PEER REVIEW JOURNAL
FOR
RESEARCH AND READINGS IN
APPLIED STATISTICS

ISSN 2321-0877

SANKHYA VIGNAN
सांख्य विज्ञान

NEW SERIES (NSV 17) JUNE 2021 NO. 2

CONTENTS

Published by
GUJARAT STATISTICAL ASSOCIATION

http://www.sankhyavignan.org

SANKHYA VIGNAN
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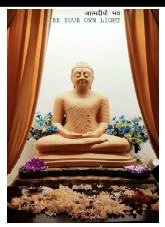

We are extremely happy to present this issue (NSV 17, June 2021, No. 2) to our readers.

This one is our second new venture for publishing our issue in digital form rather than print issue as earlier.

We express our very sincere thanks to all our contributors, evaluators, readers and well wishers for their continious and consistent support which always helps us to achieve our goal.

This issue contains **two papers for the section of Management and Statistics, four research articles, one research study article, one biography, one book review and other sections of SV News letter as well as readers forum**. You will find the details about the same in the contents given on the converpage.

We are highly indebted to our following referees who have spared no nerves for evaluation of the articles / papers submitted fo this issue. **(Names of referees are given one by one in the order of their appearance.)**

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We have our website **www.sankhyavignan.org**, where you can give your feedback and suggestions.

We express our sincere thanks to our **Research Team** for this work. In particular we thank **Shree Dinesh Darji for DTP work and Shree Ashish Bhatt for website.**

We shall forward Digital copy of this issue to all those readers whose email ID/whatsapp Nos are with us.

Our contributors will get digital copy and official certificates. We have made an effort to place this issue in yours hands on this **auspicious statistics day** to celebrate **memory of late Prof. P. C. Mahalanobis.**

Our best wishes and blessings to you all for your health, progress and prosperity on this auspicious day.

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SANKHYA VIGNAN

PEER REVIEWED REFREED BI-ANNUAL JOURNAL (ISSN : 2321-0877)

(Journal of Research and Readings in Applied Statistics)

*** Listed at International ISSN Directory, PARIS**

Average Circulation Rate : 850

Average Journal Evaluation Score : 7.50 (As on 31st March 2021)

PUBLISHED BY GSA

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SANKHYA VIGNAN is a peer reviewed refereed Bi-Annually journal that published empirical, conceptual and review papers of exceptional quality that contribute to Statistics Theory and enriched Applications of Statistical Techniques in various fields. The objective ofthe Journal is to disseminate knowledge, which ensures good practice of professional management and its focal point is on research and reflections relevant to academicians and practitioners in the field of **Applied Statistics.**

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MANAGEMENT AND STATISTICS
ARTICLE

STATISTICS IN BUSINESS ANALYTICS

A. C. BRAHMBHATT*

ABSTRACT

This article describes in breif about different fields related to business applications of statistics. Here modern concepts and ideologies are very well represented in nutshell. The concepts presented here are thought proviking for the researchers.

KEY WORDS

ANALYTICS, VOLUME, VARIETY, VELOCITY, VARIABILITY

STATISTICS IN BUSINESS ANALYSTICS

We live in the digital era and Business Analytics, Machine Learning, Artificial Intelligence, Deep Learning, Robotics and Cloud have almost revolutionalized the way we look, process and utilize information. Gone are the days when we were contended with the collection of demographic, socio-economic, psychographic kind of data , analyzed them and drew inferences. This is an era of Big Data that is characterized by 4 Vs—Volume, Variety, Velocity and Variability. New York Stock Exchange generates about one terabyte of new trade data per day! 500+ terabytes of new data get ingested into Facebook daily ! The ability to make intelligent decision based on such data is critical for both managers and firms world over. Business

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Analytics utilizes Big Data, statistical measures and analytical tools to predict future trends , patterns and outcomes ; helps gain insights and improves strategic decision making .

Business Analytics is a blend of statistical techniques, technologies, computer science, skills and practices to use data to gain insights and make data-driven decisions in future.Business Analytics helps us form hypothesis, while statistics lets you test them. Business Analysts specialize in exploring what is their in data, Statisticians focus more on what is beyond it.Gartner , Inc., world's leading IT research and advisory company defines Business Analytics as '' Business Analytics is comprised of solutions used to build analysis models and simulations to create scenarios, understand realities and predict future states.''

Business Analytics is almost ubiquitous these days. Organizations look up to analytics , using more data to dig deeper insights faster for more people.It is all pervasive across all functional areas of management: Marketing, Finance, Operations and HR.

In Marketing , it is widely used for varieties of purposes such as knowing buying patterns of consumers, understanding their buying behavior and intention, analyzing trends,identifying target audiences, employing advertising techniques that appeal to consumers, building loyal customer base etc.

In Finance it is used in investment banking, portfolio management, financial planning, budgeting, studying trends on the performance of a particular stock and advise client on whether to retain it or sell it etc.

In Operations, it is used in supply chain management, inventory management, risk mitigation plans, improving efficiency on the basis of product data etc.

In HR, it is used in recruitment and selection of employees, reducing employee attrition rate, measuring the performance of the employees to build up a career

advancement scheme etc.

In the three different types of Business Analytics –Descriptive, Predictive and Prescriptive analysis , various statistical measures and analytical techniques are used. For example, in descriptive analysis, measures of central tendency and measures of dispersion , sample size estimation, hypothesis testing etc. are widely used. In Predictive analysis, the techniques like multiple regression, logistic regression, poisson regression, structural equation modeling etc. are used. The Prescriptive analysis , calls for application all Optimizaion techniques —mathematical programming techniques.

Nearly 90% of small, middle and large organizations have set up business analytical capabilities over last 7 years in an effort to stay relevant and vibrant in the market and draw value out of the insights that Big Data recorded in the digital age can furnish.

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MANAGEMENT AND STATISTICS
RESEARCH ARTICLE

CO-OPERATIVE SOCIETIES AND THEIR ROLE TO ACCOUNTABILIGY OF MICRO FINANCE FOR PERCEIVING INTEREST FREE CONCEPT

PRATHEEPKUMAR R.[1], SOMESEKHARAN T.M.[2] AND R. ANANTHI[3]

ABSTRACT

The co-operative based microfinance is rare and may endow with remedy to the existing tribulations in the microfinance industry. Co-operative model mostly trust on the members' own savings as the source of funds fills a major gap found in the case of other models. A financing system based on profit or loss sharing with the micro- entrepreneurs should be able to provide interest free loan, while still leaving a sufficient amount in the reserves to demonstrate as profit. The paper is looking into co-operative society's role in micro finance and role of interest and Interest free.

KEYWORDS

INTEREST FREE, CO-OPERATIVE SOCIETY, SUSTAINABLE DEVELOPMENT.

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Banking the deprive people have a right to be overhaul affordably, appropriately and conveniently, comprehend the UN Sustainable Goal campaign

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(rcd. April21 / rvd. May21)

named end poverty. The microfinance has emerged as a key line of attack in accomplishing the Sustainable Development Goals and as an effective tool in the provision of financial services to down-trodden. The claims of awe-inspiring success of microfinance in financial inclusion and poverty alleviation, the microfinance sector in India has faced some critical harms highlighting shortcomings in the institutional or well-designed framework.

The reports recommend that the efforts of the existing microfinance institutions be bound for mainly towards growth and outreach. The industry also lacks rigorous analysis of the institutional, incentive features and benefit-cost or statistical studies of impacts tends to be prejudiced towards women, lack features of fund mobilization. The funding of the crisis in Andhra Pradesh reveal similar harms like lack of credit discipline, rapid expansion of credit in highly concentrated markets, lack of proper incentives for sound underwriting, lack of internal controls, reliance on credit-only services, and dependence on basic bank debt.

The single biggest predicament with conformist microfinance, and for that substance all interest-based finance, is that the borrower has to make his interest payments even if he is unable to meet them. At a time when a young business should be concerned with innovation and expansion, interest payments loom manifestly large at the end of the month. Further, interest based transactions tend to focus attentions on the process-oriented task of repayment rather than on the result- oriented task of increasing profit. Loans are taken for personal consumption or to meet special purpose. The fundability of money, which is routinely overlooked by microfinance proponents, blurs the distinction between investment and consumption borrowing. This results in increased problems for the poor rather than alleviating it.

The reason for the inveterate tribulations of same nature across the industry is that a majority of microfinance models the same pattern or a distorted method of cluster lending. Recently David Hume coined that the industry must make every “effort to keep its costs as low as possible, expand incrementally and focus

as much on savings services as on loans". The above pattern is deeply embedded in the co-operative structure.

The financial co-operatives habitually non-profit and endow with financial services: savings, checking accounts, loans, insurance, and fund transfer services (although the weaker and smaller ones are not capable of offering transfers). Cooperatives generally have higher levels of cost recovery and fund a greater proportion of their loan portfolios with mobilized deposits than another models followed by NGO. The co-operative society structure helps to provide supervision, liquidity management, refinancing, and/or technical support, and much effective than other models or difficult to available in local level. Small financial cooperatives naturally have the legal and institutional capacity to provide a wider range of more flexible financial services and the membership in a federation provides them good liquidity pool than other model

Al Khair Cooperative model allows the member to engage in participatory finance. The system of participatory finance permits the members to make a decision on finance and related financial products especially on liquidity, interest, and cost of loans. This model allows great scope of innovation in of financial products and gives space for experiment. The members can take a call on interest free borrowings, lending, Innovative financial products like micro-ventures is to be introduced, and the profit or loss sharing is used for cross collapsing the transaction cost incurred in the consumption loans.

The co-operative composition offers the obligatory institutional structure for cuisine the needs of deprive people. The indeed risks of non-repayment, due to business failures but the same exists in any collateral free loan lest the institution resorts to any illegal means. The interest free deposits given by members would be a boon for the microfinance institutions, that do not have any other option but to charge a higher interest rate from the poor clients to meet their cost. There is a strong need to study such institutions for more detailed insights and the potential for their mainstreaming. Positively, the interest free co-

operative microfinance may endow with the much required vary the industry is fervently waiting for the big change.

ACKNOWLEDGEMENTS

Wet thank the referee for reviewing our paper which has helped us in finally revising our earlier draft.

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RESEARCH ARTICLE

A MATHEMATICAL MODEL FOR CORONA VIRUS DISEASE (COVID-19) CONTAINING ISOLATION CLASS WITH INCIDENCE RATE

D. JASMINE*

ABSTRACT

In the absence of effective vaccine/antiviral strategies for reducing the burden of the coronavirus disease 2019 (COVID-19) pandemic in India, the main focus has been on basic non-pharmaceutical interventions (NPIs), such as nationwide lockdown (travel restrictions and the closure of schools, shopping malls, and worshipping and other gathering places), quarantining of exposed individuals, and isolation of infected individuals. Nationwide lockdown with high efficiency can diminish COVID-19 cases drastically, but combined NPIs may accomplish the strongest and most rapid impact on the spreading of COVID-19 in India.

The deadly coronavirus continues to spread across the globe and mathematical models can be used to show suspected, recovered, and deceased coronavirus patients, as well as how many people have been tested. Here, we develop a mathematical model to present the dynamical behavior of COVID-19 infection by incorporating isolation class. First, the formulation of model is proposed; then, positivity of the model is discussed. The local stability and global stability of proposed model are presented, which depended on the basic reproductive R_0. For $R_0<1$, the system has a globally asymptotically stable disease free equilibrium, while for $R_0>1$, the system has one unstable disease free equilibrium and a unique locally stable endemic equilibrium. For the numerical solution of the proposed model, the nonstandard finite

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(rcd. April '21/ rvd. June '21)

difference (NSFD) scheme and Runge-Kutta fourth order method are used. Our findings show that human to human contact is the potential cause of outbreaks of COVID-19. Therefore, isolation of the infected human overall can reduce the risk of future COVID-19 spread.

KEYWORDS

Covid-19, Endemic, Basic Reproduction Number, Asymptotically Stable, Virus Free Equilibrium, Isolation Class, Incidence Rate.

1. INTRODUCTION

Coronavirus disease is likely to emerge as a watershed moment in the history of the planet. COVID-19, the abbreviation of coronavirus disease (2019), is caused by a severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) [1], which hit the globe with a bang. In December 2019, the first outbreak was noticed in Hubei province, Wuhan, China [2]. On 30 January, 2020, the World Health

Organization (WHO) revealed the COVID-19 to be a public health emergency and identified it as a pandemic on 11 March, 2020. The symptoms of COVID-19 are not specific, and many cases showed that an infected person might be asymptomatic. The majority of the cases have two common symptoms which include dry cough (68%) and fever (88%). Some of the cases have symptoms that include fatigue, muscle and joint pain, respiratory sputum production (phlegm), sore throat, loss of the sense of smell, headache or chills, and the shortness of breath. Moreover, the growth of this infection can further proceed to acute respiratory distress syndrome, severe pneumonia, and death. The COVID-19 virus spreads to large extent between people in close contact with each other (within approximately 2 m). The common incubation period ranges from 1 to 14 days [3]. In the absence of a definitive treatment modality like a vaccine, physical distancing has been accepted globally as the most efficient strategy for reducing the severity of disease and gaining control over it. The

concealment of physical contact in working environments, schools and other open circles is the objective of such preventive measures.

Researchers have been tracking the spread of the virus, have mobilized to speed innovative diagnostics, and are working on a number of vaccines to protect against COVID-19. Cao et al. [6, 7] studied the clinical features of coronavirus and discussed the short-term outcomes of 18 patients and 102 patients with COVID-19 in intensive care units. Coronaviruses are typically transmitted from person to person through respiratory droplets and close contact. The majority of the transmission is happening through respiratory droplets that we may inhale from close contact with one another. Nesteruk [4] developed an SIR (susceptible, infected, and recovered) epidemic model and discussed statistically the parameters used in the proposed model and showed how to control this infection. A modified SIR epidemic model is presented in [5] to project the actual number of infected cases and the specific burdens on isolation wards and intensive care units. Further research should focus on updating the predictions with the use of up-to-date data and using more complicated mathematical models. The virus can easily spread in dense places. Social distancing or low contact rate refers to measures that are taken to increase the physical space between people to slow down the spread of the virus. Several researchers developed different models of COVID-19 and studied dynamical behaviour. From the above discussion, it was concluded that human to human contact is the potential cause of outbreaks of COVID-19. Therefore, isolation of the infected human overall can reduce the risk of future COVID-19 spread. Ming et. al. [9] developed the mathematical modelling for controlling the novel coronavirus outbreak in Wuhan, China. Okhuese [10] discussed the mathematical predictions for coronavirus as a global pandemic.

In order to do this, we divided the total population into five compartments: susceptible, exposed, infected, isolated, and recovered from the disease. The author Anwar Zeb et. al. [11] formulated a Mathematical Model for Corona virus Disease 2019 (COVID-19) Containing Isolation Class. Here we consider a mathematical model

for corona virus containing isolation class with incidence rate. This study will lead to the mathematical model formulation in which the interaction of the exposed population and infected population occurred to the susceptible populations. The infected individuals, the individuals showing no symptoms apparently but have the disease in weak form inside their bodies, must be sent to isolated class in different rates. The local stability and global stability of model is discussed, by using the approach of the basic reproductive. For the numerical solution of the proposed model, the nonstandard finite difference (NSFD) scheme and Runge-Kutta fourth order method are used. Finally, some graphical results are presented. Our findings show that human to human contact is the potential cause of outbreaks of COVID-19.

The paper is organised as follows: Section 2 is related to the model formulation keeping in mind the assumptions that exposed and infected individuals are making contacts with susceptible individuals at the same rate. Section 3 is concerned with the local stability and existence of positive equilibrium solution. Some numerical simulations are executed to illustrate the analytical results in Section 4. Finally, conclusions are presented in section 5.

