick decline in the last two quarter (i.e. Q2 and R3 2008) Hong Kong showed decline only in the last quarter Q3 2008.

As compared to money supply, the CPI shows only moderate rise in most of the Asian countries. The short term interest rare also remains moderate.

III. The Third Group

The third group includes Latin America and other countries.

In the case or real GDP shows very moderate change. Brazil and Russia show

rise in real GDP. The performance of Hungary was poor but other economies showed fair degree of consistency.

The unemployment also showed declined for many economies Poland had a high unemployment in Q1 2006 was 17.93 declined to 9.3 in Q3 2008.

The industrial production index showed considerable degree of fluctuations in the case of Chile, Mexico, Hungary and Poland.

The broad money growth is a unique behaviour. Here it has remained fairly high and consistent. The growth was 30% in Russia and 15 to 20% in Argentina in Q2 1986 it was 82.14%.

The CPI shows comparative consistency across the economies or a moderate rise in the case of Hungary and Poland.

This does not show any significant causality between money growth and CPI. In Russia only CPI remained relatively high.

Empirical findings

The impact of globalisation can be examined by considering the real GDP of 19 economies which includes developed economics and developing economies only. The performance of the economies is examined by considering the data 2006 quarter 1 to 2008 quarter 3 (11 quarters). Here we focus on the highest averages and the lowest variations and on the other side the highest fluctuating economies can also be analysed if the economy possesses relatively high mean and low fluctuations (C.V) then such economies are comparatively stable economies

The lowest value of the mean real GDP for Italy is 1.045455 and the c.v. for the same economy is 113.2874%.

On the other hand China and India show relatively high real GDP (China :10.81818, India : 8.690909) the c.v. for both the economies are 7.786886% for China and 6.141691% for India this emphasizes that India shows relatively better stability (consistency) in terms of real GDP growth because the c.v. value is lowest among all 19 economies covered.

CPI reflects a market stability, in this case most of the economies show relatively low mean, Japan shows 0.595455 as an average CPI but the fluctuations that is c.v. is highest among all the economies. The Indonesia shows relatively high mean and c. v. is comparatively moderate (42.58795%) in this case India shows relatively high price i.e. 6.568182 but the fluctuations are relatively low.

Section - II Impact on Indian Economy

In this section we try to explain the impact of globalization on Indian economy in terms of only selected parameter like GDP. The GDP is the most important parameter to show the incidence of globalization

The analysis is covered in two parts (i) impact on overall growth on GDP with reference to different time period and (ii) the impact on the share on the GDP in the different time periods

Table 1: Real GDP, India.

		Real GDP		
		period 1	period 2	period 3
		1950-51 to 2007-08	1950-51 to 1999-2000	1999-2000 to 2007-08
Linear	constant	-229635.4 (-2.86)	-10775.6 (-0.243)	-7030000 (-9.423)
	β	38315.69 (16.6)	26865.67 (17.72)	167011.45 (12.55)
	R^2	0.829 (275.6)	0.867 (314.12)	0.957 (157.49)
Exponential	constant	192939.54 (47.54)	201337.62 (54.311)	42953.6 (4.844)
	β	0.043 (70.558)	0.04 (64.448)	0.071 (19.256)
	R^2	0.989 (4.978E3)	0.989 (4153)	0.981 (371.15)

Source: Economic Survey 2008-09

The real GDP has exponential growth for all three periods the growth was 4.3% for the whole period 1950-51 to 2007-08. The growth of real GDP was 4% for the

sub period 1950-51 to 1999-2000, which rises substantial to 7.1% during the period 1999-2000 to 2007-08. This reflects that there is no negative impact of global recession on Indian economy (Table 1).

The next aspect is to examine the impact of global recession on sectoral share of GDP.

Table 2: Sectoral share of GDP-Agriculture

		Agriculture		
		period 1	period 2	period 3
		1950-51 to 2007-08	1950-51 to 1999-2000	1999-2000 to 2007-08
Linear	Constant	58.378 (174.98)	57.918 (168.39)	76.051 (24.389)
	β	-0.618 (-3.876)	-0.594 (-50.563)	-0.955 (-16.862)
	R^2	0.986 (4080)	0.982 (2557)	0.976 (284.31)
Exponential	Constant	62.604 (65.323)	60.178 (102.883)	230.002 (7.552)
	β	-0.016 (-36.93)	-0.014 (-42.83)	-0.041 (-17.209)
	R^2	0.960 (1364)	0.975 (1834)	0.977 (296.148)

The sectoral share of the agriculture- GDP declines as per expectation. In the process of economic growth usually the share of agriculture declines very fast, and on the other hand the share of other sector rises. Here the results are almost similar. The sectoral share declines at a faster rate in the period 3 (1999-2008) (table 2).

Table 3: Sectoral share of GDP-Manufacturing

		Manufacturing		
		period 1	period 2	period 3
		1950-51 to 2007-08	1950-51 to 1999-2000	1999-2000 to 2007-08
Linear	constant	15.88 (43.664)	15.239 (41.57)	11.943 (5.560)
	β	0.164 (15.648)	0.198 (15.85)	0.211 (5.496)
	R^2	0.811 (244.87)	0.84 (251.165)	0.812 (30.208)
Exponential	constant	15.903 (46.1)	15.321 (45.04)	14.449 (10.974)
	β	0.009 (13,61)	0.01 (13.852)	0.009 (5.45)
	R^2	0.765 (185.211)	0.8 (191.88)	0.809 (29.7)

The share of manufacturing sector shows better results in the second sub period (1999-2008). For the entire period (1951-2008) it shows linear growth (16.4%). In the second sub period (1999-2008) it was little higher than first sub period (1951-2000). In the second sub period there is a shift from linear to exponential growth (Table 3).

Table 4: Sectoral share of GDP-Trade

		Trade		
		period 1	period 2	period 3
		1950-51 to 2007 - 08	1950-51 to 1999 - 2000	1999-2000 to 2007-08
Linear	constant	10.222 (37.036)	10.958 (82.925)	-19.884 (-16.372)
	β	0.237 (29.868)	0.199 (44.136)	0.799 (36.864)
	R^2	0.940 (892.1)	0.976 (1948)	0.995 (1359)
Exponential	constant	11.203 (93.722)	11.404 (103.6)	4.043 (15.504)
	β	0.014 (44.315)	0.013 (38.542)	0.032 (28.302)
	R^2	0.972 (1964)	0.969 (1486)	0.991 (801.03)

From table 4 it is observed that the sectoral share of trade expands more rapidly in the second sub period (1999-2008) as compared to the earlier periods.

Table 5: Sectoral share of GDP-Finance

		Finance		
		period 1	period 2	period 3
		1950-51 to 2007-08	1950-51 to 1999 - 2000	1999-2000 to 2007-08
Linear	constant	5.335 (17.541)	5.776 (17.673)	3.616 (2.084)
	β	0.132 (15.107)	0.109 (9.75)	0.179 (5.774)
	R^2	0.8 (228.208)	0.664 (95.01)	0.826 (33.337)
Exponential	constant	6.01 (32.651)	6.21 (28.91)	6.57 (8.1)
	β	0.013 (15.211)	0.012 (9.869)	0.013 (5.893)
	R^2	0.802 (231.378)	0.67 (97.405)	0.832 (34.72)

From table 5 it is observed that the sectoral share of finance remains almost stable in the entire period and there is a marginal rise in the second sub period (1999-2008) as compared to the earlier periods.

Table 6: Sectoral share of GDP-Public administration

		Public Administration		
		period 1	period 2	period 3
		1950-51 to 2007-08	1950-51 to 1999-2000	1999-2000 to 2007-08
Linear	constant	10.323 (69.248)	10.108 (77.358)	27.319 (19.522)
	β	0.077 (17.865)	0.088 (19.623)	-0.233 (-9.33)
	R^2	0.848 (319.155)	0.889 (385.1)	0.926 (87.11)
Exponential	constant	10.409 (81.34)	10.208 (90.391)	35.912 (9.628)
	β	0.006 (17.535)	0.007 (19.074)	-0.016 (-8.903)
	R^2	0.844 (307.492)	0.883 (363.81)	0.919 (79.26)

The sectoral share of public administration shows little funny results. Here the share of it declines in the second sub period (1999-2008) when the growth of economy is fastest. Actually here the role should go up (table 6)

Conclusion Remarks

The global recession across the developed and developing countries was much severe. It was believed that India being the largest economy the impact of the recession will be much penetrating. The real GDP growth of India when compared with other economies in section I of this paper suggest that India is least affected by the global recession. In fact India and China showed much better performance as compared to many developed economy.

The impact of global recession on the specific Indian economy is also much less severe. The impact of recession is examined by taking the key indicator like real GDP and it is found that especially in the 1999-2000 onwards the growth of the economy was much better rather than poor. The sectoral share of GDP during the entire period or sub period is not adversely affected.

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Appendix: Tables

Table 1

Real GDP									
year/quarter		Germany	France	Italy	Denmark	Swden	Eurozone	U.K.	Canada
2006	Q1	1.4	1.5	1.5	3.5	4.1	2	2.2	3.2
	Q2	0.6	1.3	0.1	2.8	5.1	1.1	2.6	2.9
	Q3	2.8	1.9	1.7	3.2	4.4	2.7	2.7	2.5
	Q4	3.7	2.5	2.8	2.9	4.7	3.3	3	2.3
2007	Q1	3.6	2.1	2.3	2.3	3	3	2.8	2
	Q2	2.5	1.3	1.8	0.6	3.5	2.5	3	2.5
	Q3	2.5	2.1	1.9	1.8	2.5	2.7	3.2	2.9
	Q4	1.8	2.1	0.1	1.9	2.8	2.2	2.8	2.9
2008	Q1	2.6	2	0.3	0.7	2.2	2.1	2.5	1.7
	Q2	1.7	1.1	-0.1	0.9	0.7	1.4	1.5	0.7
	Q3	0.8	0.6	-0.9	-1.2	0	0.7	0.3	0.5
		Germany	France	Italy	Denmark	Swden	Eurozone	U.K.	Canada
	Mean	2.181818	1.681818	1.045455	1.763636	3	2.154545	2.418182	2.190909
	SD	1.023541	0.563592	1.184368	1.405897	1.61059	0.810387	0.841211	0.899394
	C. V.	46.9123	33.5109	113.2874	79.71579	53.68633	37.6129	34.78693	41.05117

Real GDP							
year/quarter		U.S.	Japan	India	China	Hongcong	Indonesia
2006	Q1	3.5	3.1	9.3	10.2	8.2	4.6
	Q2	3.6	1.4	8.9	11.3	5.2	5.5
	Q3	3	1.6	9.2	10.4	6.8	5.5
	Q4	3.1	2.3	8.6	10.7	7	6.1
2007	Q1	1.9	2.6	9.1	11.1	5.6	6
	Q2	1.9	1.6	8.9	11.9	6.6	6.3
	Q3	2.8	1.9	8.9	11.5	6.2	6.5
	Q4	2.5	2	8.4	11.2	6.7	6.3
2008	Q1	2.5	1.3	8.8	11.6	6.8	6.3
	Q2	2.1	0.7	7.9	10.1	4.2	6.4
	Q3	0.8	-0.1	7.6	9	1.7	6.1
		U.S.	Japan	India	China	HongKong	Indonesia
	Mean	2.518182	1.672727	8.690909091	10.81818	5.909091	5.963636
	SD	0.81709	0.881012	0.533768762	0.842399	1.744967	0.560844
	C. V.	32.44762	52.66918	6.141690772	7.786883	29.53021	9.404388

Real GDP						
year/quarter		Malayasia	Philippines	S.Korea	Singapore	Thailand
2006	Q1	5.3	5.5	6.2	10.6	6
	Q2	5.9	5.5	5.3	8.1	4.9
	Q3	5.9	4.8	4.6	7.2	4.7
	Q4	5.7	6.5	4	6.6	4.2
2007	Q1	5.3	6.9	4	6.1	4.3
	Q2	5.7	7.5	4.9	8.6	4.4
	Q3	6.7	6.6	5.2	8.6	4.9
	Q4	7.3	7.3	5.5	5.4	5.7
2008	Q1	7.1	5.1	5.7	7.2	6
	Q2	6.3	4.6	4.8	1.9	5.3
	Q3	4.7	4.6	3.9	-0.5	4
		Malayasia	Philippines	S.Korea	Singapore	Thailand
	Mean	5.990909	5.9	4.918182	6.345455	4.945455
	SD	0.796812	1.091788	0.752088	3.169342	0.717445
	C. V.	13.30035	18.50487	15.29199	49.94665	14.50716