2. MODEL FORMULATION

In this section, we develop a mathematical model for corona virus containing isolation class with incidence rate. The proposed model is

$$\begin{aligned}
\frac{dS(t)}{dt} &= A - \mu S(t) - \beta(N)S(t)E(t) - \frac{\delta(N)S(t)I(t)}{1+\alpha I},\\
\frac{dE(t)}{dt} &= \beta(N)S(t)E(t) + \frac{\delta(N)S(t)I(t)}{1+\alpha I} - \pi E(t) - (\mu+\gamma)E(t)\\
\frac{dI(t)}{dt} &= \pi E(t) - \sigma I(t) - \mu I(t),\\
\frac{dQ(t)}{dt} &= \gamma E(t) + \sigma I(t) - \theta Q(t) - \mu Q(t),\\
\frac{dR(t)}{dt} &= \theta Q(t) - \mu R(t),
\end{aligned} \tag{1}$$

where the parameters are similar to Table 1 [1]. Here we have considered different rate at which the susceptible population moves to infected (δ).

As the first four equations are independent of R(t), without loss of generality we modify the system as follows:

$$\frac{dS(t)}{dt} = A - \mu S(t) - \beta(N)S(t)E(t) - \frac{\delta(N)S(t)I(t)}{1+\alpha I},$$

$$\frac{dE(t)}{dt} = \beta(N)S(t)E(t) + \frac{\delta(N)S(t)I(t)}{1+\alpha I} - \pi E(t) - (\mu+\gamma)E(t),$$

$$\frac{dI(t)}{dt} = \pi E(t) - \sigma I(t) - \mu I(t),$$

$$\frac{dQ(t)}{dt} = \gamma E(t) + \sigma I(t) - \theta Q(t) - \mu Q(t). \qquad (2)$$

Let $N = A/\mu$, $s = S/N$, $e = E/N$, $i = I/N$ and $q = Q/N$ and let us rescale the system (2) to get the normalised form as follows:

$$\frac{ds}{dt} = \mu - \mu s - \beta N s e - \frac{\delta N s i}{1+\alpha i} \qquad (3)$$

$$\frac{de}{dt} = \beta N s e + \frac{\delta N s i}{1+\alpha i} - \pi e - (\mu+\gamma)e \qquad (4)$$

$$\frac{di}{dt} = \pi e - \sigma i - \mu i \qquad (5)$$

$$\frac{dq}{dt} = \gamma e + \sigma i - \theta q - \mu q \qquad (6)$$

with the initial conditions,

$$s(0) = s_0 \geq 0, \quad e(0) = e_0 \geq 0, \quad i(0) = i_0 \geq 0,$$

$$q(0) = q_0 \geq 0, \qquad (7)$$

Theorem 1 : Under the initial conditions (7), all the solutions $(s(t), e(t), i(t), q(t))$ of system (3) remain nonnegative for $t \geq 0$

Proof: By the initial conditions (7), it is found that

$$\left.\frac{ds}{dt}\right|_{s=0} = \mu > 0$$

$$\left.\frac{de}{dt}\right|_{e=0} = \frac{\delta Nsi}{1+\alpha i} \geq 0$$

$$\left.\frac{di}{dt}\right|_{i=0} = \pi e \geq 0$$

$$\left.\frac{dq}{dt}\right|_{q=0} = \gamma e + \sigma i \geq 0 \qquad (8)$$

3. LOCAL STABILITY AND EXISTANCE OF POSITIVE EQUILIBRIUM POINT

The existence of unique positive equilibrium and stability of system (3) depends on the basic reproductive number $\Re_0$ on free equilibrium point (FEP) C_0, which is determined with the help of the next generation matrix method [2]. The free corona virus equilibrium point is $C_0 = (s, e, i, q) = (1,0,0,0)$.

Consider the following matrices for finding $\Re_0$:

$$F = \begin{pmatrix} \beta Nse + \dfrac{\delta Nsi}{1+\alpha i} \\ 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \pi e + (\mu+\gamma)e \\ -\pi e + (\sigma+\mu)i \end{pmatrix} \qquad (9)$$

Now Jacobian of F and V at C_0 are

$$F = \begin{pmatrix} \beta N & \beta N \\ 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} \pi+\mu+\gamma & 0 \\ -\pi & \sigma+\mu \end{pmatrix} \qquad (10)$$

Thus, we can get the value of FV^{-1}.

$$FV^{-1} = \begin{pmatrix} \beta N & \beta N \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dfrac{1}{(\pi+\mu+\gamma)} & 0 \\ \dfrac{\pi}{(\sigma+\mu)(\pi+\mu+\gamma)} & \dfrac{1}{(\sigma+\mu)} \end{pmatrix}$$

$$\therefore FV^{-1} = \begin{pmatrix} \dfrac{(\sigma+\mu)\beta N + \pi\delta N}{(\sigma+\mu)(\pi+\mu+\gamma)} & \dfrac{\delta N}{(\sigma+\mu)} \\ 0 & 0 \end{pmatrix}$$

Thus, the dominant eigen value of FV^{-1} represents $\Re_0 = \rho(FV^{-1})$, which is given by $\Re_0 = \frac{(\sigma+\mu)\beta N + \pi\delta N}{(\sigma+\mu)(\pi+\mu+\gamma)}$ (11)

Theorem 2 : The system (3) is locally stable related to virus-free euilibrium point C_0, $\Re_0 < 1$ and unstable if $\Re_0 > 1$.

Proof: For local stability at C_0, the Jacobian of system (3) is

$$J = (E_0) = \begin{pmatrix} -\mu & -\beta N & -\delta N & 0 \\ 0 & \beta N - (\pi+\mu+\gamma) & \delta N & 0 \\ 0 & \pi & -(\sigma+\mu) & 0 \\ 0 & \gamma & \sigma & -(\theta+\mu) \end{pmatrix} \quad (12)$$

which follows the eigen values, $\lambda_2 < 0$, $\lambda_3 < 0$ and $\lambda_4 < 0$, if $\Re_0 < 1$. So the system (3) is locally stable if $\Re_0 < 1$ and unstable if $\Re_0 > 1$.

Theorem 3 : There exists a unique positive virus equilibrium point $C^*(s^*, e^*, i^*, q^*)$ for system (3), if $\Re_0 > 1$.

Proof: By equating the rights hand side of the equations of system (3) to zero, we get

$$\begin{aligned} &\mu - \mu s - \beta N s e - \frac{\delta N s i}{1+\alpha i} = 0 \\ &\beta N s e + \frac{\delta N s i}{1+\alpha i} - \pi e - (\mu+\gamma)e = 0 \\ &\pi e - \sigma i - \mu i = 0 \\ &\gamma e + \sigma i - \theta q - \mu q = 0 \end{aligned} \quad (13)$$

implies that

$$s^* = \frac{\pi\mu(1+\alpha i^*)}{\pi\mu(1+\alpha i^*) + \beta N(1+\alpha i^*)(\sigma+\mu)i^* + \pi\delta\beta N i^*}$$

$$e^* = \frac{(\sigma+\mu)}{\pi} i^*$$

$$i^* = \mu\left[\frac{[K_2 - K_1K_3] \pm \sqrt{(K_1K_3 - K_2)^2 - 4\frac{K_1K_2}{\mu}K_4}}{2K_1K_3}\right]$$

$$q^* = \frac{\gamma(\sigma+\mu)+\pi\sigma}{\pi(\theta+\mu)} i^* \tag{14}$$

where,

$$K_1 = \frac{(\sigma+\mu)(\pi+\mu+\gamma)}{\pi},$$

$$K_2 = \frac{\alpha\beta N(\sigma+\mu)}{\pi\mu},$$

$$K_3 = \frac{\alpha\mu\pi+\beta N(\sigma+\mu)+\delta N}{\pi\mu},$$

$$K_4 = \frac{\pi\delta N+\beta N(\sigma+\mu)}{\pi}$$

From the value of i^*, it is obvious that all the values of (s^*, e^*, q^*) are positive if $\Re_0 > 1$.

Theorem 4 : If $\Re_0 < 1$, then the system (3) globally stable.

Proof: Let us construct the Lyapunov function L as

$$L = (e - e_0) + \frac{\beta N+\delta N}{(\sigma+\mu)}(i - i_0) \tag{15}$$

Differentiating with respect to time, we get

$$L' = e' + \frac{\beta N+\delta N}{(\sigma+\mu)} i',$$

$$L' = \beta N s e + \frac{\delta N s i}{1+\alpha i} - (\pi+\mu+\gamma)e$$

$$+\frac{\beta N+\delta N}{(\sigma+\mu)}(\pi e - \sigma i - \mu i)$$

$$L' = \beta N s e + \frac{\delta N s i}{1+\alpha i} - (\pi+\mu+\gamma)e$$

$$+\frac{\beta N+\delta N}{(\sigma+\mu)}(\pi e) - (\beta N+\delta N)i$$

$$L' \le \beta N e + \frac{\pi\delta N e}{(\sigma+\mu)} + \frac{\pi\beta N e}{(\sigma+\mu)} - (\pi+\mu+\gamma)e + \frac{\delta N s i}{1+\alpha i} - (\beta N+\delta N)i$$

$$= (R_0 - 1)e - \frac{\beta N}{(\pi+\mu+\gamma)} i + \frac{\delta N s i}{1+\alpha i} - (\beta N+\delta N)i$$

$$\therefore L' \le (R_0 - 1)e \tag{16}$$

Therefore, if $R_0 < 1$, then $L' < 0$, which implies that the system (3) is globally stable or $R_0 < 1$.

4. NUMERICAL SIMULATIONS

We have

$$\Re_0 = \frac{(\sigma+\mu)\beta N + \pi\delta N}{(\sigma+\mu)(\pi+\mu+\gamma)}$$

Let the parametric values be $\gamma = 0.01$, $\sigma = 0.02$, $\mu = 0.02$, $N = 1.02$ and giving different values of β, π and δ, we obtain the following table.

Table 1: Simulation of R_0

Valueof of β	Valueof of π	Valueof of δ	R_0	
			Old Model	Proposed Model
0.08	0.03	0.05	2.3800	1.9975
0.06	0.03	0.03	1.8360	1.5300
0.03	0.02	0.02	0.9180	0.8160
0.02	0.01	0.01	0.6375	0.5738
0.01	0.01	0.005	0.3188	0.2869

From the numerical table, we conclude that the proposed model gives the better value than the model of [11].

5. CONCLUSIONS

In this paper, we proposed the pandemic problem of the COVID-19. Also this paper is devoted to implement the coronavirus mathematical model containing isolation class with incidence rate. Our model shows that the coronavirus spreads through contact and describes how fast something changes by counting the number of people who are infected and the likelihood of new infections. Those new infections are what induce the epidemic. For this reason, we think that this research may lead to better guessing of the spread of this pandemic in the future.

In this paper, the incidence rate acts as a controlling factor. The control measures suggest that the infection will be eradicated rapidly once the control strategies will be implemented in a true manner. The reproductive number R_0 related stability is discussed, which showed the impact of interaction of infected people to susceptible

population and proved numerically that if we control this contact rate, the control of the current disease is possible, otherwise.

State and territory governments have different restrictions in place for public gatherings. Therefore, citizens need to follow the directions from time to time to minimize the health risk. In the future study, the dynamics of COVID-19 pandemic along with the effect of some control measures by including more classes into the present model will be proposed. These compartments include symptomatic but not traced population, asymptomatic and quarantined individuals. The model will be an extended version of the present model and proposed using memory features and non-locality.

6. ACKNOWLEDGEMENTS

I thank Dr. P. Maiappan for his help in preparing this paper. I also thank the referee for reviewing this paper referee which has helped to revise it as needed.

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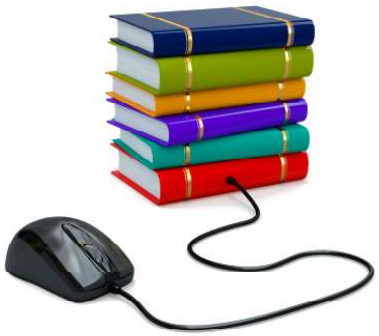

RESEARCH ARTICLE

A MODIFIED EPIDEMIOLOGICAL MODEL AND ANALYSIS OF COVID-19 PANDEMIC.

F. YOSHUVA GENESIS[1] AND D. JASMINE[2]

ABSTRACT

Susceptible – Exposed – Infected – Recovered – Susceptible (SEIRS) mathematical model for COVID 19 pandemic is formulated and analyzed by many authors. In this paper, a modified SEIR model for COVID 19 is proposed. The positivity, boundedness of solutions of the model are proved. The local stability and global stability of the endemic equilibrium points are proved in the analysis part. The basic reproduction number (R_0) for the Novel corona virus pandemic is computed. It is observed that if the basic reproduction number (R_0) is less than one, then the number of cases decrease over time and eventually the disease dies out. If R_0 is greater than one, then the number of cases increase over time gets worth. On the other hand, if R_0 is equal to one, then the number of cases is stable. Numerical simulations are calculated to illustrate analytical results.

KEYWORDS

COVID 19 PANDEMIC, REPRODUCTION NUMBER, SIMULATION, LOCAL STABILITY, GLOBAL STABILITY.

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(rcd. April '21 / rvd. May '21)

1. INTRODUCTION

Coronavirus disease (COVID-19) is an infectious disease caused by a newly discovered coronavirus. Most people infected with the COVID-19 virus will experience mild to moderate respiratory illness and recover without requiring special treatment. Older people, and those with underlying medical problems like cardiovascular disease, diabetes, chronic respiratory disease, and cancer are more likely to develop serious illness. The best way to prevent and slow down transmission is to be well informed about the COVID-19 virus, the disease it causes and *how it spreads*. Protect yourself and others from infection by washing your hands or using an alcohol-based rub frequently and not touching your face. The COVID-19 virus spreads primarily through droplets of saliva or discharge from the nose when an infected person coughs or sneezes, so it is important that you also practice respiratory etiquette (for example, by coughing into a flexed elbow). Mathematical models are useful to understand the behaviour of an infection when it enters a community and investigate under which conditions it will be wiped out or continued. Currently, COVID-19 is of great concern to research, governments, and all people because of the high rate of the infection spread and the significant number of deaths that occurred. In December 2019, coronavirus first reported in Wuhan, China. Researchers have been tracking the spread of the virus, have mobilized to speed innovative diagnostics, and are working on a number of vaccines to protect against COVID – 19. COVID-19 (SARS-CoV-2) is rapidly spreading in South Asian countries, especially in India. India was the fourth most COVID-19 affected country at present i.e., until July 10, 2020. With limited medical facilities and high transmission rate, the study of COVID-19 progression and its subsequent trajectory needs to be analysed in India. Epidemiologic mathematical models have the potential to predict the epidemic peak of COVID-19 under different scenarios. *Lockdown* is one of the most effective mitigation policies adopted worldwide to control the transmission rate of COVID-19 cases. In this study,

we use an improvised four compartment mathematical model, i.e., Susceptible (S)-Exposed (E)-Infected (I)-Recovered (R) (SEIR) to investigate the progression of COVID-19 and predict the epidemic peak under the impact of lockdown in India. The aim of this study is to provide a more precise prediction of epidemic peak and to evaluate the impact of lockdown on epidemic peak shift in India.

2. MATHEMATICAL MODEL FORMULATION

The mathematical model (SEIR) is constructed as follows. The total population is divided into four compartments.

- The class of people who are capable of becoming infected is called Susceptible class, denoted by S.
- The class of people who are infected but not infectious and waiting for a short period of time is called Exposed class, denoted by E.
- The class of people who are infected with the disease and also infectious is called Infected class, denoted by I.
- The class of people who are recovered from the infectious disease is called Recovered class, denoted by R.

The mathematical model is formulated based on the following assumptions.

I. The size of the total population is assumed to be constant. That is, $N = S(t) + E(t) + I(t) + R(t)$.

II. Both the number of births and deaths may not be equal, and the population is well mixed.

III. All parameters in the model are positive.

IV. The Exposed class has short incubation period and not yet infective but moved to infective class at a rate σ.

V. All types of population suffer natural mortality rate at a rate λ.

Variables	*Description*
S(t)	Number of susceptible individuals at time
E(t)	Number of Exposed individuals at time
l(t)	Number of infected individuals at time
R(t)	Number of recovered individuals at time

Parameter	*Description*
μ λ	Natural birth, death rates respectively
α	Infection rate or Transmission rate
λ	Recovery rate
σ	Latency rate

Based on the assumptions, the notations, parameters, the system of ordinary differential equations are formulated as follows:

$$\frac{dS}{dt} = \mu N - \alpha SI - \lambda S \quad (1)$$

$$\frac{dE}{dt} = \alpha SI - \sigma E - \lambda E \quad (2)$$

$$\frac{dI}{dt} = \sigma E - \gamma I - \lambda I \quad (3)$$

$$\frac{dR}{dt} = \gamma I - \lambda R \quad (4)$$

with initial conditions, $S(0) \geq 0, E(0) \geq 0, I(0) \geq 0, R(0) \geq 0$.

3. MATHEMATICAL MODEL ANALYSIS

In this section, the analysis of the above model is presented. The analysis consists of the positivity of solutions of the model, Boundedness of solutions of the model, and the equilibrium points of the model.

To prove this model is biologically valid, it is required to show that the solutions of the system of ordinary differential equations are positive and bounded for all time.

Theorem 1: *Solutions of the model equations 1-4 together with the initial*

conditions, $S(0) \geq 0, E(0) \geq 0, I(0) \geq 0, R(0) \geq 0$ *are always positive. That is, the model variables are positive for all* t *and will remain in* R_+^4.

Proof

The model equation (1) given by $\frac{dS}{dt} = \mu N - \alpha SI - \lambda S$. Since μN is positive, this equation can be expressed without loss of generality, after eliminating the positive terms, as an inequality $\frac{dS}{dt} \geq -(\lambda + \alpha I)S$. It implies that,

$$\frac{dS}{S} \geq -(\lambda + \alpha I)dt$$

Integrating,

$$S(t) \geq e^{-(\lambda+\alpha I)t}$$

The exponential function is always nonnegative irrespective of the sign of the exponent. Hence $S(t) \geq 0$.

The model equation (2) given by $\frac{dE}{dt} = \alpha SI - \lambda E - \sigma E$. Since αSI is positive, this equation can be expressed without loss of generality, after eliminating the positive terms, as an inequality $\frac{dE}{dt} \geq -(\lambda + \sigma)E$. It implies that,

$$\frac{dE}{E} \geq -(\lambda + \sigma)dt$$

Integrating,

$$E(t) \geq e^{-(\lambda+\sigma)t}$$

The exponential function is always nonnegative irrespective of the sign of the exponent. Hence $E(t) \geq 0$.

The model equation (3) given by $\frac{dI}{dt} = \sigma E - \gamma I - \lambda I$. Since σE is positive, this equation can be expressed without loss of generality, after eliminating the positive terms, as an inequality $\frac{dI}{dt} \geq -(\lambda + \gamma)I$. It implies that,

$$\frac{dI}{I} \geq -(\lambda + \gamma)dt$$

Integrating,

$$I(t) \geq e^{-(\lambda+\gamma)t}$$

The exponential function is always nonnegative irrespective of the sign of the exponent. Hence $I(t) \geq 0$.

The model equation (4) given by $\frac{dR}{dt} = \gamma I - \lambda R$. Since γI is positive, this equation

can be expressed without loss of generality, after eliminating the positive terms, as an inequality $\frac{dR}{dt} \geq -(\lambda)R$. It implies that,

$$\frac{dR}{R} \geq -(\lambda)dt$$

Integrating,

$$R(t) \geq e^{-(\lambda)t}$$

The exponential function is always nonnegative irrespective of the sign of the exponent. Hence $R(t) \geq 0$.

Theorem 2: *The positive solutions of the system of model equations 1-4 are bounded. That is the variables are bounded for all* t.

Proof

Each compartment is bounded if and only if the total population size is bounded. It is enough to prove that the total population size $N = S(t) + E(t) + I(t) + R(t)$ is bounded for all t. The derivative of total population with respect to time t is given by,

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dR}{dt}$$

The summation of all the four model equations is,

$\frac{dN}{dt} = \mu N - \alpha SI - \lambda S + \alpha SI - \sigma E - \lambda E + \sigma E - \gamma I - \lambda I + \gamma I - \lambda R$. This can be simplified as,

$$\frac{dN}{dt} = \mu N(t) - \lambda N(t)$$

That is, $\frac{dN}{N} = (\mu - \lambda)dt$. Integrating, $N(t) = e^{(\mu-\lambda)t}$. If $\mu > \lambda$, then it is bounded. Clearly, in any population the birth rate is higher than the death rate. Hence the model variables are bounded for all t.

DISEASE FREE EQUILIBRIUM POINT

Disease free equilibrium points are steady state solutions where there is no disease in the population. This implies that, $I(t) = R(t) = 0$. Thus form equation (1), $\mu N - \lambda S = 0$. It implies that., $S = \frac{\mu N}{\lambda}$. Thus the disease-free equilibrium point of the model

is given by,

$$E(S,E,I,R) = (\frac{\mu N}{\lambda}, 0,0,0)$$

ENDEMIC EQUILIBRIUM POINT

The Endemic equilibrium point $E^*(S^*, E^*, I^*, R^*)$ in the feasible region is a steady state solution where the disease persists in the population. It is obtained by setting the rates of change of variables with respect to time in the model to zero. That is, setting

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dR}{dt} = 0$$

Therefore, the model equations can be written as a system of nonlinear equations.

$$\mu N - \alpha SI - \lambda S = 0 \quad (5)$$

$$\alpha SI - \sigma E - \lambda E = 0 \quad (6)$$

$$\sigma E - \gamma I - \lambda I = 0 \quad (7)$$

$$\gamma I - \lambda R = 0 \quad (8)$$

Solving equations (7) and (8), will give the following expressions $I = \frac{\sigma}{\gamma+\lambda}E$ and $R = \frac{\gamma\sigma}{\lambda(\gamma+\lambda)}E$. Now substituting this in (6) will give the following expression,

$$S^* = \frac{\lambda(\sigma+\lambda)(\gamma+\lambda)}{\alpha\gamma\sigma}$$

Now substituting S^* in (5) will give the following expression,

$$E^* = \frac{\lambda(\gamma+\lambda)}{\sigma\alpha} - \frac{\mu N\gamma}{\lambda(\sigma+\lambda)}$$

Now substituting E^* in (7) will give the following expression,

$$I^* = \frac{\lambda}{\alpha} - \frac{\sigma\mu\gamma N}{\lambda(\sigma+\lambda)(\gamma+\lambda)}$$

Now substituting I^* in (8) will give the following expression,

$$R^* = \frac{\gamma}{\alpha} - \frac{\gamma^2\mu\sigma N}{\lambda^2(\sigma+\lambda)(\gamma+\lambda)}$$

Therefore, the endemic equilibrium point is given by,

$$E^*(S^*, E^*, I^*, R^*) = \left(\frac{\lambda(\sigma+\lambda)(\gamma+\lambda)}{\alpha\gamma\sigma}, \frac{\lambda(\gamma+\lambda)}{\sigma\alpha} - \frac{\mu N\gamma}{\lambda(\sigma+\lambda)}, \frac{\lambda}{\alpha} - \frac{\sigma\mu\gamma N}{\lambda(\sigma+\lambda)(\gamma+\lambda)}, \frac{\gamma}{\alpha} - \frac{\gamma^2\mu\sigma N}{\lambda^2(\sigma+\lambda)(\gamma+\lambda)} \right)$$

BASIC REPRODUCTIVE NUMBER

The basic reproduction number R_0 is the most crucial parameter in the SEIRD mathematical model. It determines how the disease is transmitting over the population during a particular time interval. The value of $R_0 > 1$ implies the disease is in outbreak state, $R_0 = 1$ indicates that the disease is still in system but in a stable state while $R_0 < 1$ represents the disease has diminished completely. In practice, the effective reproductive number R (t) measures the number of secondary cases generated by an infectious case once an epidemic is ongoing. To get the general reproduction number for the formulated model under the discussion of primary results, we use next generation matrix method.

Let $x = (E, I) = (x_1, x_2)$ and $\mathcal{F} = \begin{bmatrix} \alpha SI \\ 0 \end{bmatrix}$ and $\mathcal{V} = \begin{bmatrix} (\sigma+\lambda)E \\ -\sigma E + (\gamma+\lambda)I \end{bmatrix}$.

$$\frac{dx}{dt} = \mathcal{F}(x,y) - \mathcal{V}(x,y) = \begin{bmatrix} \alpha S x_2 \\ 0 \end{bmatrix} - \begin{bmatrix} (\sigma+\lambda)x_1 \\ -\sigma x_1 + (\gamma+\lambda)x_2 \end{bmatrix}$$

Now, $F = \begin{bmatrix} \frac{\partial \mathcal{F}_1}{\partial x_1} & \frac{\partial \mathcal{F}_1}{\partial x_2} \\ \frac{\partial \mathcal{F}_2}{\partial x_1} & \frac{\partial \mathcal{F}_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0 & \alpha N \\ 0 & 0 \end{bmatrix}$.

Now, $V = \begin{bmatrix} \frac{\partial \mathcal{V}_1}{\partial x_1} & \frac{\partial \mathcal{V}_1}{\partial x_2} \\ \frac{\partial \mathcal{V}_2}{\partial x_1} & \frac{\partial \mathcal{V}_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} (\sigma+\lambda) & 0 \\ -\sigma & (\gamma+\lambda) \end{bmatrix}$.

Computing FV^{-1} will give the matrix, $K = \begin{bmatrix} \frac{N\alpha\sigma}{(\sigma+\lambda)(\gamma+\lambda)} & \frac{\alpha}{\gamma+\lambda} \\ 0 & 0 \end{bmatrix}$.

- K_{12} is the expected number of secondary infections produced in exposed compartment by an individual initially in infected compartment over the course of its infection.
- K_{11} is the expected number secondary infections produced in exposed compartment by an infected individual originally in exposed compartment.

- The eigen values of the matrix K are $0, \frac{N\alpha\sigma}{(\sigma+\lambda)(\gamma+\lambda)}$. The spectral radius of the matrix is $\frac{N\alpha\sigma}{(\sigma+\lambda)(\gamma+\lambda)}$ Hence $R_0 = \frac{N\alpha\sigma}{(\sigma+\lambda)(\gamma+\lambda)}$

4. STABILITY ANALYSIS

In absence of the infectious disease, the model equations have a unique disease free steady state E_0. It is given by $E(S,E,I,R) = (\frac{\mu N}{\lambda}, 0,0,0)$

Now local stability analysis of DFEP is given in the following theorem.

Theorem 3: *The model equations 1-4 is locally asymptotically stable at disease free equilibrium point*

Proof

Consider the right-hand side of the equations as functions of S,E,I,R.

$$\frac{dS}{dt} = \mu N - \alpha SI - \lambda S = f(S,E,I,R)$$

$$\frac{dE}{dt} = \alpha SI - \sigma E - \lambda E = g(S,E,I,R)$$

$$\frac{dI}{dt} = \sigma E - \gamma I - \lambda I = h(S,E,I,R)$$

$$\frac{dR}{dt} = \gamma I - \lambda R = k(S,E,I,R)$$

Therefore,

$$J(S,E,I,R) = \begin{bmatrix} -\alpha I - \lambda & 0 & -\alpha S & 0 \\ \alpha I & -\sigma - \lambda & \alpha S & 0 \\ 0 & \sigma & -\gamma - \lambda & 0 \\ 0 & 0 & \gamma & -\lambda \end{bmatrix}$$

$$J\left(\frac{\mu N}{\lambda}, 0,0,0\right) = \begin{bmatrix} -\lambda & 0 & -\frac{\alpha\mu N}{\lambda} & 0 \\ 0 & -\sigma - \lambda & \frac{\alpha\mu N}{\lambda} & 0 \\ 0 & \sigma & -\gamma - \lambda & 0 \\ 0 & 0 & \gamma & -\lambda \end{bmatrix}$$

This can be written as

$$J\left(\frac{\mu N}{\lambda},0,0,0\right)=\begin{bmatrix} -a & 0 & -\frac{\alpha\mu N}{a} & 0 \\ 0 & -b & \frac{\alpha\mu N}{a} & 0 \\ 0 & \sigma & -c & 0 \\ 0 & 0 & \gamma & -a \end{bmatrix}$$

if $a=\lambda, b=(\sigma+\lambda), c=(\gamma+\lambda)$.

Now the characteristic equation is computed by using $\det[J(E_0)-\pi I]=0$.

That is,

$$|J(E_0)-\pi I|=\begin{bmatrix} -a-\pi & 0 & -\frac{\alpha\mu N}{a} & 0 \\ 0 & -b-\pi & \frac{\alpha\mu N}{a} & 0 \\ 0 & \sigma & -c-\pi & 0 \\ 0 & 0 & \gamma & -a-\pi \end{bmatrix}$$

This can be reduced,

$$|J(E_0)-\pi I|=(-a-\pi)\begin{bmatrix} -b-\pi & \frac{\alpha\mu N}{a} & 0 \\ \sigma & -c-\pi & 0 \\ 0 & \gamma & -a-\pi \end{bmatrix}$$

Furthermore, reduced form,

$$|J(E_0)-\pi I|=(-a-\pi)(-a-\pi)\begin{bmatrix} -b-\pi & \frac{\alpha\mu N}{a} \\ 0 & -c-\pi \end{bmatrix}$$

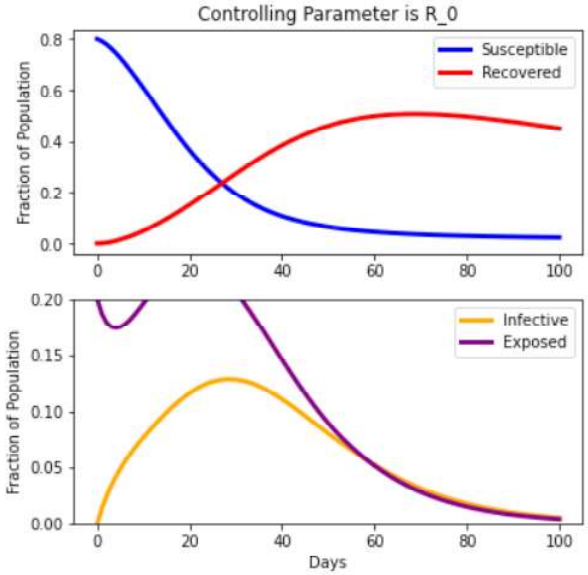
Hence, $(-a-\pi)(-a-\pi)(-b-\pi)(-c-\pi)=0$. Hence all the eigen values are negative. Hence the theorem.