Consumer price Index									
year/quarter		Germany	France	Italy	Denmark	Swden	Eurozone	U.K.	Canada
2006	Q1	2.03	1.8	2.13	2.03	0.77	2.3	1.9	2.4
	Q2	1.95	1.8	2.2	2	1.5	2.45	2.25	2.45
	Q3	1.53	2	2.3	2.13	1.3	2.13	2.47	1.73
	Q4	1.33	1.33	1.83	1.67	1.53	1.8	2.7	1.3
2007	Q1	1.7	1.13	1.73	1.9	1.93	1.87	2.87	1.83
	Q2	1.87	1.2	1.6	1.63	1.83	1.9	2.57	2.2
	Q3	2.1	1.27	1.6	1.17	1.97	1.9	1.83	2.13
	Q4	2.77	2.5	2.37	2.17	3.17	2.93	2.1	2.43
2008	Q1	2.87	2.93	3.03	3.03	3.23	3.33	2.4	1.8
	Q2	2.9	3.3	3.57	3.47	3.9	3.67	3.37	2.33
	Q3	3.1	3.27	4	4.17	4.37	3.8	4.77	2.43
		Germany	France	Italy	Denmark	Swden	Eurozone	U.K.	Canada
	Mean	2.195455	2.048182	2.396364	2.306364	2.318182	2.552727	2.657273	2.093636
	SD	0.611529	0.827705	0.806428	0.888237	1.162048	0.754216	0.828361	0.37861
	C. V.	27.85431	40.41172	33.65214	38.51246	50.12758	29.5455	31.17335	18.08384

Table 2

Consumer price Index							
year/quarter		U.S.	Japan	India	China	Hongcong	Indonesia
2006	Q1	3.67	0.43	4.8	1.2	2	16.87
	Q2	3.9	0.7	6.33	1.37	2.07	15.5
	Q3	3.33	0.6	5.63	1.07	2.3	14.87
	Q4	1.93	0.33	6.83	2.03	2.17	6.07
2007	Q1	2.43	-0.15	7	2.73	1.73	6.37
	Q2	2.67	-0.2	6.33	3.6	1.27	6.03
	Q3	2.4	-0.2	6.73	6.1	1.57	6.53
	Q4	3.97	0.53	5.5	6.63	3.47	6.73
2008	Q1	4.1	0.97	6.3	8.03	4.6	7.67
	Q2	4.37	1.37	7.77	7.77	5.73	10.13
	Q3	5.3	2.17	9.03	5.27	4.63	11.57
		U.S.	Japan	India	China	Hongcong	Indonesia
	Mean	3.460909	0.595455	6.568181818	4.163636	2.867273	9.849091
	SD	1.014273	0.717891	1.145389176	2.685045	1.496376	4.194526
	C. V.	29.30654	120.5618	17.43845112	64.48797	52.18814	42.58795

Consumer price Index						
year/quarter		Malayasia	Philippines	S.Korea	Singapore	Thiland
2006	Q1	3.73	7.3	2.37	1.4	5.73
	Q2	4.25	6.9	2.33	1.2	6.03
	Q3	3.57	6.57	2.53	0.73	3.63
	Q4	3.07	4.8	2.13	0.57	3.27
2007	Q1	2.6	2.9	2.03	0.53	2.43
	Q2	1.43	2.33	2.43	0.97	1.87
	Q3	1.77	2.57	2.27	2.73	1.63
	Q4	2.2	3.27	2.37	4.07	2.9
2008	Q1	2.6	5.57	3.8	6.6	5
	Q2	4.83	9.77	3.83	7.5	7.57
	Q3	8.4	12.2	5.53	6.53	7.2
		Malayasia	Philippines	S.Korea	Singapore	Thiland
	Mean	3.495455	5.834545	2.874545	2.984545	4.296364
	SD	1.927996	3.147988	1.075912	2.719663	2.112606
	C. V.	55.15722	53.95429	37.42895	91.12487	49.17196

Economic Indicators in the EU, USA and Japan

Real Gross Domestic Product

y-o-y percentage change

Year/ Quarter		Ger-many	France	Italy	Den-mark	Swe-den	Euro-Zone	UK	Canada	US	Japan
2006	Q1	1.40	1.50	1.50	3.50	4.10	2.00	2.20	3.20	3.50	3.10
	Q2	0.60	1.30	0.10	2.80	5.10	1.10	2.60	2.90	3.60	1.40
	Q3	2.80	1.90	1.70	3.20	4.40	$_p$2.70	2.70	2.50	3.00	1.60
	Q4	3.70	2.50	2.80	2.90	4.70	3.30	3.00	2.30	3.10	2.30
2007	Q1	3.60	2.10	2.30	2.30	3.00	3.00	2.80	2.00	1.90	2.60
	Q2	2.50	1.30	1.80	0.60	3.50	2.50	3.00	2.50	1.90	1.60
	Q3	2.50	2.10	1.90	1.80	2.50	2.70	3.20	2.90	2.80	1.90
	Q4	1.80	2.10	0.10	1.90	2.80	2.20	2.80	2.90	2.50	2.00
2008	Q`	2.60	2.00	0.30	0.70	2.20	2.10	2.50	1.70	2.50	0.70
	Q2	1.70	1.10	-0.10	0.90	0.70	1.40	1.50	0.70	2.10	0.70
	Q3	0.80	0.60	-0.90	-1.20	0.00	0.70	0.30	0.50	0.80	-0.10

Economic Indicators in some Developing Countries: Asia

Real Gross Domestic Product

y-o—y percentage change

Year/	Quarter	India	China	Hong Kong	Indo-nesia	Mala-ysia	Philip-pines	S Korea	Singa-pore	Thai-land
2006	Q1	9.30	10.20	8.20	4.60	5.30	5.50	6.20	10.60	6.00
	Q2	8.90	11.30	5.20	5.50	5.90	5.50	5.30	8.10	4.90
	Q3	920	10.40	6.80	5.50	5.90	4.80	4.60	7.20	4.70
	Q4	8.60	10.70	7.00	6.10	5.70	6.50	4.00	6.60	4.20
2007	Q1	9.10	11.10	5.60	6.00	5.30	6.90	4.00	6.10	4.30
	Q2	8.90	11.90	6.60	6.30	5.70	7.50	4.90	8.60	4.40
	Q3	8.90	11.50	6.20	6.50	6.70	6.60	5.20	8.60	4.90
	Q4	8.40	11.20	6.70	6.30	7.30	7.30	550	5.40	570
2008	Q1	8.80	10.60	6.80	6.30	7.10	5.10	5.70	7.20	6.00
	Q2	7.90	10.10	4.20	6.40	6.30	4.60	4.80	1.90	5.30
	Q3	7.60	9.00	1.70	6.10	4.70	4.60	3.90	-0.50	4.00

Economic indicators in some Developing Countries: Latin America & Others

Real Cross Domestic Product

y-o-y *percentage change*

Year/Quarter		Argen-tina	Brazil	Chile	Mexico	Czech Repub	Russia	Hungary	Poland
2006	Q1	8.60	3.40	5.10	13.50	7.40	5.50	4.50	5.20
	Q2	7.90	1.20	4.50	4.70	6.20	7.40	4.10	5.50
	Q3	8.70	3.20	2.90	4.60	5.80	6.50	3.80	5.80
	Q4	8.60	3.80	4.30	4.30	5.80	7.70	3.20	6.40
2007	Q1	8.00	4.30	5.80	2.60	6.10	7.90	2.70	7.40
	Q2	8.70	5.40	6.10	2.80	6.00	7.80	1.20	6.40
	Q3	8.70	5.70	4.10	3.70	6.00	7.70	1.00	7.40
	Q4	9.10	6.20	4.00	3.80	6.60	9.50	0.70	6.10
2008	Q1	8.40	5.80	3.00	2.60	5.30	8.50	1.70	6.10
	Q2	7.50	6.10	4.30	2.80	4.60	7.80	2.00	5.80
	Q3	6.20	6.80	4.80	1.60	4.70	6.20	0.80	4.80

Unemployment Rate

Percent of Labour Force

Year/	Quarter	Ger many	France		Den mark	Swe- den	Euro	Zone UK	Canada	US	Japan
2003	Q1	11.35	9.57	7.40	4.93	5.80	8.20	5.10	6.43	4.73	4.23
	Q2	11.15	9.15	7.00	4.50	5.75	7.90	5.45	6.25	4.63	4.15
	Q3	10.60	8.90	6.80	4.31	5.51	8.07	5.27	6.53	4.87	4.13
	Q4	10.13	8.70	6.50	4.00	4.50	7.60	5.50	6.20	4.47	4.07
2007	Q1	9.33	8.43	6.20	3.87	4.97	7.30	5.50	6.13	4.50	4.00
	Q2	9.17	8.10	6.00	3.60	4.47	7.00	5.43	6.10	4.50	3.77
	Q3	8.93	7.90	5.90	3.23	4.67	7.03	5.40	5.97	4.63	3.80
	Q4	8.57	7.50	6.00	2.83	5.50	7.20	5.27	5.87	4.80	3.87
2008	Q1	7.97	7.20	6.50	2.00	6.27	7.10	5.20	5.87	4.93	3.83
	Q2	7.87	7.37	6.80	1.70	6.67	7.20	5.30	6.13	5.33	4.03
	Q3	7.67	7.73	6.70	1.60	5.63	7.43	5.67	6.10	5.97	4.07

Unemployment Rate

Percent of Labour Force

Year/ Quarter		India	China	Hong Kong	Indo- nesia	Mala ysia	Philip- pines	S Korea	Singa- pore	Thai- land
2006	Q1			5.20		3.80	8.10	3.90	2,.20	1.87
	Q2			5.00		3.40	8.20	3.37	3.60	1.67
	Q3			4.70		3.10	8.00	3.40	2.40	1.22
	Q4	7.60	9.80			3.00	7.30	3.27	2.60	1.30
2007	Q1			4.33		,3.40	7.80	3.23	2.90	1.57
	Q2			4.30		3.40	7.40	3.33	2.40	1.60
	Q3			4.13		3.10	7.80	3.27	1.70	1.20
	Q4	7.20	9.50	3.63		3.00	6.30	3.13	1.60	0.95
2008	Q1			3.37		3.60	7.40	3.03	2.00	1.50
	Q2			3.30		3.50	8.00	3.20	2.30	1.40
	Q3	6.80	9.00	3.27		3.10	7.40	3.17	2.20	1.20

Note: Figures of China and India are annual dat

Unemployment Rate

Percent of Labour Force

Year/Quarter		Agren-tina	Brazil	Chile	Mexico	Czech Rcpub	Russia	Hungary	Poland
2006	Q1		9.90	7.96		9.03		7.67	17.93
	Q2			8.76		7.93			16.57
	Q3	10.20		8.42		7.87			15.47
	Q4	8.70		6.67		7.35	6.80	7.45	14.85
2007	Q1	9.70	9.80	6.39	4.00	7.63	7.20	7.45	15.00
	Q2	8.50	9.97	6.80	3.37	6.50	6.87	7.27	13.03
	Q3	8.10	9.33	7.43	3.93	6.33	5.87	7.17	11.93
	Q4	7.50	8.10	7.40	3.60	5.80	5.93	7.50	11.30
2008	Q1	8.40	8.43	7.37	3.90	5.87	6.23	8.03	11.43
	Q2	8.00	8.07	8.00	3.47	5.07	6.40	7.67	10.03
	Q3	7.80	7.77	8.13	4.23	5.30	5.30	7.57	9.20

Industrial Production Index: Manufacturing

y-o-y percentage change

Year/ Quarter		Germany	France	Italy	Den-mark	Swe-den	Euro-Zone	UK	Canada	US	Japan
	Q1	4.50	0.57	2.87	5.10	2.60	3.17	-0.80	1.47	3.33	2.97
	Q2	5.25	2.75	3.25	3.00	4.70	4.60	-0.80	1.03	4.60	4.30
	Q3	4.37	0.87	2.40	1.67	1.17	3.03	-067	023	5.07	5.40
	Q4	5.75	0.63	3.63	4.53	6.37	3.37	0.63	-2.63	3.90	5.70
2007-	Q1	7.73	1.03	-0.73	2.63	6.63	3.83	0.17	-1.47	2.77	3.00
	Q2	4.40	0.20	0.53	-1.73	4.07	2.53	0.60	-0.17	1.63	2.33
	Q3	523	7.07	1.65	-1.00	2.90	3.83	0.43	0.37	1.67	2.77
	Q4	4.70	2.63	-3.43	-1.77	2.13	2.60	0.70	007	180	2.77
2008	Q1	5.83	187	-0.93	-0.80	2.17	2.97	0.63	-2.90	1.63	2.30
	Q2	2.40	0.13	-1.30	5.10	-2.10	0.50	-1.00	-5.00	0.03	1.50
	Q3	-0.40	-2.17	-4.73	0.10	-1.93	-1.60	-2.13	-3.03	-2.03	-1.37