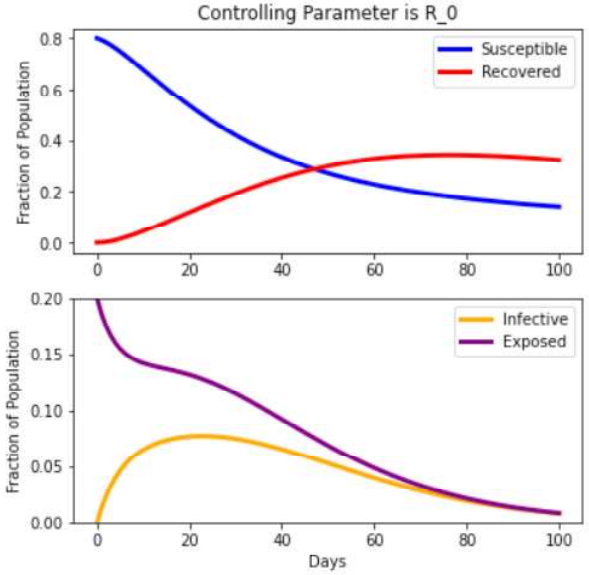
5. NUMERICAL SIMULATION USING PYTHON

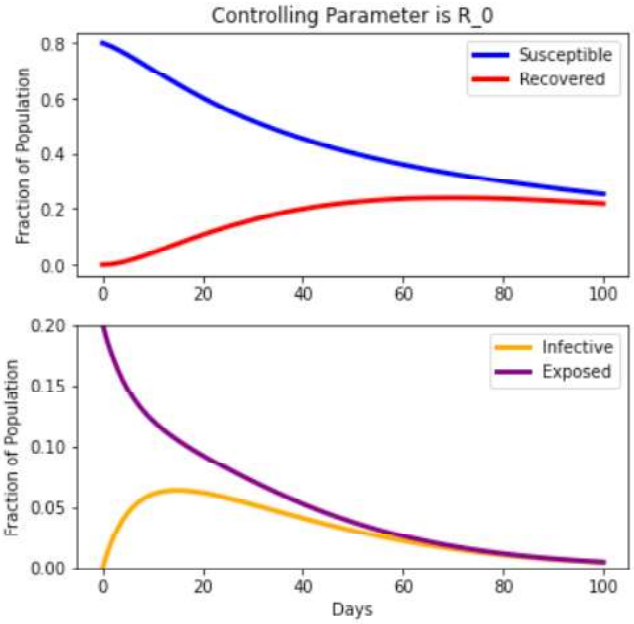
The numerical simulation is carried out using the Python programming language. For simulation purpose, a set of meaning full values are assigned to the model parameters. The values of the parameters are given below.

Parameters	*Values*
α	$R_0 + \gamma$
λ	0.00730
σ	0.07142
γ	0.11111

In this simulation, the controlling parameter was set to be the reproductive number. For the different values of this parameter the simulation was run, and the graphical results are given below.

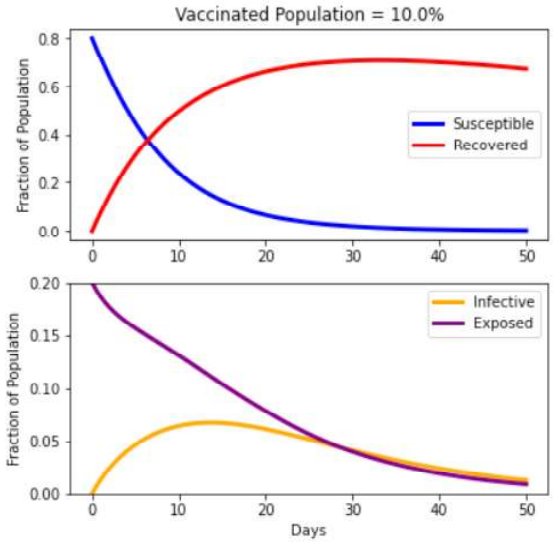


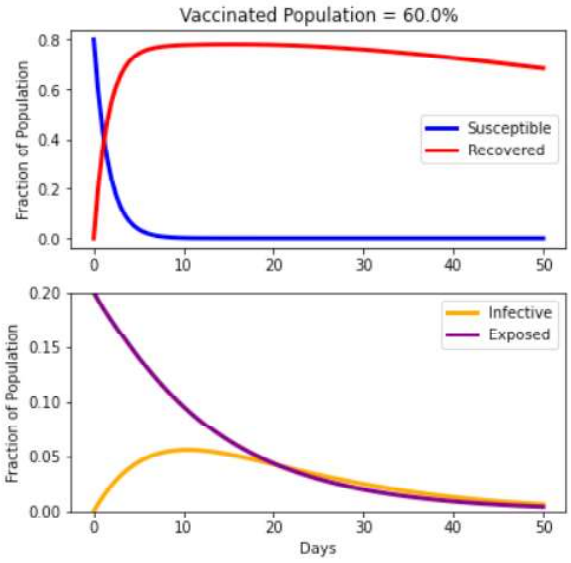




If the population gets greater the number of infections, then will be the less the amount of recovery. That means the reproductive number is high. The reproductive number of our model is given by, $R_0 = \frac{N\alpha\sigma}{(\sigma+\lambda)(\gamma+\lambda)}$ This value depends on, without loss of generality, α, γ the infection rate and the recovery rate respectively. That is, $R_0 \approx \frac{\alpha}{\gamma}$. R_0 will be less than one if the infection rate is less than the recovery rate.

Therefore, infection rate can be reduced by social distancing and through vaccination campaigns. These two factors may play a vital role in the spread of the infectious disease. If a part of the population is vaccinated, then the number of individuals in the susceptible compartment decreases. They move all the directly to the recovered compartment. A simulation was run using the above python program by modifying the model and the parameters and the graphical results are given below.





A drastic change in the curve is detected which means vaccinating the [illegible] of the population will be an effective measure to get rid of this infectious disease.

6. CONCLUSION

The SEIR mathematical model describing the dynamics of the infectious disease is formulated and analyzed. The model is developed based on biologically reasonable assumptions. Moreover, existence, positivity and boundedness of the solution of the model is shown to clarify the model is biologically meaningful and mathematically well posed. Stability analysis of the model is checked with help of next generation matrix. We observe that, infection rate plays a vital role spreading the disease, which is true for any infectious disease, not only COVID-19. If the recovery rate is higher then the disease dies out. But the recovery rate depends on various factors. It depends on each and every person. But we can control the infection rate. It can be controlled by social distancing and by vaccinating the people.

7. ACKNOWLEDGEMENTS

We thank the referee for reviewing our paper which has helped us in revising the earlier draft.

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RESEARCH ARTICLE

COBB-DOUGLAS PRODUCTION FUNCTION UNDER NON-NEUTRAL TECHNICAL PROGRESS FOR THE INDUSTRIAL SECTOR OF INDIA

M. K. DAVE[1] AND S. G. RAVAL[2]

ABSTRACT

Technical changes occur due to advancements in research and development for any industrial concern. Technical progress has an important role in the growth of industries. This suggests to consider non-neutral technical progress for measuring the growth pattern for industrial concerns. This paper deals with Cobb-Douglas production function model under non-neutral technical progress for industrial sector of all India during the period 1981-82 to 2017-18. The proposed model is well fitted at current as well as at constant prices and it may be useful for further prediction which can help in industrial planning strategies.

KEYWORDS:

CRTS, TECHNICAL PROGRESS, CD FUNCTION, ANOVA

1. INTRODUCTION

The traditional theory of production postulates that the quantity of output can be produced as a function of the quantities of the various input factors used, which means the production function explains a basic technological relationship between output and input. As defined by Walters (1963) "production function is a technological

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(rcd. March '21 / rvd. May '21)

relationship confronting a firm. It is the entrepreneur who chooses factor proportions and output levels". The function may have linear or non-linear relationship between the inputs and output and between one input and another. The role of production function is very vital in business and economics.

The concept of production function is closely related to the concept of 'technology'. A neutral change does not affect the factors of production- i.e. capital and labor, but causes a shift in production function. A non-neutral change alters the production function relation; it can be either capital saving or labor saving.

Non-neutral technical progress changes the capital intensity of technology and substitution between factors.

Production function corresponds to a specific level of technical change and any change in this leads to a shift in production function.

Technological progress is one of the most important factors responsible for the economic growth in many sectors of the economy. In industrial sector the analysis of the role of technological progress is important. Murray- Brown (1966) classified technological progress into two broad categories namely **neutral technical progress** and **non-neutral technical progress**.

Hongsong Zhang (2019) examined the role of non-Hicks neutral technology differences across firms. He explored the importance of the non-Hicks neutral technology in driving the decline in labor share and shaping the cross sectional variation in labor share across firms in Chinese steel industries. In the sample period, Chinese steel makers experienced strong labor-saving technological changes.

Almas Heshmati(2010) estimated production functions(Cobb-Douglas and Translog functional forms) to measure the rate of technical change. Multiple time trends were used as an alternative to the single time trend representation of technical change. The models were estimated using GLS methods. He used a large panel data set from Swedish crop producers farms. The rate of technical change considered over time

in STT model was negative and steadily declining. The corresponding value in the alternative model is both positive and negative but declining.

Rakesh Kumar (2002) made an attempt to analyze the efficiency and technology under currents in Indian textile industry during the period 1973-1994. His study revealed that the industry failed to optimize gains in terms of efficiency as well as productivity parameters and continued to work in the scenario of decreasing returns associated with given technology. He observed negative growth in total factor productivity for both Slow and Kendrick TFP Indices and he concluded that productivity level and growth in the industry did not play any considerable role.

In this study, we have tried to contribute to this literature by applying Cobb-Douglas production function model under non-neutral technical progress to a time series data for the industrial sector of all India at current as well as constant prices. Table 2 and Table 4 present regression analysis for the model at current and constant prices respectively. We have also made an attempt to predict the values of output for future planning for both the series. A hypothesis of CRTS (constant returns to scale) has also been tested for CD production function under non-neutral technical change.

A graphical presentation of the labor, capital and output data is given to visualize the growth pattern of the industries. This is given in Appendix-I at the end.

2. DATA BASE

For our study, we have used the data from ASI and CIM. The data are for all India pertaining to all industries for the period 1981-82 to 2017-18. We have also considered data for Consumer price Index- Annual average and Wholesale price index - Annual average for the period 1981-82 to 2017-18 taking 1981-82 as base year from RBI bulletin, applying forward & backward splicing method to obtain series at constant prices.

3. RESEARCH METHODOLOGY

Mathematical model for production function suggests that output is determined by labor employed, capital invested under technological progress to be represented by appropriate function. We have considered model for C-D production function under non - neutral technical progress. We have also tried to fit this model at current prices as well as at constant prices (i.e. we deflated output, working capital and labor in terms of wages by wholesale price index and consumer price index by taking base year as 1981-82)

3.1 MODEL

Let us consider the following CD production function model under non-neutral technical progress with necessary equations.

$$Y_t = A(t) \cdot K_t^{\alpha} L_t^{\beta} \cdot U_t \qquad (1)$$

Where A(t) = non-neutral technical progress function

Y_t = Output for year t

K_t = Working capital for year t

L_t = Labor employed and salaries in terms of wages for year t

U_t = Disturbance term for year t

Now general form for Non-neutral technical progress function A(t) can be expressed by $A(t)=A_{0e}{}^{\delta t}$ and therefore equation (1) takes the form

$$Y_t = A_0 e^{\delta t} \cdot K_t^{\alpha} L_t^{\beta} \cdot U_t$$

Where α, β, δ and A_0 are parameters of the model. α and β are partial elasticities of capital and labor respectively and δ is the technological parameter, A_0 is a scale parameter.

The log-log form of the above model is as under

$$L_n(Y_t) = A + \delta t + \alpha L_n(K_t) + \beta L_n(L_t) + Z_t \qquad (2)$$

where $A = L_n(A_0)$ and $Z_t = L_n(U_t)$

Under normality conditions regression analysis can be considered as usual with a purpose of estimating the parameters and hence to estimate the model.

4. STATISTICAL ANALYSIS

Table: 1 exhibit estimated output for all India of all industries from year 1981-82 to 2017-18 at current prices. Estimated output is increasing throughout the period. Table: 3 shows estimated output of deflated series for India for all industries for the same period. And here for this deflated series also estimated output is increasing.

Table: 1 Estimated output (Current Prices)

Year	Value of Output	Working Capital	Wages to Workers	Estimated Output
(1)	(2)	(3)	(4)	(5)
1981-82	7363046	1505488	439417	7540742.324
1982-83	8623768	1631988	514828	8596788.808
1983-84	9353741	1850402	592078	9906006.957
1984-85	10556600	2232323	675730	11597358.64
1985-86	11945705	2379864	709209	13073002.82
1986-87	13304352	2180329	785043	14206208
1987-88	15397307	2755102	893370	16833254.72
1988-89	18434878	2724616	1029223	18711791.24
1989-90	23004199	3386365	1179567	22085249.13
1990-91	27056353	4252036	1319205	26097151.58
1991-92	29919581	4446816	1358263	29233898.43
1992-93	36861377	6249011	1683112	35815174.23
1993-94	42574425	8710857	1759741	43320722.62
1994-95	51798701	8729632	2201946	48575031.01

(1)	(2)	(3)	(4)	(5)
1995-96	67051423	10766313	2797035	57585718.8
1996-97	74180838	17165931	2655459	71731271.63
1997-98	83633644	15461658	2978167	77664238.74
1998-99	78377081	10274034	2482648	76175186.23
1999-2000	89793835	10378436	2630427	84705999.66
2000-2001	92690185	10520839	2767074	94237204.42
2001-2002	96245663	10040585	2743824	102803783.7
2002-2003	113056111	10012110	2968905	114077088.1
2003-04	128740055	11923049	3047777	132217955
2004-05	167256142	16005396	3363505	158881133.2
2005-06	190835548	18446260	3766366	183560539.2
2006-07	240854764	28218880	4429135	229223789.7
2007-08	277570904	31695306	5103023	263506811
2008-09	327279786	31123298	5977184	292670016.8
2009-10	373303593	38774453	6894071	345788575.7
2010-11	467621696	62036285	8564552	438449871.9
2011-12	577602354	58879446	9985579	482485653.2
2012-13	602594536	218026022	11089620	758001863.5
2013-14	655,525,116	66,268,577	12,649,644	616881369.8
2014-15	688,381,205	64,084,031	14,048,488	680119800.9
2015-16	686,235,375	74,052,998	15,600,116	785901884
2016-17	726,551,423	66,308,287	17,353,716	849146327.1
2017-18	808,167,115	64,411,890	19,280,076	937330139

Table: 2 gives regression analysis carried out for this proposed model considering at current prices.

Table: 2 Regression Analysis for all Industries of All India (Current Prices)

Model: $L_n(Y_t) = A + \delta t + \alpha L_n(K_t) + \beta L_n(L_t) + Z_t$

Regression Statistics	
Multiple R	0.998318557
R Square	0.99663994
Adjusted R Square	0.99633448
Standard Error	0.089919851
Observations	37

ANOVA

	df	SS	MS	F	Significance F
Regression	3	79.1437079	26.38124	3262.75141	7.15993E-41
Residual	33	0.26682412	0.008086		
Total	36	79.410532			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	11.26946772	1.14810779	9.815688	2.5743E-11	8.933624864	13.6053106
time	0.099859792	0.00860653	11.6028	3.4507E-13	0.082349678	0.11736991
lncap	0.26376628	0.05015788	5.25872	8.5991E-06	0.1617193	0.36581326
lnwage	0.062678407	0.10320534	0.607317	0.54779675	-0.147294431	0.27265124

From the above Table: 2, we observed that value of R=0.99831 which shows that there is strong association between output with capital and labor. It is also clear that R^2= 0.99663 which is highly significant and the value of R^2 exhibits that more

than 99% variation in output is explained by model. Adjusted R^2 is 0.99633 which also indicates similar behavior. From table of regression coefficients it is observed that all regression coefficients are statistically significant at 5% level of significant.

F-value is also highly significant which shows that regression model may be considered to be good fit for the data.

The predicted value of output for subsequent years are Rs.1098250 lakhs and Rs.1182841 lakhs in 2018-19 and 2019-20 respectively. These predicted values are based upon the estimated labor and capital components given by AGR of the respective series.