Industrial Production Index

y-o-y percentage change

Year/ Quarter		India	China	Hong Kong	Indo-nesia	Mala-ysia	Philip-pines	S Korea	Singa-pore	Thai-land
2006	Q1	8.27	17.00	7.00	-10.90	4.00	-7.07	12.13	21.23	8.93
	Q2	9.70	17.57	5.30	-1.40	5.73	-7.87	10.67	12.03	7.40
	Q3	10.40	16.30	-0.60	8.23	5.00	-8.57	10.43	10.90	6.17
	Q4	10.57	14.77	-1.40	14.27	4.23	-14.60	4.40	7.10	6.70
2007	Q1	11.66	15.10	-1.50[1]	9.93	-0.20	-5.30	3.37	3:9	5.60
	Q2	11.50	18.30	-2.30	7.90	1.33	-4.37	6.97	9.70	5.97
	Q3	8.07	18.13	-2.10	2.27	2.13	-1.70	8.60	10.87	9.70
	Q4	8.23	17.53	-0.30	0.73	4.37	-2.13	13.67	-0.77	11.20
2008	Q1	5.63	15.27	-4.40	4.63	5.53	1.07	10.63_t	11.27	12.43
	Q2	5.40	15.90	-4.20	3.43	2.80	5.00	8.50	-5.33	10.60
	Q3	4.40	12.87	-6.70	1.13	0.33	9.03	5.70	-10.57	7.80

Industrial Production Index

y-o-y percentage change

Year/Quarter		Argentina	Brazil	Chile	Mexico	Czech Repub	Russia	Hungary	Poland
2006	Q1	5.20	4.60	6.77	7.03	14.93	3.17	11.30	12.00
	Q2	6.60	0.77	3.90	3.93	8.77	6.10	8.57	13.90
	Q3	7.67	2.77	0.17	4.30	8.73	4.20	10.30	12.83
	Q4	5.55	3.93	3.03	3.67	7.80	3.65	11.80	10.67
2007	Q1	2.80	3.80	4.50	0.80	12.63	8.33	9.33	13.27
	Q2	3.70	5.83	4.97	0.90	9.47	7.40	8.40	8.70
	Q3	1.30	6.30	3.10	1.80	6.07	4.87	10.10	8.20
	Q4	6.35	7.80	4.20	1.70	6.00	5.77	5.57	8.37
2008	Q1	9.13	6.47	3.03	1.20	6.17	6.27	6.90	18.87
	Q2	7.10	6.37	0.37	1.23	5.93	5.60	3.97	8.13
	Q3	4.57	6.77	1.03	6.95	2.50	4.73	2.00	2.97

Consumer Price Index

y o y percentage change

Year/ Quarter		Ger-many	France	Italy	Den-mark	Swe den	Euro Zone	UK	Canada	US	Japan
2006	Q1	2.03	1.80	2.13	2.03	0.77	2.30	1.90	2.40	3.67	0.43
	Q2	1.95	1.80	2.20	2.00	1.50	2.45	2.25	2.45	3.90	0.70
	Q3	1.53	2.00	2.30	2.13	1.30	2.13	2.47	1.73	3.33	0.60
	Q4	1.33	1.33	1.83	1.67	1.53	1.80	2.70	1.30	1.93	0.33
2007	Q1	1.70	1.73	1.13	1.90	1.93	1.87	2.87	1.83	2.43	0.15
	Q2	1.87	1.20	1.60	1.63	1.83	1.90	2.57	2.20	2.67	0.20
	Q3	2.10	1.27	1.60	1.17	1.97	1.90	1.83	2.13	2.40	0.20
	Q4	2.77	2.50	2.37	2.17	3.17	2.93	2.10	2.43	3.97	0.53
2008	Q1	2.87	2.93	3.03	3.03	3.23	3.33	2.40	1.80	4.10	0.97
	Q2	2.90	3.30	3.57	3.47	3.90	3.67	3.37	2.33	4.37	1.37
	Q3	3.10	3.27	4.00	4.17	4.37	3.80	4.77	3.43	5.30	2.17

Consumer Price Index

y o y percentage change

Year/ Quarter		India	China	Hong kong	Indo-nesia	Mala-ysia	Philip pines	S Korea	Singa-pore	Thai-land
2006	Q1	4.80	1.20	2.00	16.87	3.73	7.30	2.37	1.40	5.73
	Q2	6.33	1.37	2.07	15.50	4.25	6.90	2.33	1.20	6.03
	Q3	5.63	1.07	2.30	14.87	3.57	6.57	2.53	0.73	3.63
	Q4	6.83	2.03	2.17	6.07	3.07	4.80	2.13	0.57	3.27
2007	Q1	7.00	2.73	1.73	6.37	2.60	2.90	2.03	0.53	2.43
	Q2	6.33	3.60	1.27	6.03	1.43	2.33	2.43	0.97	1.87
	Q3	6.73	6.10	1.57	6.53	1.77	2.57	2.27	2.73	1.63
	Q4	5.50	6.63	3.47	6.73	2.20	3.27	3.37	4.07	2.90
2008	Q1	6.30	8.03	4.60	7.67	2.60	5.57	3.8O	6.60	5.00
	Q2	7.77	7.77	5.73	10.13	4.83	9.77	4.83	7.50	7.57
	Q3	9.03	5.27	4.63	11.57	8.40	12.20	5.53	6.53	7.20

Consumer Price Index

y o y percentage change

Year/Quarter		Argen Una	Brazil	Chile	Mexico	Czech Repub	Russia	Hungary	Poland
2006	Q1	11.57	5.50	4.07	13.67	2.77	10.77	2.50	0.60
	Q2	11.37	4.27	3.77	3.20	2.90	9.53	2.63	0.90
	Q3	10.53	4.60	3.83	3.37	2.73	10.57	3.40	1.50
	Q4	10.10	3.13	2.27	4.17	1.50	9.13	6.40	1.33
2007	Q1	9.47	3.00	2.67	4.10	1.57	7.63	8.53	2.03
	Q2	8.80	3.30	2.90	4.00	2.47	7.97	8.63	2.40
	Q3	8.63	4.00	4.77	3.97	2.50	9.00	7.70	2.03
	Q4	8.47	4.27	7.23	3.80	4.80	11.40	7.07	3.53
2008	Q1	8.47	4.63	8.03	3.90	7.37	12.87	6.90	4.20
	Q2	9.10	5.57	8.90	4.93	6.77	14.87	6.77	4.33
	Q3	8.93	6.30	9.33	5.50	6.67	14.93	6.30	4.79

Broad Money Growth

y o y percentage change

Year/ Quarter		Ger many	France	Italy	Den mark	Swe den	Euro Zone	UK	Canada	US	Japan
2006	Q1				18.43	9.93	8.82	11.97	5.46	4.66	1.45
	Q2				10.25	10.97	8.36	13.35	6.10	4.73	1.20
	Q3				7.60	13.03	8.67	14.32	7.90	4.50	0.52
	Q4						9.20	12.66	8.73	5.02	0.66
2007	Q1				7.60	12.70	10.23	12.87	9.40	5.73	1.07
	Q2				10.37	12.63	10.67	13.37	10.77	6.40	1.43
	Q3				13.53	12.17	11.53	13.10	11.77	6.50	2.10
	Q4				13.10	13.77	12.03	11.73	12.15	6.20	2.03
2008	Q1				12.63	14.63	11.03	12.37	12.47	6.57	2..20
	Q2				10.07	11.53	10.20	10.90	13.30	6.30	2.07
	Q3				5.23	11.20	8.90	11.63	11.83	5.97	2.23

Broad Money Growth

y-o-y percentage change

Year/Quarter		India	China	Hong-Long	Indo-nesia	Mala 1ia	Philip-pines	S Korea	Singapore	Thailand
2006	Q1	16.92	17.52	8.09	17.41	7.68		7.17	7.69	7.53
	Q2	18.14	17.38	13.26	16.59	6.87			10.91	8.63
	Q3	19.61	16.33	12.95	13.57	8.29			12.14	7.40
	Q4	18.92	15.61	16.59		11.39			16.57	6.83
	Q1	21.54	16.95	16.00	14.62	14.51	23.28	11.50	21.83	7.83
	Q2	20.36	16.97	14.27	14.60	13.71	22.94	10.97	23.00	8.05
	Q3	21.29	18.34	18.85	17.35	12.47	16.46	10.77	21.32	7.47
	Q4	22.74	17.87	24.58	17.00	10.15	11.39	10.64	16.61	6.35
2008	Q1	22.63		14.67	16.48	11.63	6.09	13.29	12.90	5.75
	Q2	21.52		10.44	17.01	12.88	3.89	15.29	9.75	4.76
	Q3	19.99		5.18	14.76	13.48	9.07		9.60	3.98

Broad Money Growth

y o y percentage change

Year/Quarter		Argen tiva	Brazil	Chile	Mexico	Czech Repub	Russia	Hungary	Poland
2006	Q1	20.40	19.09	12.35	16.03	11.92	.38.41	15.63	11.87
	Q2	82.14	19.57	10.76	17.62	11.98	39.80	14.81	11.16
	Q3	21.33	18.59	10.99	14.25	13.12	44.66	15.14	13.18
	Q4	22.85	18.94	11.11	12.98	13.15	46.68	12.44	14.46
2007	Q1	25.19	18.92	12.88	11.68	14.35	49.37	8.27	18.45
	Q2	29.81	21.48	14.07	9.33	15.52	56.68	5.33	16.16
	Q3	25.10	22.19	14.86	11.96	14.91	51.32	5.98	I5.19
	Q4	21.18	21.82	l25.25	11.56	15.80	49.07	8.71	13.58
2008	Q1	21.68	19.90	13.69	12.98	14.72	47.63	12.84	13.35
	Q2	15.43	18.99	14.16	13.01	12.09	14.32	12.28	15.48
	Q3	16.61	19.76	15.34	11.74	12.99	30.50	10.46	16.98

References :

[1] Central Bank of respective countries and their websites (The economist)

[2] Economic Survey of India: Government of India 2008-09.

[3] ICRA Bulletin: Money and Finance : March 2009.

RESEARCH ARTICLE

A PARAMETER UNIFORM CONVERGENCE FOR A SYSTEM OF 'n' PARTIALLY SINGULARLY PERTURBED DIFFERENTIAL EQUATIONS WITH DISCONTINUOUS SOURCE TERMS AND ROBIN INITIAL CONDITIONS

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ABSTRACT

In this paper, a system of partially singularly perturbed Robin type Initial Value Problem with discontinuous source terms is considered . The derivative of first m equations in the system is multiplied by a same singular perturbation parameter ε and the remaining n-m equations are unperturbed. A piecewise uniform Shishkin mesh is constructed and used, in conjunction with a classical finite difference scheme, to form a numerical method for solving this problem. It is proved that the numerical approximations generated by this method are essentially first order convergent in the maximum norm at all points of the domain uniformly with respect to the singular perturbation parameter. Numerical results are presented in support of the theory.

KEYWORDS:

Partially singularly perturbed problems, Robin initial conditions, Finite difference schemes, Discontinuous source term, Shishkin mesh, Parameter uniform convergence

1.1 Introduction

Consider a system of singularly perturbed robin type initial value problems with discontinuous source terms on the unit interval $\Omega = (0, 1]$, assume a single

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discontinuity in the source term at a point $d \in \Omega$. Let $\Omega^- = (0, d)$ and $\Omega^+ = (d, 1)$ and the jump at d in any function is given as $[\omega](d) = \omega(d+) - \omega(d-)$. The corresponding initial value problem is to find $u_1, u_2, \ldots u_n \in \mathbb{D} = \mathbb{C}^0(\bar{\Omega}) \cap \mathbb{C}^1(\Omega^- \cup \Omega^+)$, such that

$$\vec{L}\vec{u}(x) = E\vec{u}'(x) + A(x)\vec{u}(x) = \vec{f}(x), \quad x \in \Omega^- \cup \Omega^+ \tag{1}$$

with the prescribed initial conditions

$$\vec{\beta}\vec{u}(0) = \vec{u}(0) - E\vec{u}'(0) = \vec{\varphi} \tag{2}$$

where, $E = diag(\varepsilon, \varepsilon, \ldots, \varepsilon (m \text{ terms}) 1, 1, \ldots, (n-m) \text{ terms})$,

$\vec{u}(x) = (u_1(x), u_2(x), \ldots, u_n(x))^T$, $A(x) = (a_{ij}(x))_{n \times n}$ and $\vec{f}(x) = (f_i(x))_{n \times 1}$.

The problem (1) and (2) can also be written in the operator form

$$\vec{L}\vec{u} = \vec{f} \text{ on } \Omega \tag{3}$$

with

$$\vec{\beta}\vec{u}(0) = \vec{\varphi} \tag{4}$$

where the operators $\vec{L}, \vec{\beta}$ are defined by

$$\vec{L} = ED + A, \qquad \vec{\beta} = I - ED$$

where I is the identity operator, $D = \frac{d}{dx}$ is the first order differential operator.

Assumption 1.1.

The functions $a_{ij}, f_i \in \mathbb{C}^{(2)}(\bar{\Omega})$, $i, j = 1(1)n$ satisfy the following positivity conditions

$$\left.\begin{array}{l} (i)\, a_{ii}(x) > \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}(x)| \text{ for } i = 1(1)n \\ (ii)\, a_{ij}(x) \leq 0 \text{ for } i \neq j \text{ and } i = 1(1)n \end{array}\right\} \forall\, x \in \bar{\Omega}. \tag{5}$$

Assumption 1.2.