Table: 3 Estimated Output, Deflated series (constant price)

Year	WPI	CPI	Deflated Output	Deflated Capital	Deflated Wages	Estimated Output
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1981-82	100	100	73630.46	15054.88	4394.17	71256.45
1982-83	104.9	107.8	82209.4185	15557.56	4775.77	77742.27
1983-84	112.8	121.29	82923.2358	16404.273	4881.507	83596.06
1984-85	120.1	128.26	87898.418	18587.202	5268.439	92097.19
1985-86	125.4	136.95	95260.8054	18978.182	5178.598	97561.66
1986-87	132.7	148.91	100258.87	16430.512	5271.929	102202.2
1987-88	143.5	161.96	107298.307	19199.317	5515.992	112064.9
1988-89	154.3	177.17	119474.258	17657.913	5809.24	119421
1989-90	165.7	188.04	138830.41	20436.723	6272.958	131938.8
1990-91	182.7	209.78	148091.697	23273.322	6288.517	142493.7
1991-92	207.8	238.04	143982.584	21399.5	5706.028	145677.8
1992-93	228.7	260.87	161177.862	27324.053	6451.919	165067.3
1993-94	247.8	280.43	171809.625	35152.772	6275.152	179534.2
1994-95	279.02	308.7	185645.119	31286.761	7132.964	194612.8
1995-96	301.32	340.22	222525.631	35730.496	8221.254	218577

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1996-97	315.2	371.72	235345.298	54460.441	7143.708	235404
1997-98	329.08	397.83	254143.807	46984.496	7486.029	248483.9
1998-99	348.065	450	225179.438	29517.573	5516.996	228447.9
1999-2000	360.05	465.22	249392.682	28824.985	5654.157	243450
2000-2001	385.82	482.6	240242.043	27268.776	5733.68	257634.2
2001-2002	399.7	503.26	240794.754	25120.303	5452.1	266909.2
2002-2003	413.33	523.91	273525.055	24223.042	5666.823	285095.3
2003-04	435.88	543.48	295356.646	27353.971	5607.892	306469.5
2004-05	464.21	565.22	360302.755	34478.783	5950.789	340621.4
2005-06	485.09	589.13	393402.354	38026.469	6393.098	373500.9
2006-07	517.59	629.35	465338.905	54519.755	7037.634	426290.1
2007-08	541.26	603.25	512823.604	58558.375	8459.218	480652.6
2008-09	584.9	641.86	559548.275	53211.315	9312.286	517758
2009-10	607.18	699.77	614815.364	63859.898	9851.91	571405.4
2010-11	665.21	786.64	702968.53	93258.197	10887.51	654398.6
2011-12	724.63	868.68	797099.698	81254.497	11495.12	693068.9
2012-13	768.08	941.07	784546.578	283858.48	11784.05	868251.5
2013-14	815.18	1037.54	804147.693	81293.183	12191.96	794035.7
2014-15	825.31	1138.94	834088.046	77648.436	12334.7	840781.6
2015-16	794.88	1211.28	863319.463	93162.487	12879.03	924480.7
2016-17	808.65	1278.89	898474.523	81998.747	13569.36	979801.2
2017-18	832.56	1331.97	970701.349	77366.064	14474.86	1051240

Table: 4 gives regression analysis carried out for the proposed model considering at constant prices.

Table: 4 Regression Analyses (Constant Prices)

Model: $L_n(Y_t) = A + \delta t + \alpha L_n(K_t) + \beta L_n(L_t) + Z_t$

Regression Statistics	
Multiple R	0.997421945
R Square	0.994850536
Adjusted R Square	0.994382403
Standard Error	0.06061321
Observations	37

ANOVA

	df	SS	MS	F	Significance F
Regression	3	23.42309622	7.8076987	2125.145	8.20142E-38
Residual	33	0.121240721	0.003674		
Total	36	23.54433694			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	7.620734067	0.485666363	15.691295	7.29E-17	6.632638426	8.60882971
TIME	0.059784481	0.002028502	29.472236	2.78E-25	0.055657463	0.0639115
LN DE CAPITAL	0.126840089	0.038236347	3.3172648	0.002221	0.049047657	0.20463252
LN DE WAGES	0.278155624	0.072167505	3.8543057	0.000508	0.131329732	0.42498152

From Table: 4 we can deduce that for the deflated series also R=0.99742 which indicates that there is strong association between output with capital and labor. Here for deflated series R^2=0.99485 which shows that more than 99% variation in output is explained by the model. Adjusted R^2 =0.99438 which also

exhibit similar behavior. All the regression coefficients are statistically significant at 5% level of significance.

F-value is also highly significant which shows that regression model may be considered to be good fit for the deflated data.

The predicted values of output for subsequent years are Rs. 1192301256 lakhs and Rs.1366917604 lakhs in 2018-19 and 2019-20. These predicted values are based upon the estimated labor and capital components given by AGR of the respective series.

5. HYPOTHESIS TESTING

If there are constant returns to scale (CRTS) in industries i.e. proportional increase in inputs would bring corresponding proportional increase in output. Thus we can write for CRTS

$$\alpha + \beta = 1 \qquad (3)$$

If we want to test the linear restriction given in (3) , our hypothesis will be as follows

$H_0 : \alpha + \beta = 1$

$H_1 : \alpha + \beta \neq 1$

Let us consider model for C-D production function under non-neutral technical progress in (2)

$$L_n(Y_t) = A + \delta t + \alpha L_n(K_t) + \beta L_n(L_t) + Z_t$$

We call this model as **unrestricted model.**

From equation (3) we have

$$\alpha + \beta = 1 \quad \Rightarrow \quad \beta = 1 - \alpha \qquad .(4)$$

By using (4) we can write our C-D production function under non-neutral technical progress as

$$L_n(Y_t) = A + \delta t + \alpha L_n(K_t) + (1 - \alpha) L_n(L_t) + Z_t$$

$$\Rightarrow L_n\left(\frac{Y_t}{L_t}\right) = A + \delta t + \alpha L_n\left(\frac{K_t}{L_t}\right) + +Z_t \quad (5)$$

Where $\frac{Y_t}{L_t}$ = output labor ratio at time t

$\frac{K_t}{L_t}$ = Capital labor ratio at time t

For equation (5) we call it a **restricted model.**

We run the regression for both these models restricted as well as unrestricted.

Now we shall apply F-test as given below

$$F = \frac{(RSS_r - RSS_{un})/m}{RSS_{un}/(n-k)}$$

Where RSS_r=RSS from restricted model

RSS_{un}=RSS from unrestricted model

m=number of restriction (here 1)

k= number of coefficients in unrestricted model =4

n= number of observations=37

Here, $F = \frac{0.744716 - 0.266824/1}{0.266824/33} \Rightarrow F = 59.10$

Critical F-value with 1 df in the numerator and 33 df in denominator is 4.08

A significant value of F rejects the null hypothesis for CRTS at 5% level of significance(similarly testing at constant prices indicates that F is highly significant).Therefore hypothesis of linear restriction $\alpha + \beta = 1$ is rejected which means there are no constant returns to scale in industries. We may suggest that unrestricted model may be considered to be the best one.

6. CONCLUDING REMARKS

In this study an attempt is made to envisage the fitting procedure for CD function model under non-neutral technical progress. From this study what is observed is that the model seems to be well fitted for the entire industrial sector of All India based upon the available data and their restriction and assumptions.

This analysis may be helpful for planning purpose for the industrial sector of the economy. The short run predictions obtained based upon this academic exercise may be fruitful for the organizational set up for industrial planning for the future years.

7. ACKNOWLEDGEMENT

We thank the referee for the comments which have helped us in thoroughly revising the earlier draft of this paper.

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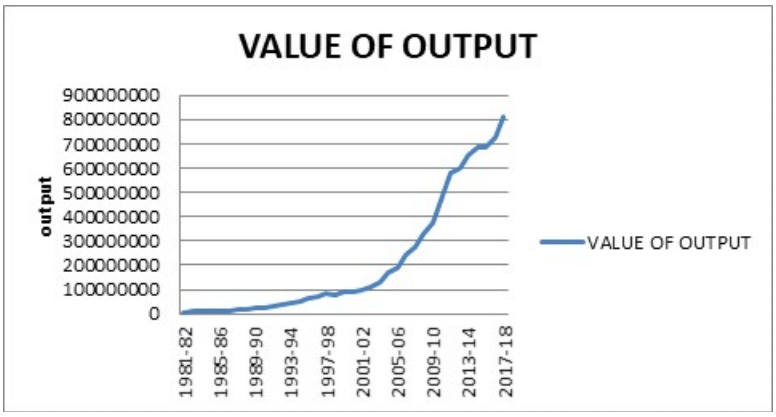
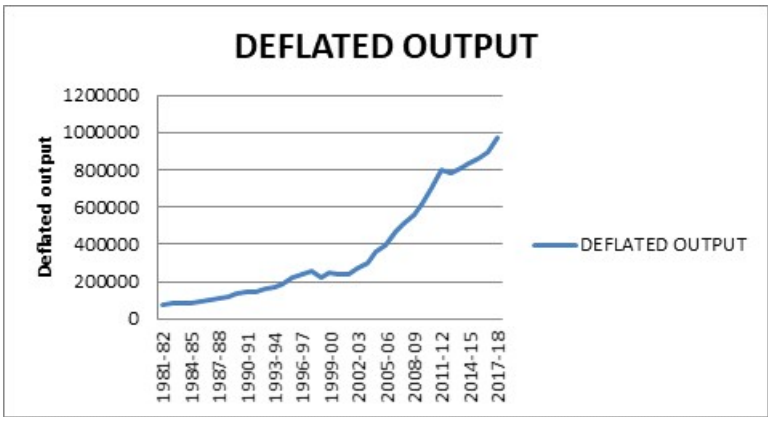
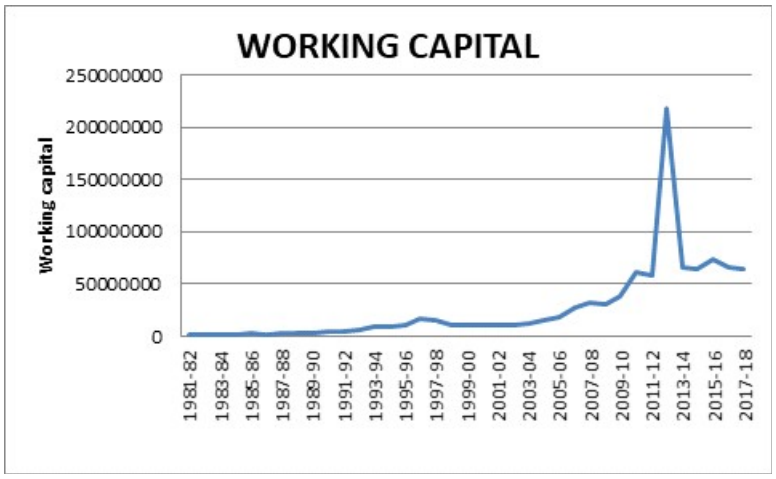
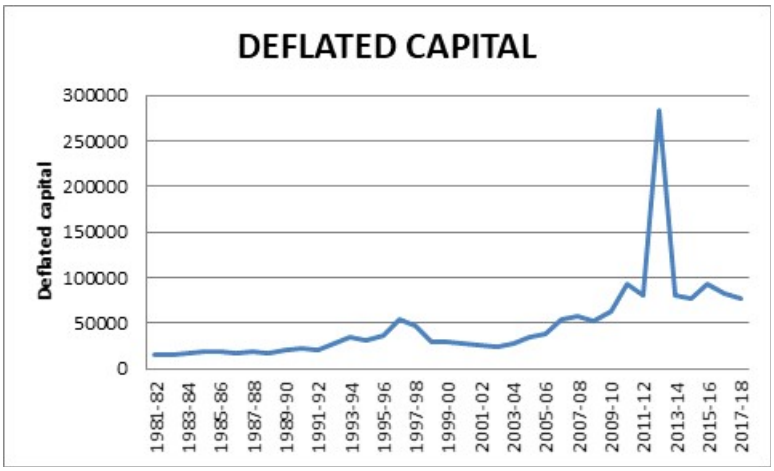
9. APPENDIX-I: GRAPHICAL REPRESENTATION








DEFLATED CAPITAL
Deflated capital
300000
250000
200000
150000
100000
50000
0
1981-82
1983-84
1985-86
1987-88
1989-90
1991-92
1993-94
1995-96
1997-98
1999-00
2001-02
2003-04
2005-06
2007-08
2009-10
2011-12
2013-14
2015-16
2017-18


WAGES TO WORKERS
Wages to workers
25000000
20000000
15000000
10000000
5000000
0
1981-82
1983-84
1985-86
1987-88
1989-90
1991-92
1993-94
1995-96
1997-98
1999-00
2001-02
2003-04
2005-06
2007-08
2009-10
2011-12
2013-14
2015-16
2017-18


DEFLATED WAGES
Deflated wages
16000
14000
12000
10000
8000
6000
4000
2000
0
1981-82
1983-84
1985-86
1987-88
1989-90
1991-92
1993-94
1995-96
1997-98
1999-00
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2015-16
2017-18

RESEARCH ARTICLE

PARAMETERIC SENSITIVITY ANALYSIS PERTAINING TO NON-NEUTRAL CDPF MODEL FOR INDUSTRIAL SECTOR OF ALL INDIA

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ABSTRACT

Technical progress is inevitable for growth and advancement of industries. For the entire industrial sectors for all India, non-neutral CD production function is found to be well fitted. A further study is carried out in this paper to visualise the sensitivities of the parameters of the fitted mode. This is achieved by total sensitivity indices (TSI) and partial sensitivity indices (PSI) for the fitted model and some useful conclusions are drawn based upon this study.

KEYWORDS:

IRTS, NON-NEUTRAL, TSI, PSI

1. INTRODUCTION

Theoretical studies related with econometric models for production functions are extremely useful for the industrial sectors for any economy. These studies have their special and specific roles in empirical econometrics. Not only that such studies specify the growth pattern in the respective economic sectors, but also visualise and hence envisage for advancements and further developments in the respective fields.

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(rcd. March.'21/rvd. May '21)

Our earlier study related with Cobb-Douglas production function model under non-neutral technical progress was found to be suitable as the model was fitted for the entire industrial sector of all India. The study was carried out at current prices as well as at constant prices with relevant test procedures pertaining to the statistical analysis and conclusions were drawn on the basis of the fitted model.

In this paper, we want to carry out the sensitivity analysis for this model. It may be useful and important to visualise as to what will happen if the partial labor elasticity , partial capital elasticity and technology parameter and all of them observe changes (increase or decrease) form certain level? This is accomplished by Total Sensitivity Indices (TSI). Similarly if only one of the parameters changes at certain level and all other remaining parameters are at fixed level, what will be its effect upon the estimated output based upon the fitted model? This is accomplished by Partial Sensitivity Indices (PSI).

2. DATA BASE

ASI and CIM data are used corresponding to the entire industrial sector of all India for the period 1981-82 to 2017-18. The series for total output, labor employed and capital invested are deflated by the respective indices to achieve the series at constant prices and wherever necessary forward and backward splicing method is used to obtain these series.

3. METHODOLOGY

3.1 MODEL

Let us consider out Non-neutral CD production function model (for the industrial sector) as given by

$$Y_t = A_0 \exp(\delta t) \cdot K_t^{\alpha} \cdot L_t^{\beta} \cdot U_t \quad (1)$$

so that $$L_n(Y_t) = A + \delta t + \alpha L_n(K_t) + \beta L_n(L_t) + Z_t \quad (2)$$

where $A = L_n(A_0)$ and $Z_t = L_n(U_t)$

Where Y_t = Output for the year t (in value terms)

K_t = Working capital for the year t (in value terms)

L_t = Labor employed for the year t(in value terms)

U_t = Disturbance term for the year t

On the basis of $\hat{A}_0, \hat{\delta}, \hat{\alpha}$ and $\hat{\beta}$, we can find estimated $\widehat{Y_t}$

A_0 is scale parameter (which may exhibit shift in the production function model. It behaves as a fixed parameter indicating change of origin).

δ is the technology parameter

α is the partial elasticity of capital

β is the partial elasticity of labor (All these parameters are positive)

We had observed that under normality conditions when fitted this model for the entire industrial sector of all India, (i) there is increasing returns to scale (IRS) and (ii) model is well fitted during the period of our study.

3.2 SENSITIVITY ANALYSIS

$$\widehat{L_n(Y_t)} = \hat{A} + \hat{\delta}_t + \hat{\alpha} L_n(K_t) + \hat{\beta} L_n(L_t) \quad (3)$$

Which gives for the fitted model as

$$\widehat{L_n(Y_t)} = 7.6207 + 0.0597t + 0.1268 L_n(K_t) + 0.2782 L_n(L_t) \quad (4)$$

We want to carry out our sensitivity analysis for the model from the equation given in (4) above.