The positive number $\propto$ satisfy the inequality

$$0 < \propto < \min_{\substack{i=1(1)n \\ x \in \bar{\Omega}}} \left\{\sum_{j=1}^{n} a_{ij}(x)\right\} \tag{6}$$

Assumption 1.3.

The singular perturbation parameter ε satisfy $0 < \varepsilon \leq 1$.

The above problem is singularly perturbed in the following sense. The reduced problem obtained by putting $\varepsilon = 0$ in the system (1) is the linear algebraic system

$$A(x)\vec{v}(x) = \vec{f}(x), \; x \in \Omega^- \cup \Omega^+ \tag{7}$$

where $A(x) = \begin{pmatrix} a_{11}(x) & a_{12}(x)\cdots & a_{1n}(x) \\ a_{21}(x) & a_{22}(x)\cdots & a_{2n}(x) \\ & \vdots & \\ a_{n1}(x) & a_{n2}(x)\cdots & a_{nn}(x) \end{pmatrix}$

$\vec{v}(x) = \left(v_1(x), v_2(x), \ldots, v_n(x)\right)^T$

The source terms $f_1(x), f_2(x), \ldots\ldots, f_n(x)$ are sufficiently smooth on $\bar{\Omega}\backslash\{d\}$. The solution components $u_1, u_2, \ldots\ldots, u_n$ of the problem (1) and (2) have overlapping initial layers at $x = 0$ and have overlapping interior layers to the right side of point of discontinuity at $x = d$.

Theorem 1.1.

Let $A(x)$ satisfy (5) and (6). The problem (1) – (2) has a solution $\vec{u} \in \mathbb{D}$.

Proof: The proof is by construction. Let $\vec{y}$ and $\vec{z}$ be the particular solutions of the differential equations

$\varepsilon y_i'(x) + \sum_{j=1}^{n} a_{ij}(x)\, y_j(x) = f_i(x), i = 1,2, \ldots\ldots, m$ for all $x \in \Omega^-$ (8)

$y_i'(x) + \sum_{j=1}^{n} a_{ij}(x)\, y_j(x) = f_i(x), i = m+1, \ldots, n$ for all $x \in \Omega^-$ (9)

and

$\varepsilon z_i'(x) + \sum_{j=1}^{n} a_{ij}(x)\, z_j(x) = f_i(x),\ i = 1,2, \ldots, m$ for all $x \in \Omega^+$ (10)

$z_i'(x) + \sum_{j=1}^{n} a_{ij}(x)\, z_j(x) = f_i(x),\ i = m+1, \ldots, n$ for all $x \in \Omega^+$ (11)

Consider the function

$$\vec{u}(x) = \begin{cases} y_i(x) + \beta_i\left(u_i(0) - y_i(0)\right)\varphi_i(x), & i = 1,2, \ldots, n, \quad x \in \Omega^- \\ z_i(x) + B_i\varphi_i(x), & i = 1,2, \ldots, n, \quad x \in \Omega^+ \end{cases} \quad (12)$$

where $\vec{\varphi}$ is the solution of

$$\varepsilon\varphi_i' + \sum_{j=1}^{n} a_{ij}(x)\,\varphi_i(x) = 0$$

$(\vec{\beta}\vec{\varphi})_i(0) = 1, \qquad i = 1,2, \ldots, n,$ for all $x \in \Omega$

Here $B_i,\ i = 1(1)n$ is chosen so that $\vec{u} \in \mathbb{D}$. In $\Omega, 0 < \vec{\varphi} \leq 1$, there can be no internal maximum or minimum for $\vec{\varphi}$ and hence $\varphi_i' < 0, i = 1(1)n$ in Ω. Choose the constants B_i such that

$\vec{y}(d^-) = \vec{z}(d^+), \vec{u}(d^-) = \vec{u}(d^+)$.

For the constants B_i to exist, it is required that

$$\frac{[u_i(0) - y_i(0)]\varphi_i(d^-)}{\varphi(d^+)} \neq 0 \quad for\, i = 1(1)n.$$

Since $\varphi_i(d^+) > 0$ is true, the existence of $\vec{\beta}$ and hence $\vec{u}$ is ensured.

Remark: Throughout this paper, we use C as a generic positive constant which is independent of the perturbation parameter ε and the discretization parameter N.

2. Analytical Results

The operator $\vec{L}$ satisfies the following maximum principle.

Lemma 2.1.

Let $A(x)$ satisfy (5) and (6). Suppose that a function $\vec{u} \in \mathbb{D}$ satisfies $\vec{\beta}\vec{u}(0) \geq \vec{0}, \vec{L}\vec{u}(x) \geq 0$ for all $x \in \Omega^- \cup \Omega^+$. Then $\vec{u}(x) \geq \vec{0}$ for all $x \in \bar{\Omega}$.

Proof: Let $u_i(p_i) = \min_{x \in \bar{\Omega}}\{u_i(x)\}, for\ 1 \leq i \leq n$. Without loss of generality assume that $u_1(p_1) \leq u_i(p_i), \text{for } 2 \leq i \leq n. If\ u_1(p_1) \geq 0,$ then there is nothing to prove. Suppose that $u_1(p_1) \geq 0$ then there is nothing to prove. Suppose that $u_1(p_1) < 0,$ then the proof is by showing that this leads to contradiction. Note that $p_1 \neq \{0\},$ so either $p_1 \in \Omega^- \cup \Omega^+ \ or\ p_1 = d.$

Case (i): $p_1 \in \Omega^- \cup \Omega^+$

$(\vec{\beta}\vec{u})_i(0) = \vec{u}(0) - E\vec{u}'(0), \quad i = 1,2,\dots,m < 0,$ a contradiction

$\left(\vec{\beta}\vec{u}(0)\right)_i = \vec{u}(0) - \vec{u}'(0), \quad i = m+1,\dots,n < 0,$ a contradiction

and

$$(\vec{L}\vec{u})_i(p_1) = \varepsilon u_i'(p_1) + \sum_{j=1}^{n} a_{1j}(p_1)u_j(p_1), i = 1,2,\dots,m < 0, \text{ which is a contradiction.}$$

$$(\vec{L}\vec{u})_i(p_1) = u_i'(p_1) + \sum_{j=1}^{n} a_{1j}(p_1)u_j(p_1), i = m+1,\dots,n < 0, \text{ which is a contradiction.}$$

Case (ii): $p_1 = d$

Since $\vec{u} \in \mathbb{C}(\Omega)$ and $u_1(d) < 0,$ then there exist a neighbourhood $N_h = (d-h, d)$ such that $u_1(x) < 0$ for all $x \in N_h$. Now choose a point $x_1 \neq d, x_1 \in N_h$ such that $u_1(x_1) > u_1(d)$. It follows from the mean value theorem that, for some $x_2 \in N_h, u_1'(x_2) = \frac{u_1(d) - u_1(x_1)}{d - x_1} < 0,$ since $x_2 \in N_h$.

Thus by similar argument of the first case, it follows that,

$$(\vec{L}\vec{u})_1(x_2) = \varepsilon u_1'(x_2) + \sum_{j=1}^{n} a_{1j}(x_2)u_j(x_2) < 0$$

which is the contradiction.

As an immediate consequence of the above lemma the stability result is established in the following.

Lemma 2.2.

Let $A(x)$ satisfy (5) and (6). Let $\vec{u}$ be the solution of (1) and (2). Then,

$$\|\vec{u}(x)\|_{\Pi} \le max\left\{\|\beta\vec{u}(0)\|, \frac{1}{\alpha}\|\vec{L}(\vec{u})\|_{\Omega^- \cup \Omega^+}\right\}.$$

Proof: Define the functions

$$\vec{\theta}^{\pm}(x) = M \pm \vec{u}(x)$$

where $M = \max\left\{\|\beta\vec{u}(0)\|, \frac{1}{\alpha}\|\vec{L}(\vec{u})\|_{\Omega^- \cup \Omega^+}\right\}$. Using the properties of $A(x)$, it is not hard to verify that $\beta\vec{\theta}^{\pm}(0) \ge \vec{0}$ and $\vec{L}\vec{\theta}^{\pm}(x) \ge \vec{0}$ on $\Omega^- \cup \Omega^+$. It follows from lemma 2.1 that $\vec{\theta}^{\pm}(x) \ge \vec{0}$ on $\bar{\Omega}$. Hence,

$$|\vec{u}(x)| \le max\left\{\|\beta\vec{u}(0)\|, \frac{1}{\alpha}\|\vec{L}(\vec{u})\|_{\Omega^- \cup \Omega^+}\right\}$$

Lemma 2.3.

Let $A(x)$ satisfy (5) and (6). Let $\vec{u}$ be the solution of (1) and (2).Then, for each $i,\ i = 1,2,\dots,n$

and $x \in \Omega^- \cup \Omega^+$. there exist a constant C such that for $i = 1,2,\dots,m$

$$|u_i(x)| \le C\varepsilon\left\{\|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+}\right\}$$

$$|u_i'(x)| \le C\varepsilon^{-1}\left\{\|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+}\right\}$$

$$|u_i''(x)| \le C\varepsilon^{-2}\left\{\|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+} + \|\vec{f}'\|_{\Omega^- \cup \Omega^+}\right\}$$

and for $i = m+1,\dots,n$ $|u_i(x)| \le C\left\{\|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+}\right\}$

$$|u_i'(x)| \le C\left\{\|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+}\right\}$$

$$|u_i''(x)| \le C\left\{\|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+} + \|\vec{f}'\|_{\Omega^- \cup \Omega^+}\right\}$$

Proof: From Lemma 2.2, it is evident that,

$$|\vec{u}(x)| \le \|\vec{\beta}\vec{u}(0)\| + \frac{1}{\alpha}\|\vec{L}(\vec{u})\|_{\Omega^- \cup \Omega^+}$$

Thus,

$|u_i(x)| \leq C\left\{\|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+}\right\}$.

Rewrite the differential equation (1), we get

$\vec{u}'(x) = E^{-1}(\vec{f} - A\vec{u})$

Hence, $|u_i'(x)| \leq C\varepsilon^{-1}\left(\|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+}\right)$ for $i = 1,2, \ldots, m$

and $|u_i'(x)| \leq C\left(\|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+}\right)$ for $i = m+1, \ldots, n$

Differentiating (1) once, we get

$$E\vec{u}''(x) + A(x)\vec{u}'(x) = \vec{f}'(x) - A'(x)\vec{u}(x).$$

Using the bounds of $\vec{u}'$ and $\vec{u}$

$|\vec{u}''(x)| \leq \varepsilon^{-1}[|\vec{f}(x)| + C\varepsilon_i^{-1}(\|\vec{\varphi}\| + \|\vec{f}\|) + C(\|\vec{\varphi}\| + \|\vec{f}\|)]$

and hence,

$|u_i''(x)| \leq C\varepsilon^{-2}\left[\|\vec{f}'\| + \|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+}\right]$, for $i = 1,2, \ldots, m$

and

$|u_i''(x)| \leq C\left[\|\vec{f}'\| + \|\vec{\varphi}\| + \|\vec{f}\|_{\Omega^- \cup \Omega^+}\right]$, for $i = m+1, \ldots, n$.

3. Estimates of derivatives

To derive sharper bounds on the derivatives of the solution, the solution is decomposed into a sum composed of a regular component $\vec{v}$ and a singular component $\vec{w}$. That is, $\vec{u} = \vec{v} + \vec{w}$. The regular component $\vec{v}$ is defined as the solution of the following problem:

$$\vec{L}\vec{v}(x) = \vec{f}(x), \quad x \in \Omega^- \cup \Omega^+ \tag{13}$$

$\vec{\beta}\vec{v}(0) = \vec{\beta}\vec{u}_0(0)$.

The singular component $\vec{w}$ is defined as the solution of the following problem

$$\vec{L}\vec{w}(x) = \vec{0}, \quad x \in \Omega^- \cup \Omega^+ \tag{14}$$

$\vec{\beta}\vec{w}(0) = \vec{\beta}(\vec{u} - \vec{v})(0), [\vec{w}](d) = -[\vec{v}](d)$.

Theorem 3.1.

Let $A(x)$ satisfy (5) and (6). Then the components $v_i, i = 1(1)n$ of the regular component $\vec{v}$ and its derivatives satisfy the bounds for all $x \in \Omega^- \cup \Omega^+$ for $i = 1(1)m$,

$\|\vec{v}^{(k)}\|_{\Omega^- \cup \Omega^+} \leq C$ for $k = 0,1$

$|[\vec{v}(d)]| \leq C, \qquad |[\vec{v}'](d)| \leq C$

$\|v_i''\|_{\Omega^- \cup \Omega^+} \leq C\varepsilon^{-1}$.

and for $i = m+1(1)n$

$\|\vec{v}^{(k)}\|_{\Omega^- \cup \Omega^+} \leq C$ for $k = 0,1$

$|[\vec{v}(d)]| \leq C$, $|[\vec{v}'](d)| \leq C$

$\|v_i''\|_{\Omega^- \cup \Omega^+} \leq C$

Proof:

Following the techniques in [28], one can arrive at the results

$\|[\vec{v}^{(k)}]\|_{\Omega^- \cup \Omega^+} \leq C$ for $k = 0,1$

Also for $i = 1,2,\ldots,m$,

$\|v_i''\|_{\Omega^- \cup \Omega^+} \leq C\varepsilon^{-1}$

and for $i = m+1,\ldots,n$

$|[v_i](d)| = v_i(d^+) - v_i(d^-) \leq |v_i(d^+)| + |v_i(d^-)| \leq C$

$\|v_i''\|_{\Omega^- \cup \Omega^+} \leq C$

Similarly, $|[\vec{v}'](d)| \leq C$, and hence the proof is completed.

Now bounds on the layer components of $\vec{u}$ are to be found. Consider the layer functions

$B_l(x) = e^{-\alpha x/\varepsilon}$, $\quad B_r(x) = e^{-\alpha(x-d)/\varepsilon}$.

Theorem 3.2.

Let $A(x)$ satisfy (5) and (6). Then the components w_i, $i = 1(1)n$ of the regular component $\vec{w}$ and its derivatives satisfy the bounds for all $x \in \Omega^- \cup \Omega^+$ and for $i = 1(1)m$

$$|w_i(x)| \leq \begin{cases} CB_l(x), & x \in \Omega^- \\ CB_r(x), & x \in \Omega^+ \end{cases} |w_i'(x)| \leq \begin{cases} C\varepsilon^{-1}B_l(x), & x \in \Omega^- \\ C\varepsilon^{-1}B_r(x), & x \in \Omega^+ \end{cases} |w_i''(x)| \leq \begin{cases} C\varepsilon^{-2}B_l(x), & x \in \Omega^- \\ C\varepsilon^{-2}B_r(x), & x \in \Omega^+ \end{cases}$$

and for $i = m+1(1)n$,

$$|w_i(x)| \leq \begin{cases} CB_l(x), & x \in \Omega^- \\ CB_r(x), & x \in \Omega^+ \end{cases} |w_i'(x)| \leq \begin{cases} CB_l(x), & x \in \Omega^- \\ CB_r(x), & x \in \Omega^+ \end{cases} |w_i''(x)| \begin{cases} CB_l(x), & x \in \Omega^- \\ CB_r(x), & x \in \Omega^+ \end{cases}$$

Proof: We have $\vec{u} = \vec{v} + \vec{w}$ and by lemma 2.2, $|\vec{w}(0)| \leq C$ and $|\vec{w}(d+)| \leq C$. Define the barrier functions

$\vec{\xi}^{\pm} = CB_l(x) \pm w_i(x)$

with C chosen sufficiently large such that $\vec{\xi} \geq |\vec{w}|$ at $x = 0, d+$, for $i = 1,2,\dots,m$

$$(\vec{L}\vec{\xi}^{\pm})_i(x) = \varepsilon[CB_i(x) \pm w_i(x)]' + CB_i(x)\sum_{j=1}^{n} a_{ij}(x) + \sum_{j=1}^{n} w_j(x)$$

$$= C\left(\sum_{j=1}^{n} a_{1j} - \alpha, \sum_{j=1}^{n} a_{2j} - \alpha, \dots, \sum_{j=1}^{n} a_{nj} - \alpha\right)B_i(x) \pm w_i(x)$$

$\geq 0 = (\vec{L}\vec{w})_i$ And for $i = m+1,\dots,n$

$$(\vec{L}\vec{\xi}^{\pm})_i(x) = [CB_i(x) \pm w_i(x)]' + CB_i(x)\sum_{j=1}^{n} a_{ij}(x) + \sum_{j=1}^{n} w_j(x)$$

≥ 0

and it is not hard to see that $\vec{\beta}\vec{\xi}(0) \geq \vec{0}$. Using maximum principle (2.1), we get the required bounds on $\vec{w}$. Now to bound first – order derivative of w_i, consider $\varepsilon w_i' + \sum_{j=1}^{n} a_{ij} w_j = 0$, for $i = 1,2,\dots,m$ together with the bound on $\vec{w}$. This implies that

$$|w_i'(x)| \leq \begin{cases} C\varepsilon^{-1}B_l(x), & x \in \Omega^- \\ C\varepsilon^{-1}B_r(x), & x \in \Omega^+ \end{cases}$$

Consider the equation $w_i' + \sum_{j=1}^{n} a_{ij} w_j = 0$, for $i = m+1,\dots,n$ we have

$$|w_i'(x)| \leq \begin{cases} CB_l(x), & x \in \Omega^- \\ CB_r(x), & x \in \Omega^+ \end{cases}$$

Now to bound second-order derivatives, differentiate $\varepsilon w_i' + \sum_{j=1}^{n} a_{ij} w_j = 0$ once and using the estimates of w_i', we get the required bounds on singular component $\vec{w}$ and its derivatives.

4. The Shishkin mesh

A piecewise uniform Shishkin mesh $\bar{\Omega}^N = \{x_j\}_{j=0}^{N}$ with N mesh-intervals constructed as follows. The interval $[0,1]$ is subdivided into 4 sub-intervals $[0,\sigma) \cup (\sigma, d] \cup (d, d+\tau] \cup (d+\tau, 1]$. Then each of the sub-intervals $[0,\sigma), (\sigma, d], (d, d+\tau]$ and $(d+\tau, 1]$ a uniform mesh of $\frac{N}{4}$ mesh points is placed. The interior points of the mesh are denoted by

$$\Omega^N = \left\{x_j : 1 \leq j \leq \frac{N}{2} - 1\right\} \cup \left\{x_j : \frac{N}{2} + 1 \leq j \leq N - 1\right\}$$

Clearly, $x_{\frac{N}{2}} = d$ and $\bar{\Omega}^N = \{x_j\}_{j=0}^{N}$. Note that this mesh is a uniform mesh when $\tau = \frac{d}{2}$

and $\sigma = \frac{1-d}{2}$. It is fitted to the singular perturbation problem by choosing τ and σ to be the following functions of N and ε.

$$\sigma = min\left\{\frac{d}{2}, \frac{\varepsilon}{\alpha}\ln N\right\}$$

$$\tau = min\left\{\frac{1-d}{2}, \frac{\varepsilon}{\alpha}\ln N\right\}$$

5. The Discrete Problem

The Initial Value Problem (1), (2) is discretised using a fitted mesh method composed of a classical finite difference operator on a piecewise uniform fitted mesh $\bar{\Omega}^N$. Then the fitted mesh method for solving the system (1) and (2) is, for $i = 1,2,\dots,m$, $j = 1,2,\dots,N$.

$$(\vec{L}^N\vec{U})_i(x_j) = ED^-\vec{U}(x_j) + A(x_j)\vec{U}(x_j) = \vec{f}(x_j),\ j \neq \frac{N}{2}, i = 1,2,\dots,m \quad (15)$$

$$\vec{\beta}\vec{U}(0) = \vec{U}(0) - \varepsilon D^+\vec{U}(0) = \vec{\varphi} \quad (16)$$

and

$$(\vec{L}^N\vec{U})_i(x_j) = D^-\vec{U}(x_j) + A(x_j)\vec{U}(x_j) = \vec{f}(x_j),\ j \neq \frac{N}{2}, i = m+1,\dots,n \quad (17)$$

$$\vec{\beta}\vec{U}(0) = \vec{U}(0) - D^+\vec{U}(0) = \vec{\varphi} \quad (18)$$

and at $x_{\frac{N}{2}} = d$, the scheme is given by

$$\vec{L}^N\vec{U}\left(x_{\frac{N}{2}}\right) = ED^-\vec{U}\left(x_{\frac{N}{2}}\right) + A\left(x_{\frac{N}{2}}\right)\vec{U}\left(x_{\frac{N}{2}}\right) = \vec{f}\left(x_{\frac{N}{2}} - 1\right).$$

The problem (13), (14) can also be written in the operator form

$\vec{L}^N\vec{U} = \vec{f}$ on Ω^N with

$\vec{\beta}^N\vec{U}(0) = \vec{\varphi}$

where $\vec{L}^N = ED^- + A$

with $\vec{\beta}^N = I - ED^+$

and D^+, D^- are the difference operators

$$D^-\vec{U}(x_j) = \frac{\vec{U}(x_j) - \vec{U}(x_{j-1})}{x_j - x_{j-1}}, D^+\vec{U}(x_j) = \frac{\vec{U}(x_{j+1}) - \vec{U}(x_j)}{x_{j+1} - x_j}, j = 1,2,\dots,N$$

The following discrete results are analogous to those for the continuous case.

Lemma 5.1.

Let A(x) satisfy (5) and (6). Suppose that a mesh function $\vec{Z}(x_j)$

satisfies $\vec{\beta}\vec{Z}(x_0) \geq \vec{0}$

and $\vec{L}^N\vec{Z}(x_j) \geq \vec{0}$, for all $x_j \in \Omega^N$ and $(D^+ - D^-)\vec{Z}\left(x_{\frac{N}{2}}\right) \leq \vec{0}$, implies that $\vec{Z}(x_j) \geq \vec{0}$ for all $x_j \in \bar{\Omega}$.

Proof: Let x_q be any point at which $\vec{Z}(x_q)$ attains its minimum on $\bar{\Omega}^N$. If $\vec{Z}(x_q) \geq \vec{0}$, then there is nothing to prove. Without loss of generality, suppose that $Z_1(x_q) < 0$, then clearly, $x_q \neq 0$. If $x_q = 0$, then

$\vec{\beta}^N\vec{Z}(0) = \vec{Z}(0) - \varepsilon D^+\vec{Z}(0)$

< 0, a contradiction.

Therefore, $x_q \neq 0$. $If\ q \neq N/2$, it is clear that

$D^-Z_1(x_q) \leq 0 \leq D^+Z_1(x_q)$

and hence if $x_q \in \Omega^N, q \neq N/2$, then

$$(\vec{L}^N\vec{Z})_i(x_q) = \varepsilon D^-Z_i(x_q) + a_{11}(x_q)Z_1(x_q) + \cdots + a_{1n}(x_q)Z_n(x_q) < 0, \qquad i = 1,2,\ldots,m$$

$$(\vec{L}^N\vec{Z})_i(x_q) = D^-Z_i(x_q) + a_{11}(x_q)Z_1(x_q) + \cdots + a_{1n}(x_q)Z_n(x_q) < 0, \quad i = m+1,\ldots,n$$

which is a contradiction. Hence, the only possibility is that $x_q = x_{\frac{N}{2}}$. Then

$D^-Z_1\left(x_{\frac{N}{2}}\right) \leq 0 \leq D^+Z_1\left(x_{\frac{N}{2}}\right) \leq D^-Z_1\left(x_{\frac{N}{2}}\right)$.

From the above it is observed that

$Z_1\left(x_{\frac{N}{2}-1}\right) = Z_1\left(x_{\frac{N}{2}}\right) = Z_1\left(x_{\frac{N}{2}+1}\right) < 0$

Then, $(\vec{L}^N\vec{Z})_1\left(x_{\frac{N}{2}-1}\right) < 0$, which is a contradiction. Hence the result.

Lemma 5.2.

Let $A(x)$ satisfy (5) and (6). If $\vec{U}$ be the numerical solution of (1) and (2), then

$$\|\vec{U}\|_{\bar{\Omega}^N} \leq max\left\{\|\vec{\beta}^N\vec{U}(0)\|, \frac{1}{\alpha}\|\vec{f}\|_{\Omega^{-N}\cup\Omega^{+N}}\right\}.$$

Proof: Define the two mesh functions

$$\vec{\theta}^{\pm}(x_j) = max\left\{\|\vec{\beta}^N\vec{U}(0)\|, \frac{1}{\alpha}\|\vec{f}\|_{\Omega^{-N}\cup\Omega^{+N}}\right\} \pm \vec{U}$$

Using the properties of $A(x)$, it is not hard to verify that $\vec{\beta}^N\vec{\theta}^{\pm}(0) \geq \vec{0}$ and $\vec{L}^N\vec{\theta}^{\pm} \geq \vec{0}$ on Ω^N. Applying the discrete maximum principle (Lemma 5.1) then gives $\vec{\theta}^{\pm} \geq \vec{0}$ on Ω^{-N},

and so

$$|\vec{U}| \leq max\left\{\|\vec{\beta}^N \vec{U}(0)\|, \frac{1}{\alpha}\|\vec{f}\|_{\Omega^{-N}\cup\Omega^{+N}}\right\}$$

as required.