3.2.1 TOTAL SENSITIVITY ANALYSIS

Since A (or A_0) behaves as a scale parameter, we shall apply sensitivity analysis only for the shape parameters δ, α and β.

If all these parameters observe a certain change (e.g. 5% increase or 5% decrease) from the original value, what will be the corresponding change observed in the estimated $\widehat{Y_t}$?

This is given by Total Sensitivity Indices, which are computed from the following formulae.

Total sensitivity index at time t when all the parameters (i.e. δ, α and β) increase together by 5% from the original level

$$T_1 = (TSI)_t^{1.05} = \left[\frac{\hat{Y}'_{t(1.05)} - \hat{Y}_t}{\hat{Y}_t}\right] \times 100 \qquad (5)$$

Here $\hat{Y}'_{t(1.05)}$ = Estimated value of $\hat{Y}_t$ at time t when all the parameters increase by 5% from the basic level

$\hat{Y}_t$ =Estimated (original) value of Y_t at time t

Similarly TSI at time t when all the parameters decrease simultaneously by 5% from the basic level

$$T_2 = (TSI)_t^{0.95} = \left[\frac{\hat{Y}'_{t(0.95)} - \hat{Y}_t}{\hat{Y}_t}\right] \times 100 \qquad (6)$$

Where $\hat{Y}'_{t(0.95)}$ = Estimated value of $\hat{Y}_t$ at time t when all the parameters decrease by 5% from the original level

$\hat{Y}_t$ = Estimated (original) value of Y_t at time t

3.2.2 PARTIAL SENSITIVITY ANALYSIS (PSI)

If only one of the above three parameters (i.e. δ, α and β) observe a certain change (i.e. 5% increase or 5% decrease) form the original level and other parameters are fixed, what will be the corresponding change in the estimated value of $\hat{Y}_t$ as compared to the original value of $\hat{Y}_t$?

This is accomplished by Partial Sensitivity Analysis and they are computed from the following formulae.

(A) For parameter δ

Partial Sensitivity Index at time t when only parameter δ increase by 5% (keeping α and β as fixed) from the original level.

$$P_1^{\delta} = (PSI)_t^{(\delta,1.05)} = \left[\frac{\hat{Y}_t'^{(\delta,1.05)} - \hat{Y}_t}{\hat{Y}_t}\right] \times 100 \qquad (7)$$

Here $\hat{Y}_t'^{(\delta,1.05)}$ = Estimated value of Y_t when δ only increases by 5% from the original level.

$\hat{Y}_t$ = Original estimated value of Y_t at time t.

Partial Sensitivity Index at time t when only parameter δ decreases by 5% (keeping α and β as fixed) from the original level.

$$P_2^{\delta} = (PSI)_t^{(\delta,0.95)} = \left[\frac{\hat{Y}_t'^{(\delta,0.95)} - \hat{Y}_t}{\hat{Y}_t}\right] \times 100 \qquad (8)$$

Here $\hat{Y}_t'^{(\delta,0.95)}$ = Estimated value of Y_t when δ only decreases by 5% from the original level.

$\hat{Y}_t$ = Original estimated value of Y_t at time t.

(B) For Parameter α

Partial Sensitivity Index at time t when only parameter α increase by 5% (keeping δ and β as fixed) from the original level.

$$P_3^{\alpha} = (PSI)_t^{(\alpha,1.05)} = \left[\frac{\hat{Y}_t'^{(\alpha,1.05)} - \hat{Y}_t}{\hat{Y}_t}\right] \times 100 \qquad (9)$$

Here $\hat{Y}_t'^{(\alpha,1.05)}$ = Estimated value of Y_t when α only increases by 5% form the original level.

$\hat{Y}_t$ = Original estimated value of Y_t at time t

Partial Sensitivity Index at time t when only parameter α decreases by 5% (keeping δ and β as fixed) from the original level.

$$P_4^{\alpha} = (PSI)_t^{(\alpha,0.95)} = \left[\frac{\hat{Y}_t'^{(\alpha,0.95)} - \hat{Y}_t}{\hat{Y}_t}\right] \times 100 \qquad (10)$$

Here $\hat{Y}_t^{\prime(\alpha,0.95)}$ = Estimated value of Y_t when α only decreases by 5% form the original level.

$\hat{Y}_t$ = Original estimated value of Y_t at time t.

(C) For parameter β

Partial Sensitivity Index at time t when only parameter β increase by 5% (keeping $\delta and\, \alpha$ as fixed) from the original level.

$$P_5^{\beta} = (PSI)_t^{(\beta,1.05)} = \left[\frac{\hat{Y}_t^{\prime(\beta,1.05)} - \hat{Y}_t}{\hat{Y}_t}\right] \times 100 \qquad (11)$$

Here $\hat{Y}_t^{\prime(\beta,1.05)}$ = Estimated value of Y_t when β only increases by 5% form the original level.

$\hat{Y}_t$ = Original estimated value of Y_t at time t.

Partial Sensitivity Index at time t when only parameter β decreases by 5% (keeping $\delta and\, \alpha$ as fixed) from the original level.

$$P_6^{\beta} = (PSI)_t^{(\beta,0.95)} = \left[\frac{\hat{Y}_t^{\prime(\beta,0.95)} - \hat{Y}_t}{\hat{Y}_t}\right] \times 100 \qquad (12)$$

Here $\hat{Y}_t^{\prime(\beta,0.95)}$ = Estimated value of Y_t when β only decreases by 5% form the original level.

$\hat{Y}_t$ = Original estimated value of Y_t at time t.

4. ANALYSIS

4.1 TSI

For our study based upon the fitted model , total sensitivity indices T_1 and T_2 are given in the following Table:1.

Table:1 Total Sensitivity Indices

Time	$\hat{Y}_t$	T_1	T_2	Time	$\hat{Y}_t$	T_1	T_2
0	71256.45	19.44691	-16.2827	20	266909.2	27.59223	-21.6399
1	77742.27	19.968	-16.6469	21	285095.3	28.01309	-21.898
2	83596.06	20.40386	-16.9494	22	306469.5	28.47619	-22.1803
3	92097.19	20.98797	-17.351	23	340621.4	29.15627	-22.5908
4	97561.66	21.33675	-17.5893	24	373500.9	29.75236	-22.9471
5	102202.2	21.61864	-17.7809	25	426290.1	30.61243	-23.4553
6	112064.9	22.17979	-18.1592	26	480652.6	31.39832	-23.9137
7	119421	22.56844	-18.4193	27	517758	31.88749	-24.1963
8	131938.8	23.18054	-18.8253	28	571405.4	32.53881	-24.5695
9	142493.7	23.65494	-19.1376	29	654398.6	33.44016	-25.0799
10	145677.8	23.79122	-19.2272	30	693068.9	33.82343	-25.2949
11	165067.3	24.56676	-19.7308	31	868251.5	35.33906	-26.133
12	179534.2	25.09063	-20.0678	32	794035.7	34.73571	-25.8019
13	194612.8	25.59575	-20.3898	33	840781.6	35.12121	-26.0141
14	218577	26.32681	-20.8511	34	924480.7	35.76344	-26.3648
15	235404	26.79552	-21.1447	35	979801.2	36.15818	-26.5787
16	248483.9	27.13845	-21.3579	36	1051240	36.63779	-26.8369
17	228447.9	26.60462	-21.0268				
18	243450	27.00748	-21.2779				
19	257634.2	27.36723	-21.5008				

4.2 PSI

For our study based upon the model fitted, Partial Sensitivity indices $P_1^{\delta}, P_2^{\delta}, P_3^{\alpha}, P_4^{\alpha}, P_5^{\beta}$ and P_6^{β} are given in the following Table:2

Table:2 Partial Sensitivity Indices

Time	$\hat{Y}_t$	P_1^{δ}	P_2^{δ}	P_3^{δ}	P_4^{δ}	P_5^{δ}	P_6^{δ}
0	71256.45	0.0036887	0.0036887	6.292469	-5.92207	12.37985	-11.0095
1	77742.27	0.30278809	-0.2952327	6.314224	-5.94202	12.5097	-11.1129
2	83596.06	0.60273996	-0.5933022	6.349525	-5.97405	12.54352	-11.1403
3	92097.19	0.90362308	-0.890447	6.433405	-6.04895	12.66259	-11.2348
4	97561.66	1.20536701	-1.1867417	6.447022	-6.06175	12.63518	-11.214
5	102202.2	1.50800716	-1.4821565	6.349347	-5.97611	12.66271	-11.2364
6	112064.9	1.81162365	-1.7766194	6.454041	-6.06937	12.73327	-11.2925
7	119421	2.11608491	-2.0702629	6.397143	-6.01981	12.81407	-11.3568
8	131938.8	2.42152914	-2.3629594	6.495422	-6.1073	12.93428	-11.4517
9	142493.7	2.7277557	-2.6549055	6.582725	-6.18519	12.93765	-11.4552
10	145677.8	3.03493283	-2.9459457	6.525565	-6.13557	12.78457	-11.3358
11	165067.3	3.34315104	-3.236001	6.690426	-6.28142	12.97713	-11.4874
12	179534.2	3.65218129	-3.5252917	6.860535	-6.43162	12.93301	-11.4536
13	194612.8	3.96217513	-3.8136811	6.781229	-6.36272	13.13405	-11.6116
14	218577	4.2731458	-4.1011627	6.870829	-6.44196	13.35735	-11.7863
15	235404	4.58491957	-4.3879012	7.156301	-6.69244	13.13554	-11.6141
16	248483.9	4.89767999	-4.6737328	7.055609	-6.60528	13.20877	-11.672
17	228447.9	5.21124485	-4.9588282	6.740021	-6.32969	12.72858	-11.2967
18	243450	5.5258667	-5.2429635	6.723519	-6.31595	12.76664	-11.3273
19	257634.2	5.84144991	-5.526231	6.685558	-6.28329	12.78811	-11.3449
20	266909.2	6.1579397	-5.8086848	6.629612	-6.23484	12.70866	-11.2832
21	285095.3	6.47543158	-6.0902449	6.60463	-6.21353	12.76881	-11.3311
22	306469.5	6.79382028	-6.3710096	6.686371	-6.28629	12.75194	-11.3186

Time	$\hat{Y}_t$	P_1^{δ}	P_2^{δ}	P_3^{δ}	P_4^{δ}	P_5^{δ}	P_6^{δ}
23	340621.4	7.11322311	-6.6508808	6.842671	-6.42427	12.84465	-11.3921
24	373500.9	7.43358452	-6.9299125	6.908649	-6.48279	12.95684	-11.4808
25	426290.1	7.75490703	-7.2081077	7.152744	-6.69682	13.10746	-11.5993
26	480652.6	8.07721266	-7.4854524	7.200941	-6.73947	13.39692	-11.8255
27	517758	8.40046928	-7.7619792	7.135508	-6.68307	13.54817	-11.9436
28	571405.4	8.72465751	-8.0377094	7.259078	-6.79146	13.63674	-12.0129
29	654398.6	9.04984457	-8.3125906	7.516518	-7.01565	13.79444	-12.1354
30	693068.9	9.37599012	-8.5866621	7.422243	-6.9346	13.88002	-12.2021
31	868251.5	9.70307201	-8.8599467	8.277121	-7.67128	13.91889	-12.2328
32	794035.7	10.0311634	-9.1323884	7.421732	-6.9356	13.9724	-12.2746
33	840781.6	10.3602011	-9.4040444	7.390061	-6.90887	13.99039	-12.2892
34	924480.7	10.6902508	-9.6748654	7.513743	-7.01684	14.05845	-12.3422
35	979801.2	11.0212984	-9.944868	7.426375	-6.94179	14.1409	-12.4062
36	1051240	11.3533471	-10.214055	7.386386	-6.90775	14.2431	-12.4852