6. The Local Truncation Error

From Lemma 5.2, it is seen that in order to bound the error $\|\vec{U}-\vec{u}\|$, it suffices to bound $\vec{L}^N(\vec{U}-\vec{u})$. Notice that, for $x_i \in \Omega^N$.

$$\vec{L}^N(\vec{U}(x_j)-\vec{u}(x_j)) = E(D^- - D)\vec{u}(x_j)$$

and

$$((\vec{L}-\vec{L}^N)u)_i(x_j) = \varepsilon(D^- - D)v_i(x_j) + \varepsilon(D^- - D)w_i(x_j)$$

which is the local truncation of the first derivative. Then, by the triangle inequality,

$$\left|(\vec{L}^N(\vec{U}-\vec{u}))_i(x_j)\right| \leq \left|\varepsilon(D^- - D)v_i(x_j)\right| + \left|\varepsilon(D^- - D)w_i(x_j)\right|$$

Analogous to the continuous case, the discrete solution $\vec{U}$ can be decomposed into $\vec{V}$ *and* $\vec{W}$ which are defined to be solutions of the following discrete problems

$$(\vec{L}^N\vec{V})(x_j) = \vec{f}(x_j) \text{ on } \Omega^N, \vec{\beta}^N\vec{V}(0) = \vec{\beta}\vec{v}(0) \quad (19)$$

and

$$(\vec{L}^N\vec{W})(x_j) = \vec{0} \text{ on } \Omega^N, \vec{\beta}^N\vec{W}(0) = \vec{\beta}\vec{w}(0) \quad (20)$$

where $\vec{v}$ *and* $\vec{w}$ are the solutions of (13) and (14) respectively.

Further, for $i = 1,2,\dots,m$

$$\left|(\vec{\beta}^N(\vec{V}-\vec{v})_i(0)\right| = \left|\varepsilon(D-D^+)v_i(0)\right|$$

$$\left|(\vec{\beta}^N(\vec{W}-\vec{w})_i(0)\right| = \left|\varepsilon(D-D^+)w_i(0)\right|$$

$$\left|(\vec{L}^N(\vec{V}-\vec{v}))_i(x_j)\right| = \left|\varepsilon(D^- - D)v_i(x_j)\right|$$

$$\left|(\vec{L}^N(\vec{W}-\vec{w}))_i(x_j)\right| = \left|\varepsilon(D^- - D)w_i(x_j)\right| \quad (21)$$

For $i = m+1,\dots,n$

$$\left|(\vec{\beta}^N(\vec{V}-\vec{v})_i(0)\right| = \left|(D-D^+)v_i(0)\right|$$

$$\left|(\vec{\beta}^N(\vec{W}-\vec{w})_i(0)\right| = |(D-D^+)w_i(0)|$$

$$\left|(\vec{L}^N(\vec{V}-\vec{v}))_i(x_j)\right| = |(D^- - D)v_i(x_j)|$$

$$\left|(\vec{L}^N(\vec{W}-\vec{w}))_i(x_j)\right| = |(D^- - D)w_i(x_j)| \quad (22)$$

The error at each point $x_j \in \bar{\Omega}^N$ is denoted by $\vec{U}(x_j)-\vec{u}(x_j)$. Then the local truncation error $\vec{L}^N(\vec{U}(x_j)-\vec{u}(x_j))$ has the decomposition

$$\vec{L}^N(\vec{U}-\vec{u})(x_j) = \vec{L}^N(\vec{V}-\vec{v})(x_j) + \vec{L}^N(\vec{W}-\vec{w})(x_j)$$

By a Taylor expansion on regular and singular components, we have

$$\left|\varepsilon\left(\frac{d}{dx}-D^-\right)v_k(x_j)\right| \le C\varepsilon\frac{(x_j-x_{j-1})}{2}|v_k|_2 \le CN^{-1} \quad (23)$$

And

$$\left|\varepsilon\left(\frac{d}{dx}-D^-\right)w_k(x_j)\right| \le \begin{cases} C\varepsilon\frac{(x_j-x_{j-1})}{2}|w_k|_2 \\ C\varepsilon\max_{[x_j,x_{j-1}]}|w_k'| \end{cases} \quad (24)$$

where $k = 1,2,\dots,n, j \neq \frac{N}{2}$.

The error in the smooth and singular components are bounded in the following section.

7. Error Analysis

The proof of the theorem on the error estimate is split into two parts. First, a theorem concerning the error in the smooth component is established. Then the error in the singular component is established.

Theorem 7.1.

Let $A(x)$ satisfy (5) and (6). Let $\vec{v}$ denote the smooth component of the solution of (1), (2) and $\vec{V}$ denote the smooth component of the solution of the problem (13), (14). Then

$$\left|\left(\vec{L}^N(\vec{V}-\vec{v})\right)_i(x_j)\right| \le CN^{-1}.$$

Proof: From the expression (23),

$$\left|\vec{\beta}^N(\vec{V}-\vec{v})_i(0)\right| \le C\varepsilon(x_1-x_0)\max_{s\in[x_0,x_1]}|v_i''(s)|$$

$$\le CN^{-1}, \text{for } i = 1,2,\dots,m \quad (25)$$

$$\left|\vec{\beta}^N(\vec{V}-\vec{v})_i(0)\right| \le C(x_1-x_0)\max_{s\in[x_0,x_1]}|v_i''(s)|$$

$$\le CN^{-1}, \text{ for } i=m+1,\dots,n \qquad (26)$$

It is not hard to find that

$$|\varepsilon(D^- - D)v_i(x_j)| \le Ch_j \max_{s\in I_j}|\varepsilon v_i''(s)|$$

$$\le Ch_j$$

$$\le CN^{-1}, \text{ for } i=1,2,\dots,m$$

$$|(D^- - D)v_i(x_j)| \le Ch_j \max_{s\in I_j}|v_i''(s)|$$

$$\le Ch_j$$

$$\le CN^{-1}, \text{for } i=m+1,\dots,n$$

As required.

Lemma 7.1.

Let $A(x)$ satisfy (5) and (6). Let $\vec{w}$ denote the singular component of the solution of (1), (2) and $\vec{W}$ denote the singular component of the solution of the problem (13), (14). Then

$$\left|\left(L^N(\vec{W}-\vec{w})\right)_i(x_j)\right| \le CN^{-1}\ln N, i=1,2,\dots,n$$

Proof: For the proof of this theorem, we have to evaluate the error estimates for the singular components on different subintervals considered as follows:

Case (i): The solution argument depends on whether the transition parameter $\sigma=\frac{d}{2}$ or $\sigma=\frac{\varepsilon}{\alpha}\ln N$

Sub-case (i): When $\sigma=\frac{d}{2}$, the mesh is uniform and it satisfies $\frac{\varepsilon}{\alpha}\ln N \ge \frac{d}{2}$.

From the expression of (23),

$$|(\vec{\beta}^N(\vec{W}-\vec{w}))_i(0)| \le C\varepsilon(x_1-x_0)\max_{s\in[x_0,x_1]}|\vec{w}''(s)| \le CN^{-1}\ln N, \text{ for } i=1,2,\dots,m$$

and for $i=m+1,\dots,n$

$$|(\vec{\beta}^N(\vec{W}-\vec{w}))_i(0)| \le C(x_1-x_0)\max_{[x_0,x_1]}|w_i''| \le CN^{-1}\ln N$$

The solution argument used above then yields

$\left|\vec{L}^N(\vec{W}-\vec{w})_i(x_j)\right| \le C\varepsilon(x_j - x_{j-1})|\vec{w}|_2$, for $i = 1,2,\dots,m$

Since $x_j - x_{j-1} \le 2N^{-1}$, the estimate for $|\vec{w}|_2$ obtained which gives

$\left|\vec{L}^N(\vec{W}-\vec{w})_i(x_j)\right| \le \frac{\varepsilon}{2}(x_j - x_{j-1})|\vec{w}|_2 \le \varepsilon N^{-1} C\varepsilon^{-2}\theta^{\frac{-\alpha x}{\varepsilon}} \le C\varepsilon^{-1}N^{-1}$.

Therefore,

$\left|\vec{L}^N(\vec{W}-\vec{w})_i(x_j)\right| \le CN^{-1}\ln N$, since$\varepsilon^{-1} \le \frac{2d\ln N}{\alpha}$.

Similarly for $i = m+1,\dots,n$

$\left|\vec{L}^N(\vec{W}-\vec{w})_i(x_j)\right| \le CN^{-1}\ln N$.

Sub-case (ii): When $\sigma = \frac{\varepsilon}{\alpha}\ln N$ the mesh is piecewise uniform, with the mesh spacing $\frac{4\sigma}{N}$ in the subinterval $[0,\sigma]$ *and* $\frac{4(d-\sigma)}{N}$ in the subinterval $(\sigma, d]$.

The proof of this sub-case is similar to the proof of Theorem 3.2.2 in section 1 of [28].

Case (ii):

The solution argument depends on whether the transition parameter $\tau = \frac{1-d}{2}$ or $\tau = \frac{\varepsilon}{\alpha}\ln N$

Sub-case (i): When $\tau = \frac{1-d}{2}$, the mesh is uniform and it satisfies $\frac{\varepsilon}{\alpha}\ln N \ge \frac{1-d}{2}$.

Thus, following the same logic as in the first case, the proof follows.

Sub-case (ii): When $\tau = \frac{\varepsilon}{\alpha}\ln N$, the mesh is piecewise uniform, with the mesh spacing $\frac{4\tau}{N}$ in the subinterval $[d, d+\tau]$ and $\frac{4(1-(d+\tau))}{N}$ in the subinterval $(d+\tau, 1]$.

The proof of the sub-case is similar to the proof of theorem 3.2 in section 1 of chapter [28].

Therefore, we concluded that

$\left|\vec{L}^N(\vec{U}-\vec{u})_i(x_j)\right| \le CN^{-1}\ln N, j \ne \frac{N}{2}$.

Now at the point $x_{\frac{N}{2}} = d$, for $i = 1,2,\dots,n$

$\left|\left(\vec{L}^N(\vec{U}-\vec{u})\right)_i(x_j)\right| \le C\varepsilon h^+ \max_{\left[x_{\frac{N}{2}}, x_{\frac{N}{2}+1}\right]}|y''(\eta)| + C\varepsilon h^- \max_{\left[x_{\frac{N}{2}-1}, x_{\frac{N}{2}}\right]}|y''(\theta)|$ where $\begin{cases} x_{\frac{N}{2}} < \eta < x_{\frac{N}{2}+1} \\ x_{\frac{N}{2}-1} < \theta < x_{\frac{N}{2}} \end{cases}$

$$\leq C\sigma N^{-1}\frac{B_r(\eta)}{\varepsilon} + CN^{-1}\frac{B_l(\theta)}{\varepsilon}$$

$$\leq C\frac{\sigma N^{-1}}{\varepsilon} + CN^{-1}\frac{B_l(\theta)}{\varepsilon}$$

$$\leq CN^{-1}\ln N.$$

Similarly this proof is true for $i = m+1, \ldots, n$.

We conclude this section with the following main result which follows by using the error analysis for the regular and singular components, and the discrete maximum principle.

Theorem 7.2.

Let $\vec{u}$ be the solution of the continuous problem (1), (2) and $\vec{U}$ be the solution of the discrete problem (13), (14). Thus, for N sufficiently large,

$$\left\|\left(\vec{L}^N(\vec{U}-\vec{u})\right)\right\| \leq CN^{-1}\ln N$$

where C is a constant independent of ε and N.

Proof: Consider the two mesh functions

$$\theta_i^{\pm}(x_j) = \begin{cases} CN^{-1}\ln N(1+2x_j) \pm \vec{L}^N\left(U_i(x_j)-u_i(x_j)\right), & j \leq \dfrac{N}{2} \\ CN^{-1}\ln N(d+x_j) \pm \vec{L}^N\left(U_i(x_j)-u_i(x_j)\right), & j > \dfrac{N}{2} \end{cases}$$

where C is suitably chosen sufficiently large constant. Hence for $j < \frac{N}{2}$, it is not hard to verify that $\left(\vec{\beta}^N\vec{\theta}^{\pm}\right)_i(0) \geq \vec{0}$ and

$$\left(\vec{L}^N\vec{\theta}^{\pm}\right)_i(x_j) = C\varepsilon N^{-1}\ln N + CN^{-1}\ln N(1+2x_j)\sum_{p=1}^{n} a_{ip}(x_j) \pm \vec{L}^N\left(U_i(x_j)-u_i(x_j)\right)$$

$$> CN^{-1}\ln N\sum_{p=1}^{n} a_{ip}(x_j) \pm \vec{L}^N\left(U_i(x_j)-u_i(x_j)\right) > CN^{-1}\ln N\alpha \pm CN^{-1}\ln N \geq 0$$

and for $j > \frac{N}{2}$,

$$\left(\vec{L}^N\vec{\theta}^{\pm}\right)_i(x_j) = C\varepsilon N^{-1}\ln N + CN^{-1}\ln N(d+x_j)\sum_{p=1}^{n} a_{ip}(x_j) \pm \vec{L}^N\left(U_i(x_j)-u_i(x_j)\right)$$

$$> CN^{-1}\ln N\sum_{p=1}^{n} a_{ip}(x_j) \pm \vec{L}^N\left(U_i(x_j)-u_i(x_j)\right) > CN^{-1}\ln N\alpha \pm CN^{-1}\ln N \geq 0$$

and for $j=\frac{N}{2}$

$$(\vec{L}^N\vec{\theta}^{\pm})_i\left(x_{\frac{N}{2}}\right)=CN^{-1}\ln N\frac{\left(d+x_{\frac{N}{2}}+h^+-1-2x_{\frac{N}{2}}\right)}{h^+}-CN^{-1}\ln N\frac{\left(1+x_{\frac{N}{2}}\right)-\left(1+x_{\frac{N}{2}}-h^-\right)}{h^-}$$

$$=CN^{-1}\ln N\frac{(h^+-1)}{h^+}-CN^{-1}\ln N\pm CN^{-1}\ln N$$

$\leq 0.$

Thus, for N sufficiently large,

$\|\vec{U}-\vec{u}\|\leq CN^{-1}\ln N$

which completes the proof.

Numerical Illustration

The numerical method proposed above is illustrated through an example presented in this section.

Example 8.1.

Consider the following singularly perturbed robin type initial value problem with discontinuous source terms

$$\varepsilon u_1'(x)+(2+x)u_1(x)-u_2(x)-u_3(x)=f_1(x),x\in\Omega^-\cup\Omega^+$$

$$\varepsilon u_2'(x)-u_1(x)+4u_2(x)-u_3(x)=f_2(x),x\in\Omega^-\cup\Omega^+$$

$$\varepsilon u_3'(x)-u_1(x)-u_2(x)+(4+e^x)u_3(x)=f_3(x),x\in\Omega^-\cup\Omega^+$$

with

$$\beta u_1(0)=1,\ \beta u_2(0)=1,\ \beta u_3(0)=1.$$

where

$$f_1(x)=\begin{cases}1 & for\ 0\leq x\leq 0.5\\ 1 & for\ 0.5\leq x\leq 1,\end{cases}$$

$$f_2(x)=\begin{cases}3 & for\ 0\leq x\leq 0.5\\ 0.5 & for\ 0.5\leq x\leq 1,\end{cases}\quad f_3(x)=\begin{cases}2 & for\ 0\leq x\leq 0.5\\ 1 & for\ 0.5\leq x\leq 1.\end{cases}$$

The exact solution of the test example is not known. Therefore, we estimate the error for $\bar{U}$ by comparing it to the numerical solution $\bar{U}$obtained on the mesh $\tilde{x}_i$ that contains the mesh points of the original and their midpoints. For different values of N and the parameter ε, we compute

$$D_\varepsilon^N=\left\|\vec{U}-\tilde{\vec{U}}(x_j)\right\|_\Pi$$

The numerical solution obtained by applying the fitted mesh method (13) and

(14) to the Example is shown in Figure 1. The order of convergence and the error constant are calculated and are presented in Table 1.

Table 1

Maximum pointwise errors $D_\varepsilon^N, D^N, p^N, p^*$ and C_p^N generated for the example

η	Number of mesh points N			
	72	144	288	576
0.100E+01	0.719E-01	0.420E-01	0.229E-01	0.120E-01
0.250E+00	0.135E+00	0.107E+00	0.710E-01	0.418E-01
0.625E-01	0.106E+00	0.942E-01	0.726E-01	0.499E-01
0.156E-01	0.106E+00	0.938E-01	0.724E-01	0.498E-01
0.391E-02	0.106E+00	0.938E-01	0.724E-01	0.497E-01
D^N	0.135E+00	0.107E+00	0.726E-01	0.499E-01
p^N	0.329E+00	0.561E+00	0.542E+00	
C_p^N	0.270E+01	0.270E+01	0.230E+01	0.198E+01
The order of uniform convergence p^* = 0.3290984E+00				
Computed $\vec{\varepsilon}$-uniform error constant, $C_{p^*}^N$= 0.2696402E+01				

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References

[1] **E.P. Doolan, J.J.H. Miller,** W.H.A. Schilders, Uniform Numerical Methods for Problems with Initial and Boundary Layers, Boole Pres, 1980.

[2] **P.A. Farrell, A. Hegarty, J.J.H. Miller, E. ORiordan, G.I. Shishkin,** Robust computational Mathematical Computation, Chapman & Hall/CRC Press, 2000.

[3] **R.Janet, J.J.H.Miller, and S.Valarmathi,** A Parameter-Uniform Essentially First Order Convergent Numerical Method for a System of Two Singularly Perturbed Differential Equations of Reaction-Diffusion Type with Robin Boundary Conditions, International Journal Of Numerical Analysis And Modeling Volume 1, Number 1, Pages 1-18.

[4] **R.E. O'Malley.** Introduction to singular Perturbations. Academic Press, New York, 1974.

[5] **P. Maragatha Meenakshi, S. Valarmathi, and J.J.H Miller.** Solving a partially singularly perturbed initial value problem on shishkin meshes. Applied Mathematics and Computation. 215: 3170-3180,2010.

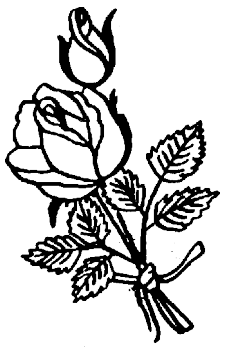
[6] **S. Matthews, E. ORiordan, G.I. Shishkin.** Numerical methods for a system of singularly perturbed reactiondiffusion equations, J. Comput. Appl. Math. 145 (2002) 151166.

[7] **J.J.H. Miller, E. O'Riordan, and G.I. Shishkin.** Fitted numerical methods for singular perturbation problems. Error estimates in the maximum norm for linear problems in one and two dimensions. World Scientific publishing Co.Pvt.Ltd., Singapore, 1996.

[8] **Paramasivam Mathiyazhagan, Valarmathi Sigamani, and John J.H. Miller** Second order parameter-uniform convergence for a fnite difference method for a singularly perturbed linear reaction-diffusion system, Mathematical Communications, Vol. 15, No.2, pp 587-612 (2010).

[9] **H.G. Roos, M.Stynes and L. Tobiska.** Numerical Method for Singularly Perturbed Differential Equations. Springer Verlag, 1996.

[10] **G.I. Shishkin, Grid Approximations** of Singularly Perturbed Elliptic and Parabolic Equations, Ural Branch of Russian Academy of Sciences, 1992.



BIOGRAPHY

J. CARL FRIEDRICH GAUSS*

H. D. BUDHBHATTI**

Johann Carl Friedrich Gauss (30 April 1777 – 23 February 1855) was a German mathematician and physicist who made significant contributions to many fields in mathematics and science

Gauss's intellectual abilities attracted the attention of the **Duke of Brunswick**, who sent him to the Collegium Carolinum (now **Braunschweig University of Technology**), which he attended from 1792 to 1795, and to the **University of Göttingen** from 1795 to 1798. While at university, Gauss independently rediscovered several important theorems. His breakthrough occurred in 1796 when he showed that a regular polygon can be constructed by compass and straightedge if the number of its sides is the product of distinct Fermat primes and a power of construction problems had occupied mathematicians since the days of the **Ancient Greeks,** and the discovery ultimately led Gauss to choose mathematics instead of philology as a career. Gauss was so pleased with this result that he requested that a regular **heptadecagon** be inscribed on his **tombstone.** The **stonemason** declined, stating that the difficult construction would essentially look like a circle.

The year 1796 was productive for both Gauss and number theory. He discovered a construction of the heptadecagon on 30 March. He further advanced **modular**

* Adapted from wikipedia (the free encyclopedia) and other related sources.

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arithmetic, greatly simplifying manipulations in number theory. On 8 April he became the first to prove the quadratic reciprocity law. This remarkably general law allows mathematicians to determine the solvability of any quadratic equation in modular arithmetic. The **prime number theorem,** conjectured on 31 May, gives a good understanding of how the **prime numbers** are distributed among the integers.

Gauss's diary entry related to sum of triangular numbers (1796)

Gauss also discovered that every positive integer is representable as a sum of at most three **triangular numbers** and then jotted down in his diary the note: "ÅÕÑÇÊÁ! num = Ä + Ä + Ä". On 1 October he published a result on the number of solutions of polynomials with coefficients in finite fields, which 150 years later led to the **Weil conjectures.**

Gauss remained mentally active into his old age, even while suffering from gout and **general unhappiness.** For example, at the age of 62, he taught himself Russian.

In 1840, Gauss published his influential ***Dioptrische Untersuchungen*****,** in which he gave the first systematic analysis on the formation of images under a **paraxial approximation** (Gaussian optics). Among his results, Gauss showed that under a **paraxial approximation** an optical system can be characterized by its **cardinal points** and he derived the **Gaussian lens formula.**

In 1845, he became an **associated member of the Royal Institute of the Netherlands;** when that became the Royal Netherlands Academy of Arts and Sciences in 1851, he joined as a foreign member.

He was elected as a member of the **American Philosophical Society in 1853.**

Gauss was an ardent **perfectionist** and a hard worker. He was never a prolific writer, refusing to publish work which he did not consider complete and above criticism. This was in keeping with his personal motto *pauca sed matura* ("few, but ripe"). His personal diaries indicate that he had made several important mathematical discoveries years or decades before his contemporaries published them

Though he did take in a few students, Gauss was known to dislike teaching.

It is said that he attended only a single scientific conference, which was in Berlin in 1828. Several of his students became influential mathematicians, among them **Richard Dedekind and Bernhard Riemann.**

Gauss usually declined to present the intuition behind his often very

In his 1799 doctorate in absentia, *A new proof of the theorem that every integral rational algebraic function of one variable can be resolved into real factors of the first or second degree*, Gauss proved the **fundamental theorem of algebra** which states that every non-constant single-variable polynomial with complex coefficients has at least one complex root. Mathematicians including Jean le Rond d'Alembert had produced false proofs before him, and Gauss's dissertation contains a critique of d'Alembert's work. Ironically, by today's standard, Gauss's own attempt is not acceptable, owing to the implicit use of the Jordan curve theorem. **However, he subsequently produced three other proofs, the las**t one in 1849 being generally rigorous. His attempts clarified the concept of complex numbers considerably along the way.

Gauss also made important contributions to **number theory** with his 1801 book *Disquisitiones Arithmeticae* (Latin, Arithmetical Investigations), which, among other things, introduced the triple bar symbol a" for congruence and used it in a clean presentation of modular arithmetic, contained the first two proofs of the law of quadratic reciprocity, developed the theories of **binary and ternary quadratic forms,** stated the class number problem for them, and showed that a regular heptadecagon (17-sided polygon) can be constructed with straightedge and compass. It appears that Gauss already knew the class number formula in 1801.

From 1989 through 2001, Gauss's portrait, a **normal distribution curve** and some prominent Göttingen buildings were featured on the German ten-mark banknote

- the **normal distribution,** also known as the **Gaussian distribution, the most common bell curve in statistics;**

- the **Gauss Prize,** one of the highest honors in mathematics;
- **gauss, the CGS unit for magnetic field.**

Writings

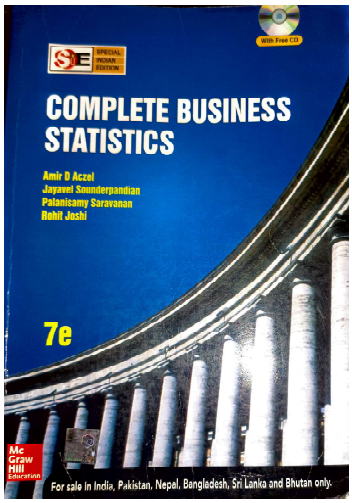
- 1799: Doctoral dissertation on the fundamental theorem of algebra, with the title: *Demonstratio nova theorematis omnem functionem algebraicam rationalem integram unius variabilis in factores reales primi vel secundi gradus resolvi posse* ("New proof of the theorem that every integral algebraic function of one variable can be resolved into real factors (i.e., polynomials) of the first or second degree")
- 1801: *Disquisitiones Arithmeticae* (Latin). A German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second ed.). New York: Chelsea. 1965.* English translation by Arthur A. Clarke *Disquisitiones Arithmeticae (Second, corrected ed.). New York: Springer. 1986.*
- 1808: *"Theorematis arithmetici demonstratio nova". Göttingen: Commentationes Societatis Regiae Scientiarum Gottingensis. 16.* German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second ed.). New York: Chelsea. 1965.*
- 1809: *Theory of the Motion of Heavenly Bodies Moving about the Sun in Conic Sections* (English translation by C.H. Davis), reprinted 1963, Dover, New York.
- *Theoria motus corporum coelestium in sectionibus conicis solem ambientium (in Latin). Hamburg: Friedrich Perthes & Johann Heinrich Besser. 1809.*
- 1811: *"Summatio serierun quarundam singularium". Göttingen: Commentationes Societatis Regiae Scientiarum Gottingensis.* German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second ed.). New York: Chelsea. 1965.*
- 1812: *Disquisitiones Generales Circa Seriem Infinitam* {\displaystyle 1+{\frac {\alpha \beta }{\gamma .1}}+{\mbox{etc.}}}
- 1818: *"Theorematis fundamentalis in doctrina de residuis quadraticis demonstrationes et amplicationes novae". Göttingen: Commentationes Societatis*

Regiae Scientiarum Gottingensis. . German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second ed.). New York: Chelsea. 1965.*