Diargam-1

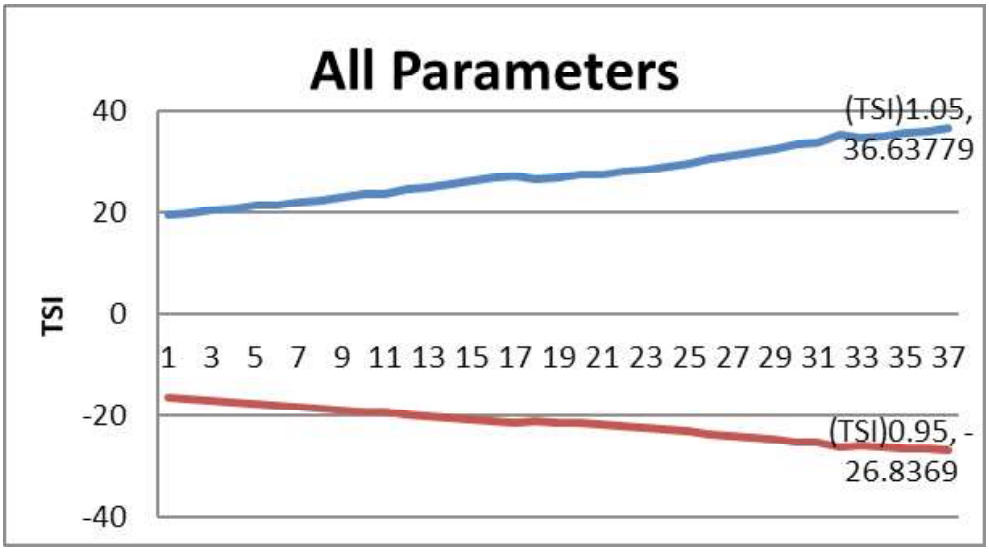


Diagram-2

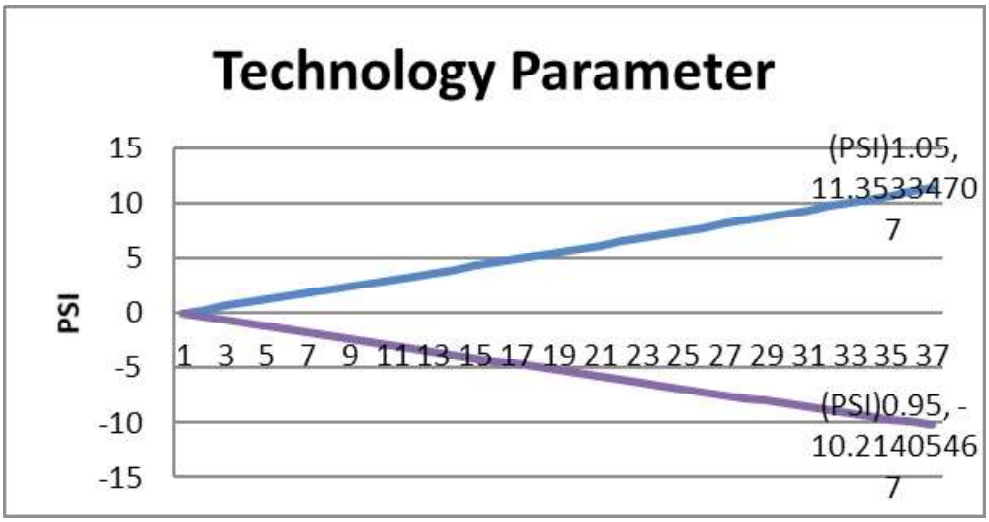


Diagram-3

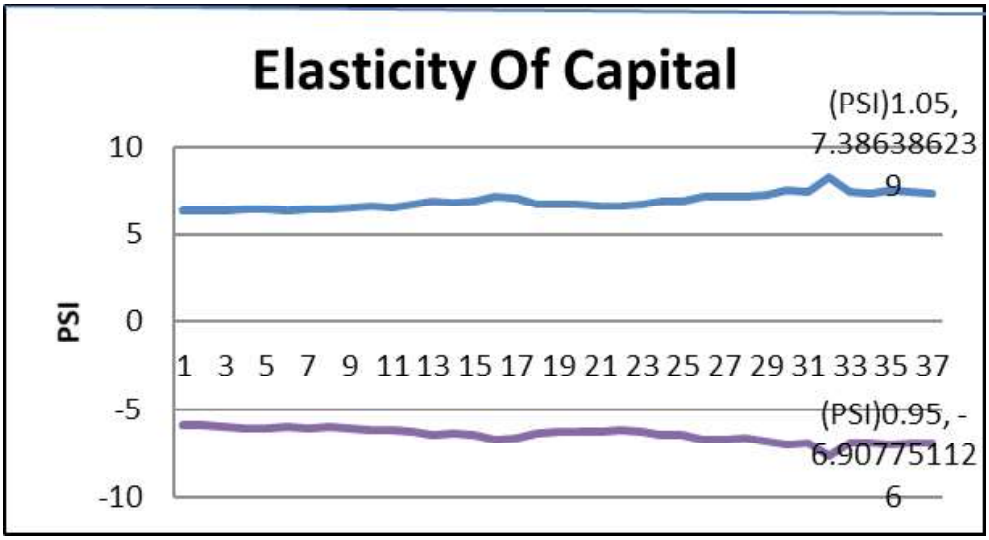
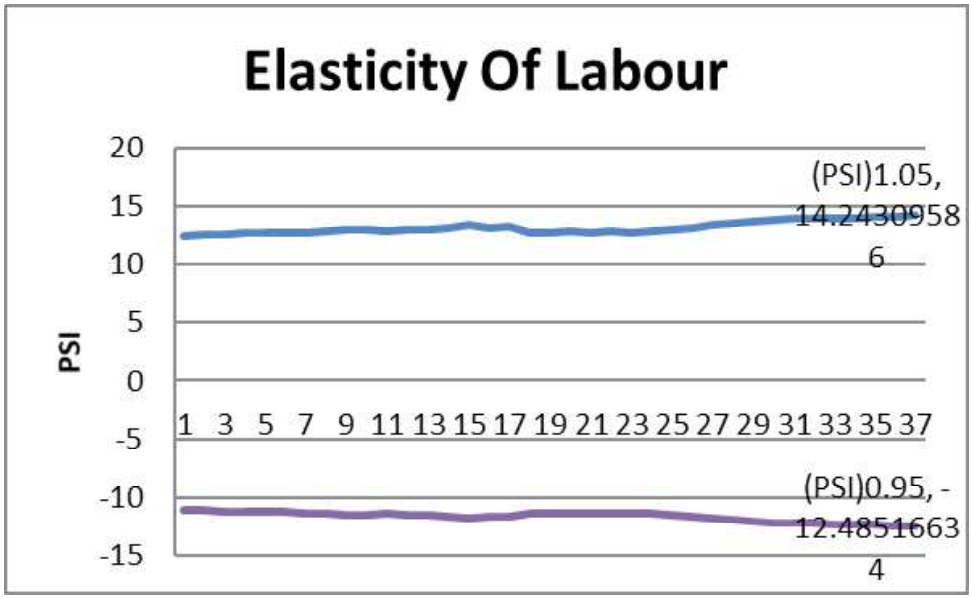


Diagram-4



5. CONCLUSIONS

5.1 TSI

From TSI indices given in Table:1 , we find that TSI for the entire series observes almost increase during the period of our study for 5% increase in all the parameter values .It is 19.45% for the year 1981-82 and it reaches to 36.64% in the year 1917-18. Similarly TSI for the base year is -16.28 and it reaches to minimum value -26.83 for 5% decrease in all the parameter values. This series is also found almost uniformly decreasing throughout the period. We also find that for 5% increase in all the parameters , the average values of TSI during successive 5 years period are 20.39%, 20.64%, 25.06%, 27.08%, 28.67%, 32.02% and 35.23%. Also, the overall average for the whole series is found to be 27.73%. Similarly for 5% decrease in all the parameter values the averages for successive 5 years are -16.94%,-18.46%,-

20.04%,-21.32%, -22.29%,-24.27%,-26.07% respectively. Here the overall average for the whole series is -21.60%.

5.2 PSI

5.2.1 TECHNOLOGY PARAMETER

When only technology parameter δ increases by 5% and all other parameters are fixed, we find that PSI series almost steadily increases during the period of study .It is 0.0037% in the base year and reaches to 10.21% in the year 2017-18 .For 5% decrease in the technology parameter δ ,the base year has the same steady value 0.0037%. But it almost steadily decreases during the subsequent periods and reaches to minimum -10.21% in the year 1917-18.

For five yearly averages during successive periods for PSI series corresponding to δ , we find the values are 0.61%, 2.12%, 3.65%, 5.21%, 6.8%, 8.4% and 10.36%.The overall average for the whole series for 5% increase is 5.58%.

For 5% decrease the five yearly averages for technology parameter δ are -0.59%, -2.07%, -3.52%, -4.96%, -6.37%, -7.76% and -9.4% respectively. The overall average for the whole series is found to be -5.19%.

5.2.2 PARTIAL ELASTICITY OF CAPITAL

PSI series for changing parameter α when all other parameters are fixed shows that for 5% increase in the parameter value the series is almost uniformly increasing. It is 6.29% in the base year and 7.39% in the final year. It becomes 8.28% in the year 2012-13. Similarly for 5% decrease in parameter α keeping all the parameters as fixed, we find that the series is almost uniformly decreasing during the period. It is -5.92% in the base year and -6.91% in the final year. This series has its minimum value -7.67% in the year 2012-13.

Five yearly averages for the PSI series of α (other parameters are fixed) are 6.37%, 6.47%, 6.70%, 6.92%, 6.77%, 7.99% and 7.40% respectively for 5% increase

in the parameter value. Similarly for 5% decrease in the parameter α alone, we find five yearly averages as -5.99% , -6.08%, -6.29% ,-6.49%, -6.36%, -6.86%, -6.92% respectively. The overall average for 5% increase in α alone is 6.89% and 5% decrease in α alone is -6.46%.

5.2.3 PARTIAL ELASTICITY OF LABOUR

For PSI indices corresponding to 5% increase in parameter β (other parameters as fixed) , we find that this series is almost steadily increasing. It is 12.38% in the base year and 14.24% in the last year. For 5% decrease in the parameter β alone (all the other parameters kept fixed) PSI indices indicate that there is almost uniformly decreasing pattern during the period of study. It is -11.01% in the base year and reaches to -12.49% in the last year.

If we observe PSI indices on the basis of five yearly averages for 5% increase in the value of parameter β alone(others kept fixed) it is found that the values are 12.51%,12.80%,13.07%,12.96%,12.83%,13.45% and 14.06%. Similarly for 5% decrease in the parameter β alone(others kept fixed), the five yearly averages are -11.11%,-11.35%,-11.56%, -11.48%,-11.38%,-11.87% and -12.34%. The overall average for the whole period is found to be 13.14% for 5% increase in parameter β alone and it is -11.62% for the whole series for 5% decrease in parameter β alone.

Diagrammatic representation of TSI and PSI is given in Diagram-1, 2, 3 and 4.

6. CONCLUDING REMARKS

Our study carried out in this paper was meaningful in the sense that we may be able to highlight the sensitivity of the parameters of the model. It is observed for TSI indices that the overall increase is about 28% whereas overall decreases -22%. On the basis of PSI indices, overall averages for the respective series for increase as well as decrease in the parameter values indicate that the most sensitive parameter

is β(partial elasticity of labor). Next sensitive parameter is α (partial elasticity of capital) and next is the technology parameter δ.

It seems that labor elasticity is prominent as compared to the other elasticities, it may be convicted that employment in skilled labor could be more effective as there is tremendous advancement in technology in recent times. Change in capital elasticity can also play a significant role in the over all development for the industrial sector as a whole. This may perhaps can induce the technology parameter elasticity also to rise subsequently, thus influencing positive growth. Such an analysis may be fruitful to visualize the growth pattern and for planning strategies corresponding to the industrial sectors of the economy.

7. ACKNOWLEDGEMENT

We thank Dr. B. B. Jani for his guidance and discussion to prepare this paper. We also thank the referee (anonymous) for reviewing this paper which has helped us to revise the earlier draft.

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RESEARCH STUDY ARTICLE

EFFECTIVENESS OF TEACHING TECHNIQUES WITH RESPECT TO INDIVIDUAL SOCIAL RESPONSIBILITY OF STUDENTS

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ABSTRACT

Individual Social Responsibility, its requirements and benefits are growing as a concept gradually from individual to society. ISR has grown out of leading concept of CSR. It can be seen through results of research that Students and teachers recognize the needs of those skills which are top required job skills and still been lacking in students.

These skills an Individual social responsibility is expected from employees in organizations and also as the outcome of education. The study shows the suggestions and requirements for inclusion of ISR, its special teaching techniques and tool to measure ISR in education system and Organizations

KEYWORDS

Social responsibility, teachings, techniques, students responsiveness

1. INTRODUCTION

This report presents the study on 'Effectiveness of teaching techniques with respect to Individual social responsibility of students'. The study having the variables; teaching technique that requiring teachers inputs, students to do with their

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(rcd. April'20 / rvd. May'20)

own views and ISR that takes us to its parent concept CSR (corporate social responsibility) leading us to manager's opinions.

So based on this analysis with the main objective of understanding the gaps and requirements of soft-skills and how can it all compile up to individual social responsibility, three different questionnaires were formed. The skills were researched from various sources most used for acquiring jobs like linkedin, indeed and naukri.com. Best teaching methods were compiled through different research articles. National education policy and India Skills report guided the researcher throughout the study.

Following are the concepts, thoughts and background to support and direct the study:

The first teacher of the child is the mother, the second teacher is the father, and the third teacher is the teacher at school. These gurus open the eye of the - mind of the student to the vast world of knowledge, with its two integral dimensions of the secular and the religious. "Human existence depends upon compassion and curiosity leading to knowledge, but curiosity and knowledge without compassion is inhuman and compassion without curiosity and knowledge is

ineffectual." —Victor Weisskopf, nuclear physicist (Shapiro, 2011). The research tries to:

- To identify the effectiveness of teaching techniques with respect to individual social responsibility of students.

- To identify the importance of individual social responsibility of students as an employee for managers in organizations.

- To identify the individual social responsibility of students.

The major findings of this study has concluded that Individual Social Responsibility carries an important role in getting and doing a good job in any organization. Managers suggest the lacking skills in students, few years of inclusion of ISR and these skills in school curriculum. Students shows the lack in those skills and teachers suggested

various factors affecting this gap. Those factors includes education, student's efforts, teaching methods and lacking resources for the same. This also leads us to the assumption of lacking Individual responsibility in teachers themselves.

This study also bounds it's results with various limitations like lack of time and favorable situations for selection of respondents. Lack of previous studies and Individual social responsibility as newly emerging concept; created the demand for a new tool. Being in experimentation stage it lacks the reliability.

2. REVIEW OF LITERATURE:

India skills report (2021) suggest the firsthand lack of work training can and has done to an individual's ability to feed their family as the oldest of 16 children to graduate from high school and college. Even now, more than five years after graduating from college, I am surrounded by my peers, the world's largest youth cohort of 1.8 billion people. I'm aware that we lack the requisite skills to realise our ideas and the promises made to us by our parents. Currently, 263 million children and adolescents are out of school, and millions more are not receiving the education they deserve. That fact, combined with the fact that nearly half of the jobs that young people are trained for today will not be available in the future, creates a frightening environment.

Every young person needs to have the skills and training they need to be active citizens and successful participants in society. On current trends, it is estimated that by 2030, more than half of all young people will be unemployed due to a lack of necessary skills.

Gujarat Samachar, according to an article in Sandesh, Indian express, is based on an Indian skills report. The eighth edition of the India Skills Report (ISR), released today, reveals that less than half of Indian alumnae are employable. According to the survey, 45.9% of graduates will be employable in 2021, down from 46.21 percent in 2020 and 47.38 percent in 2019. [ ] [

CSR and Sustainability paper discusses:

"Social responsibility is an ethical structure that means that a person has an obligation to collaborate and work with other individuals and organizations for the benefit of society as a whole. Every individual has an obligation to carry out in order to maintain a balance between the economy and the ecosystems."

"Corporate social responsibility (CSR) is a technique used by corporate citizens to improve the living conditions in communities near their business locations. However, Individual social responsibility (ISR) is being established as a culture within the company, with employees being assigned to various community initiatives as part of their individual responsibility to society."

(Indiacsr.in) ISR new dimension towards social uplift ment discusses that

Individual Social Responsibility (ISR) is critical to achieving Corporate Social Responsibility

(CSR).

Many businesses operate solely to make money. Others are serious about giving back to society and providing a valuable service to a group or the whole planet.- of these businesses has a corporate responsibility to its shareholders, patrons, staff, the general public, and the environment in which it operates.

Corporate Social Responsibility (CSR) is achieved by Individual Social Responsibility (ISR) The Golden Rule — "Do unto others as you would have them do unto you" — is as old as the

ISR definition. ISR promotes a practical approach to positively influencing and impacting others in and outside one's immediate circle.

Since a corporation is made up of people, ISR decides the culture of social responsibility that is created. People are becoming more socially conscious, and corporations and businesses must respond by becoming more socially responsible.

Feature.theirworld.org suggest "In the wake of increasing globalization, we have grown into increasingly conscious not only of what we buy, but also of how the

products and services we buy are created," according to the International Organization for Standardization (ISO). Output that is detrimental to the environment, child labor, unsafe working practices, and other inhumane conditions are examples of problems that are being brought to light. Both businesses and organizations seeking long-term sustainability and reputation must first recognize that they must behave in accordance with moral and ethical standards."

Allowing for partial relationships between many personal variables, such as the correlation between cognition and motivation, has been used to investigate schoolchildren's success. Students' social and academic success is influenced by contextual variables such as a child's ability to assume communal responsibility. As a result, students who have more responsibilities have a more positive attitude.

Ministry of education suggest Learners are required to have greater autonomy and effort in learning processes, inspecting learning materials, and comprehending content in the educational system. Students must have skills that allow them to initiate, guide, and monitor the search for information, as well as its subsequent processing and storage, in order to develop their knowledge effectively both within and outside of the classroom find out more. They are needed for students to use in order to advance their educational application of results.

Owing to a shortage of soft skills, there has been an increase in unemployment and a reduction in work satisfaction. Workers' technical skills are no longer sufficient to compete in this intensely competitive global workplace. The value of soft skills cannot be overstated.

3. RESEARCH METHODOLOGY

Research Objectives

Main objective: Analyzing the effectiveness of teaching techniques with respect to individual social responsibility of students. This study tries to investigate, if organizations, teachers and students understand the importance of individual

social responsibility. Also what of Managers expect from the employees.

Under this descriptive type of study, with random sampling technique;

Three different questionnaires for three groups of respondents which are students, teachers and managers were formed and distributed through online links of google forms. Total 56 responses were collected among which 35 were students of (10^{th}, 12^{th} and final year), 10 were teachers and 11 were managers.

The data were further assessed, edited and presented into pie-chart, tables and graphs. These were later interpreted and described in simple words and language that can be easily comprehended.

4. FINDINGS

From the data collected from students:

1. Majority respondents are suggesting communication skills, the skills of patience, peace, satisfaction and happiness, positive attitude, confidence, helping nature, interpersonal and social skills that are required to live a good life.
2. Majority of respondents agree on the purpose of education being learning to live a good life.
3. Majority of students are not aware of their financial requirements.
4. This question tries to find out communication of students with the people around them. More than half students talk to classmates, students of other classes, subject teachers, school teachers, neighbours either most often or regularly.
5. The reason of not talking to remaining people for majority is not being allowed
6. Majority institutes provide teamwork, rarely.
7. All students liked teamwork.
8. The reason behind importance of teamwork for majority of students is the development aspect.
9. Majority students get permission to attend activity period, when they have not finished their homework.