- 1821, 1823 and 1826: *Theoria combinationis observationum erroribus minimis obnoxiae*. Drei Abhandlungen betreffend die Wahrscheinlichkeitsrechnung als Grundlage des Gauß'schen Fehlerfortpflanzungsgesetzes. (Three essays concerning the calculation of probabilities as the basis of the Gaussian law of error propagation) English translation by G.W. Stewart, 1987, Society for Industrial Mathematics.
- 1827: *Disquisitiones generales circa superficies curvas*, Commentationes Societatis Regiae Scientiarum Gottingesis Recentiores. Volume **VI**, pp. 99–146. "General Investigations of Curved Surfaces" (published 1965), Raven Press, New York, translated by J. C. Morehead and A. M. Hiltebeitel.
- 1828: *"Theoria residuorum biquadraticorum, Commentatio prima". Göttingen: Commentationes Societatis Regiae Scientiarum Gottingensis. 6.* . German translation by H. Maser
- 1828: *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second ed.). New York: Chelsea. 1965. pp. 511–533*
- 1832: *"Theoria residuorum biquadraticorum, Commentatio secunda". Göttingen: Commentationes Societatis Regiae Scientiarum Gottingensis. 7.* . German translation by H. Maser *Untersuchungen über höhere Arithmetik (Disquisitiones Arithmeticae & other papers on number theory) (Second ed.). New York: Chelsea. 1965.* [Introduces the Gaussian integers, states the law of biquadratic reciprocity, proves the supplementary law for $1 + i$]
- *"Intensitas vis magneticae terrestris ad mensuram absolutam revocata". Commentationes Societatis Regiae Scientiarum Gottingensis Recentiores.*
- *Resultate aus den Beobachtungen des magnetischen Vereins. Im Jahre 1836 (in German). Göttingen: Dieterichsch Buchhandlung. 1837.*

- *Intensitas vis magneticae terrestris ad mensuram absolutam revocata (in Italian). Milano. 1838.*
- *Resultate aus den Beobachtungen des magnetischen Vereins. Im Jahre 1837 (in German). Göttingen: Dieterichsch Buchhandlung. 1838.*
- *Resultate aus den Beobachtungen des magnetischen Vereins. Im Jahre 1838 (in German). Leipzig: Weidmannsche Verlagsbuchhandlung. 1839.*
- *Resultate aus den Beobachtungen des magnetischen Vereins. Im Jahre 1839 (in German). Leipzig: Weidmannsche Verlagsbuchhandlung. 1840.*
- *Resultate aus den Beobachtungen des magnetischen Vereins. Atlas des Erdmagnetismus nach den Elementen der Theorie enworfen (in German). Leipzig: Weidmannsche Verlagsbuchhandlung. 1840.*
- *Resultate aus den Beobachtungen des magnetischen Vereins. Im Jahre 1840 (in German). Leipzig: Weidmannsche Verlagsbuchhandlung. 1841.*
- *Resultate aus den Beobachtungen des magnetischen Vereins (in German). Leipzig: Weidmannsche Verlagsbuchhandlung. 1843.*
- 1843/44: *Untersuchungen über Gegenstände der Höheren Geodäsie. Erste Abhandlung*, Abhandlungen der Königlichen Gesellschaft der Wissenschaften in Göttingen. Zweiter Band, pp. 3–46
- 1846/47: *Untersuchungen über Gegenstände der Höheren Geodäsie. Zweite Abhandlung*, Abhandlungen der Königlichen Gesellschaft der Wissenschaften in Göttingen. Dritter Band, pp. 3–44
- *Theoria combinationis observationum erroribus minimis obnoxiae (in French). Paris: Mallet-Bachelier. 1855.*
- *Briefwechsel zwischen Gauss und Bessel (in German). Leipzig: Wilhelm Engelmann. 1880.*
- *Demonstratio nova theorematis omnem functionem algebraicam rationalem integram unius variabilis in factores reales primi vel secundi gradus resolvi posse (in German). Leipzig: Wilhelm Engelmann. 1890.*
- *Intensitas vis magneticae terrestris ad mensura absoluta revocata (in Latin). Leipzig: Wilhelm Engelmann. 1894.*

BOOK REVIEW



The objective of writing review is to bring in light some unsung books. One may find the same for **COMPLETE BUSINESS STATISTICS** (7e) authored by (1) Amir D Aczel (2) Jayavel Sounderpandian (3) Palanisamy Saravann and (4) Rohit Joshi. This book is published by Mc. Graw Hill Education and comprised 15 chapters along with more two chapters on the CD.

This book covers almost all topics of basic statistics. Every chapter starts with "Using Statistics" and ends with 'Using the Computer' and real life case study associated with particular chapter.

More or less researchers in Social Sciences, Psychology and other disciplines find difficulties in applying appropriate statistical techniques and drawing conclusions and interpretations. This book might be helpful to them.

Every chapter equipped with sufficient discussion on particular topic and followed by large proportion of the problems and examples concerned with international issues. The success of any book can be measured by how much curiosity it can ignite in its user. This book seems to be succeed in that manner.

The most attracting feature of this book is that it describes significance of particular statistical theory and justification of its application. However OR methods are not included in this book as these are modern business techniques. Incorporation of these techniques certainly make the book augmented.

Date : 31/12/2021

Dr. H.M. Dixit0
(Statistics Department)
Pilvai College (NG)

SV NEWS LETTER

Manish B. Thaker*

Last year the whole world suffered a lot due to COVID-19 pendamic. Let us hope that the coming new year 2022, will convert Pandemic into Endemic.

Corona has also given rise to many new words like Shutdown, Online and Offline, Virtual, Quarantine, Isolation, BioBubble, Sanetizer, Mask, Social Distance, Micro Contentment Zone, variant, Work from home, Webinar etc for our daily vocabulary.

Corona has also pushed for new medical and scientific inventions, world wide. It has also blessed companies for saving huge amount of expenses by means of webinars for annual meetings etc. These may be called blessings in disguise.

We shall focuss here with some brief news only pertaining to our subect matters.

* Indian science congress association conference will take place in PUNE during Jan. 3-7, 2022.
* International conference on Applied Maths and Statistics (ICA SMO) will take place on Jan. 9, 2022 at Aurangabad.
* 20th International conference for research work in Matrices and Statistics (IVMS 2021) took place at Manipal during Dec. 13-15, 2021
* International conference on Recent Advances in computer science and information technology (ICRA CSIT) will be held at New Delhi on Jan. 2, 2022 and many more...
* At our home place, Dr. M. N. Patel, HOD, Stat. Dept. Gujarat University, Ahmedabad retired on 31st Oct. 2021. In his place now Dr. Chetanaben D. Bhavsar is working as HOD from Nov. 1, 2021.
* There are good news about an essay competition to be organized on behalf **Ministry of Statistics and Programme Implemention (New Delhi)** for Statistics Day celebration on 29th June 2022. The details are as located herewtih.

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भारत सरकार/Government of India
सांख्यिकी एवं कार्यक्रम कार्यान्वयन मंत्रालय
Ministry of Statistics & Programme Implementation
प्रशिक्षण इकाई /Training Unit
******

खुर्शीद लाल भवन, जनपथ, नई दिल्ली
Khurshid Lal Bhawan, Janpath, New Delhi
दिनांक/dated: 15th December, 2021

Call for applications for **'On the Spot Essay Writing** Competition-2022 **' for Post**-Graduate Students of Statistics

As part of the celebration of Statistics Day on 29th June, 2022, the Ministry of Statistics & Programme Implementation (MoSPI) will organize an essay writing competition on subjects relevant to the Statistics/Official Statistics. The interested Post-Graduate students of Statistics currently studying in recognized universities/colleges/institutes are invited to participate in 'On the Spot Essay Writing Competition- 2022'.

2. Details of the competition are as given below:

(i) The candidates should submit their application form through email at training-mospi@nic.in as per instructions at Annex. The last date for the receipt of applications will be 24th January, 2022 . The Competition will be held on 20th February, 2022 (Sunday) from 10:00 AM to 1:00 PM.

(ii) The candidates should give two preferences for their center of examination from the drop down list while filling the application form. The Ministry reserves right to allot a center other than opted by the candidate. Each eligible candidate will be asked through a separate communication by email to come to the examination center for writing essay. No TA/DA will be paid for appearing in the competition.

(iii) Two topics of essay would be given on the spot. The essay should be written on one of the given topics in about 5000 words in three hours.

(iv) The essays will be evaluated on the basis of contents, originality of ideas, sequence of presentation and writing skills. The list of winners will be published on Ministry's website. The winners will also be notified in writing.

(v) The winners will be given certificates along with cash prize as given below:

First Prize	: ₹ 15,000/-
Second Prize (2 numbers)	: ₹12,000/- (each)
Third Prize (3 numbers)	: ₹10,000/- (each)
Consolation prizes (5 numbers)	: ₹5,000/- (each)

(vi) The winners of the competition will be invited to a function to be organized by MoSPI to receive the prizes on 29th June, 2022. The winners coming to the place of function will be entitled to receive actual fares of their to and fro journey from their permanent residence or from the university/institute they are studying through shortest route by rail in AC-III tier. Stay arrangements will also be done by the Ministry.

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READERS FORUM

Ashvin J. Patel*

* **Himanshu Desai (Ahmedabad)**
Is it statistics and management or management and statistics ?! (Both !)

* **Rakesh Pandya (Gandhinagar)**
What is the thumb rule for SV Journal ?
(To promote research work from young and energetic researchees. We feel happy and satisfied for our successful efforts)

* **P. Mariappan (Trichy)**
Can we not include articles / papers in applied maths in SVJ ?
(We do have now.)

* **Bhavin Shah (IIM, Indore)**
Why we do not have papers for analytics ?
(Let us begin with yours !)

* **Shailesh Teredesai (Ahmedabad)**
Earlier we had announced about separate topics for different issues in a year. What has happend to that?
(It depends upon the materials submitted in time for relevant issues)

* **Bipin Mehta (Ahmedabad)**
I have complained often to continue print copies for each issue. I feel sorry that this time also it is not considered since 2021.
(We have financial crunch and also administrative shortcomings. Corona is one more factor, so sorry for that !)

* **Pratheeshkumar (Kerala)**
You can invite for more application oriented papers from different fields also like commerce, economics, sociology, pyschology etc. That can enhance the further growth for SVJ.
(Thanks. We have already kept it as an open platform.)

* Head, Statistics Dept., R. H. Patel Arts & Commerce College, Vadaj, Ahmedabad.
EMAIL : ashvinjpatel@gmail.com

* **V. H. Bajaj (Aurangabad)**
Why we do not take articles / papers from agricultural fields where very prominent and useful research work is always important ?
(Sir, we do have. Please submit your materials in time)

* **Dr. Jasmine (Trichi)**
I find articles by A. C. Brahmbhatt always giving new dimensions for research. Similary some papers on data based research as well as survey work are really good. Please ensure to give more focus on application oriented research work.
(Thanks for your feedback and suggestions)

* **Vaishali Halani (Ahmedabad)**
This is now new age and period for Biostatistics, Data Science, Machine Learning, Artificial Intelligence etc. One with sound knowledge and acquaintance with softwares can be more useful and important in the coming days, SVJ can take up this as one of their next initiative step for further growth.
(Thanks, for yur suggestions, we do have this in mind for the coming days.)
(Thanks, for your suggestions, we do have this in mind for the coming days.)

SANKHYA VIGNAN

સંખ્યા વિજ્ઞાન

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EDITORIAL ADVISARY BOARD

RESEARCH TEAM

J. CARL FRIEDRICH GAUSS*

JOHANN CARL FRIEDRICH GAUSS was a German Mathematician cum Physicist. He was born on 20th April 1777 at Brunswick, Germany. He had excellent intellectual abilities right from his childhood which had attracted many dignitaries. He had independently rediscovered several theorems during his university studies and works. He worked at Braunschweig University of Technology during 1792-1795 and at the university of Gottingen from 1795-1798. He had obtained his doctorate degree in Mathematics in 1799. In 1845, he became on **Associated Member of Royal Institute of Netherlands**. He was aldo elected as a **member of American Philosophical Society in 1855.**

Gauss had expired at the age of 77, on Fe. 23, 1855 at Gottingen, Germany. Until hid death, he worked with the same zeal, sprit and vigour. He was awarded **Lalande Prize in 1809 and Copley Medal in 1838.**

Gauss is very well known for his contributions in **Number Theory, Fundamental Theorem of Algebra, Probability and Statistics** etc. He was much acquainted with mathematicians like **Dedekind, Riemann, Markov, J.R.D' Alembert** etc. His very significant work is **GAUSSIAN Distribution** (which is now known as Normal Distribution.)

Gauss Prize is the highest honours in Mathematics. **GAUSS (Units)**, the CGS unit for magnetic field are among his chief achievements.

The Following things are named after GAUSS.

- **GAUSS (Unit)**
- **GAUSSIAN Distribution**
- **GAUSS MARKOV Theorem**
- **GAUSSIAN Elimination**
- **GAUSSIAN Integer**
- **GAUSSIAN Integral**
- **GAUSSIAN Mixture Model**
- **GAUSSIAN Quadrature.**

***Brief biographicasl sketch is given inside the journal.**

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(Journal of GSA, Ahmedabad)

To,

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