10. For majority of students, 75-100% of times teachers solve their academic problems
11. Majority students have lead at least 2-4 tasks throughout their school and college events.
12. Majority students always finish their task as they have decided on time.
13. For more than majority of students, career and learning is the motivation for coming to school.
14. Majority think they can balance all the work, assignment and program.
15. This question tries to analyze the confidence of students by asking what will they do if they fail this exam, majority are okay to give exam again.
16. Majority will inform someone and leave if teacher is in meeting at the time when student has been asked to stay back for something important bus will leave if he/she stays back anymore. .
17. Majority chose to learning the lesson and apologize, the resilience skill of students
18. The above question tries to analyze the analytical skills of students where 12 out of 35 students have justified their answers. Among which majority have written their interests, 18% are suggesting their ability, 9% chooses the subject because of good teaching and remaining 25% are interested in personal growth.
19. Majority students chose to buy only one pen, this shows the lack of entrepreneurial skills of students.
20. Majority students know how to use docs and paints, this shows the IT skills of students.
21. Majority suggest the purpose of education to each how to live a good life
22. Majority students agree that the purpose of education should be to teach 'how to live life'
23. Majority are not sure if education is full-filling its purpose .
24. Majority students has heard about Individual Social Responsibility

25. 17 out of 35 has written about ISR. Out of 17, 11 has written about individual responsibility towards society

Findings from the Data collected from Teachers:

1. Majority has chosen all of the above skills required to live a good life.
2. 40% has chosen the purpose of education is 'to teach how to live a good life'
3. Majority teachers uses black board method of teaching.
4. Majority teachers uses practical method of teaching.
5. Majority has chosen yes, with the suggestion of focusing individual needs for teaching these skills.
6. Communication, Teamwork, Problem Solving, Organization has got 80% each as the skills that students learn from school based on majority of teacher's perception.
7. The above table shows what all Commercial awareness has been selected by majority respondents.
8. Skills students learn from above table through teaching techniques used by teachers are Communication, Teamwork, Perseverance and motivation, confidence, Leadership as selected by 70% respondents
9. According to majority of teachers, skills lacking in students making it the reason of not getting job are Communication, Confidence and leadership, Perseverance and motivation.
10. Majority teachers agree that ISR is an important skill to be taught in school while
11. 1% disagree on the same.
11. Majority teachers agree that ISR can be taught with better teaching practices while 30% disagree on the same.
12. Majority teachers agree that ISR can be measured with one's actions/decision taken in given situation or observation while 50% disagree on the same.

13. Majority teachers model by taking leadership to do the right thing and taking the initiative.
14. Majority selected student's maturity due to age, 62.5% selected education policy, 62.5% suggested teaching resources by teachers as the causes of gap between expected verses actual practices of ISR as suggested
15. Responsibility of different aspects in not being able to provide stimulating environment for learning these skills as suggested by teacher respondents that involves 42.9% of teaching The above pie chart shows that 14.3% respondents suggest few months to make these skills a permanent part of individuality, 14.3% suggest 1-2 years, and 57.1% suggest 3-4 years
16. Majority teachers has suggested Yes teacher for inclusion of ISR in education in school and 14.3 has suggested No.
17. 6^{th} grade is suggested by 3 out of 6 teacher respondents for inclusion of ISR.
18. This question tries to find out teaching techniques that helps in learning CSR based on following suggestions by teachers: Demonstration, field and volunteer work, teamwork, role play, practice based techniques, awareness programs and self-care practices.
19. Majority percent of teachers suggested personal model and kinesthetic learning teaching techniques for teaching ISR
20. Majority agrees, 11.1% disagree and 44.4% responds maybe to get trained in specialized techniques for ISR.

Findings from responses of HRs/Managers

1. Majority has chosen the skill of surviving with peace in all situations as an important skills for good life
2. Majority respondents chose to create employees, 36.4 suggest to teach how to live a good life, 9.1% suggest to learn to be exploratory in life; the purpose of education.

3. Majority respondents think that education is not full-filling its purpose..
4. Communication and Teamwork and confidence, each one has been selected by majority respondents. Problem solving has been selected by 90.9% respondents as topmost required skills for getting job.
5. Organization and ability to work under pressure each one has been selected has top skills for initial training of employees , by majority
6. Percent of ineffectiveness of different aspects as selected by respondents in which:
7. 9.1 For majority; family, education, teaching techniques, classroom environment, curriculum and education policy has been selected by 5,1,2,3,2,3 respectively
8. 9.2 For majority; family, education, teaching techniques, classroom environment, curriculum and education policy has been selected by 2,6,3,1,4,2 respectively.
9. 9.3 For m/ajority; family, education, teaching techniques, classroom environment, curriculum and education policy has been selected by 5,1,2,3,2,3 respectively.
10. For majority; family, education, teaching techniques, classroom environment, curriculum and education policy has been selected 2,1,1,1,2,2 respectively.
11. Majority thinks education is in failure of equipping students with these skills, 9.1 suggest the failure of family.
12. Majority of respondents think that ISR is an important skill to be taught in school.
13. Respondents suggest that 27.3% inability of teaching techniques and curriculum and 36.4% select curriculum and education policy.
14. Majority respondents suggest few months to make these skills a permanent part of individuality, 27.3% suggest 1-2 years and others suggest 2-3 years, it depends on individual, include it more on subjective basis and entire education system for many years.
15. Majority respondents suggest the inclusion of ISR as a major part of education

in school

16. Majority respondents suggest the major role of ISR in performing better at job while other 45.5% are not sure about it.
17. Majority respondents disagree that education system is serving to ISR
18. Majority respondents suggest that 'ISR tool will surely help in assessment and selection of employees.

5. CONCLUSIONS:

Individual social responsibility is an important skill to be taught in school while it is not being inculcated by Education System. It should be included as major part for minimum few months to many years in education system as it plays a major role of ISR in performing better at job.

Most of the skills need special attention through either teaching techniques or education. All

of these skills are required may it be for getting a good job or to live a good life which is the purpose of education.

Individual Social Responsibility is a concept which is still growing and it should be formally introduced in education system for minimum 6 months to 4 years. Teachers agree to get trained in special teaching techniques for ISR and Managers suggest that ISR tool will surely help in assessment and selection of employees.

6. ACKNOWLEDGEMENTS

We thank the referee for reviewing our paper. This has helped us to revise our earlier draft of this paper.

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BIOGRAPHY

HERALD CRAMÉR*

H. D. BUDHBHATTI**

Harald Cramér (25 September 1893 – 5 October 1985) was a Swedish mathematician, actuary, and statistician, specializing in mathematical statistics and probabilistic number theory. John Kingman described him as "one of the giants of statistical theory".

Harald Cramér was born in Stockholm, Sweden on 25 September 1893. Cramér remained close to Stockholm for most of his life. He entered the University of Stockholm as an undergraduate in 1912, where he studied mathematics and chemistry. During this period, he was a research assistant under the famous chemist, Hans von Euler-Chelpin, with whom he published his first five articles from 1913 to 1914. Following his lab experience, he began to focus solely on mathematics. He eventually began his work on his doctoral studies in mathematics which were supervised by Marcel Riesz at the University of Stockholm. Also influenced by G. H. Hardy, Cramér's research led to a PhD in 1917 for his thesis "On a class of Dirichlet series".

* Adapted from wikipedia (the free encyclopedia) and other related resources. (We express our sincere thanks and gratitude for this assistance)

** Ex. CSO, head, Statistics Dept., GSRTC, Ahmedabad.

(Thanks to the referee for reviewing this article)
(rcv. May '21 / rvd. June '21)

ACADEMIC PROFESSIONAL CAREER

Following his Ph.D., he served as an Assistant Professor of Mathematics at Stockholm University from 1917 to 1929. Early on, Cramér was highly involved in analytic number theory. He also made some important statistical contributions to the distribution of primes and twin primes. His most famous paper on this subject is entitled "On the order of magnitude of the difference between consecutive prime numbers", which provided a rigorous account of the constructive role in which probability applied to number theory and included an estimate for prime gaps that became known as Cramér's conjecture.

In the late 1920s, Cramér became interested in the field of probability, which at the time was not an accepted branch of mathematics. Cramér knew that a radical change was needed in this field, and in a paper in 1926 said, "The probability concept should be introduced by a purely mathematical definition, from which its fundamental properties and the classical theorems are deduced by purely mathematical operations." Cramér took an interest in the rigorous mathematical formulation of probability in the work of French and Russian mathematicians such as Kolmogorov, Lévy, Bernstein, and Khinchin in the early 1930s. Cramér also made significant development to the revolution in probability theory. Cramér later wrote his careful study of the field in his Cambridge publication *Random variables and probability distributions* which appeared in 1937. Shortly after World War II, Cramér went on to publish the influential *Mathematical Methods of Statistics* in 1946. This text was one that "showed the way in which statistical practice depended on a body of rigorous mathematical analysis as well as Fisherian intuition."

In 1929, Cramér was appointed to a newly created chair in Stockholm University, becoming the first Swedish professor of Actuarial Mathematics and Mathematical Statistics. Cramér retained this position up until 1958. During his tenure at Stockholm University, Cramér was a Ph.D. advisor for 10 students, most notably Herman Wold and Kai Lai Chung. In 1950 he was elected as a Fellow of the American

Statistical Association. Starting in 1950, Cramér took on the additional responsibility of becoming the President of Stockholm University. In 1958, he was also appointed to be Chancellor of the entire Swedish university system. Cramér retired from the Swedish university system in 1961.

ACTUARIAL CAREER

A large portion of Cramér's work concerned the field of actuarial science and insurance mathematics. During the period from 1920 to 1929, he was an actuary for the life insurance company Svenska livförsäkringsbolaget. His actuarial work during this time led him to study probability and statistics which became the main area of his research. In 1927 he published an elementary text in Swedish Probability theory and some of its applications. Following his work for Svenska livförsäkringsbolaget, he went on to work for Återförsäkringsaktiebolaget Sverige, a reinsurance company, up until 1948. He was also known for his pioneering efforts in insurance risk theory. After this period, he remained as a consultant actuary to Sverige from 1949 to 1961. Later in his life, he was elected to be the Honorary President of the Swedish Actuarial Society.

LATER YEARS

Cramér remained an active contributor to his professional career for an additional 20 years. Following his retirement in 1961, he became extremely active in research, which had been slowed due to his Chancellorship. During the years from 1961 to 1983, Cramér traveled throughout the United States and Europe to continue his research, making significant stops at Berkeley, Princeton, and at the Research Triangle Institute of North Carolina.

Cramér received an Honorary Doctorate from Heriot-Watt University in 1972.

His academic career spanned over seven decades, from 1913 to 1982.

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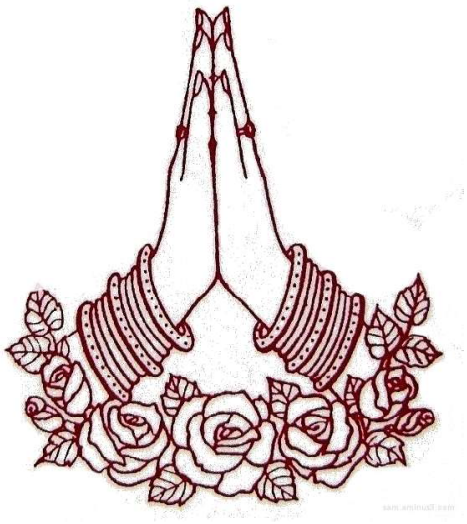
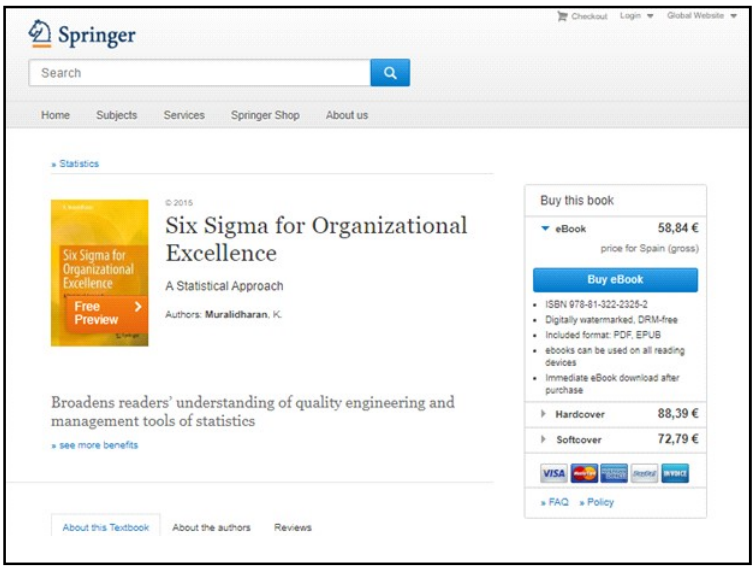


BOOK REVIEW



I am extremely happy to write this review on the book "**Six Sigma for Organizational Excellence: A Statistical Approach**" written by Prof. (Dr.) K. Muralidharan and published by Springer Nature India. It is a matter of pleasure to read a book written by my own student who has high potential of research and excelled in multidisciplinary areas of Statistics subject. The book is by now a best seller as there is a great demand for the book because of its material content and presentation. The book is a well documented material for Managers, Engineers, and professionals working in the area of Quality. We have this book in our department library recommended as a text book-cum-reference book for our M.Sc(QPM) students at the postgraduate level.

Six Sigma is a business process improvement tool to achieve customer satisfaction through a systematic problem solving approach. It is uniquely driven by close understanding of customer requirements and reinventing business processes. It facilitates people excellence as well as technical excellence in terms of creativity, collaboration, communication, dedication and above all increases the accountability of what one does in an organization. The Six Sigma philosophy works under a five-phase improvement cycle called DMAIC where D for *define*, M for *measure*, A for *analysis*, I for *improvement* and C for

* Abouth the Author
Dr. K. Muralidharan, Professor of Statistics,
Dept. of Statistics, M. S. University, VADODARA

control. Interestingly, the book is written in the same DMAIC philosophy explaining the necessity of each phase in project management.

There are about fifteen chapters that blend management and statistical tools equally for quality improvement in any aspects of organizational segments. It can apply to both process improvement and product improvement and even design redesign efforts of any organization. This book offers all those essential ingredients for business process improvements as perceived by people in industry. All the statistical concepts are explained systematically and elegantly with respect to business problems and contexts. There is a chapter exclusively for sample size determination that will help decision makers to collect appropriate data for quality assessments. The uniqueness of the book can also be viewed from many interesting chapters like: sigma level estimation, Six Sigma Marketing, case studies and so on. The book also enables one to work on projects associated with scientific decision making. Emphasis is also laid on understanding and applying the concepts of quality through project management and technical analysis using statistical methods.

I, therefore, strongly recommend this book for every statistical library and scientific institutions for professional development and career in Six Sigma.

Date : 30/05/2021

Prof. (DR). Ashok Shanubhogue
Ex. Head, Dept. of Statistics,
S. P. University, Vallabh Vidyanagar
Gujarat State.

SV NEWS LETTER

Manish B. Thaker*

We all are passing through CORONA age and experiencing CORONA effects during this crisis peirod. It has immense effects upon our social, economic and political systems. The worst effect is with our educatinal system. Eventhough online education and virtual platforums are becoming popular and mandatory, we are slowly heading towards a crisis and collapsible state. It is very difficult to predict at this stage what will be the future growth of our educational system if the same or similar circumstances go on continuing.

In my personal opinion, a time has come for our country to think about open textbook examinations as it is done in many countries abraod. Ofcourse this needs to think about appropriate infrastructural development.

* * *

* Dr. Rakesh R. Pandya has been promoted on as Director of DIES.

* Dr. Hiten S. Parekh is promoted as Director (Evaluation) at DIES.

* On the eve of the 15th Statistics Day, 2021, the department of Agricultural Statistics, N. M. College of Agriculture, NAV, Navsari proposes one day webinar on **'Statistics for Food Securing and promoting Sustainable Agriculture** on 29th June in association with NSSO and GSA. There will be seminar with discussions to highlight official statistics as well as Bigdata Science and Information Communication Technology.'

-Dr. Alok Shrivastav, M : 9424242849, 9408985065

* * *

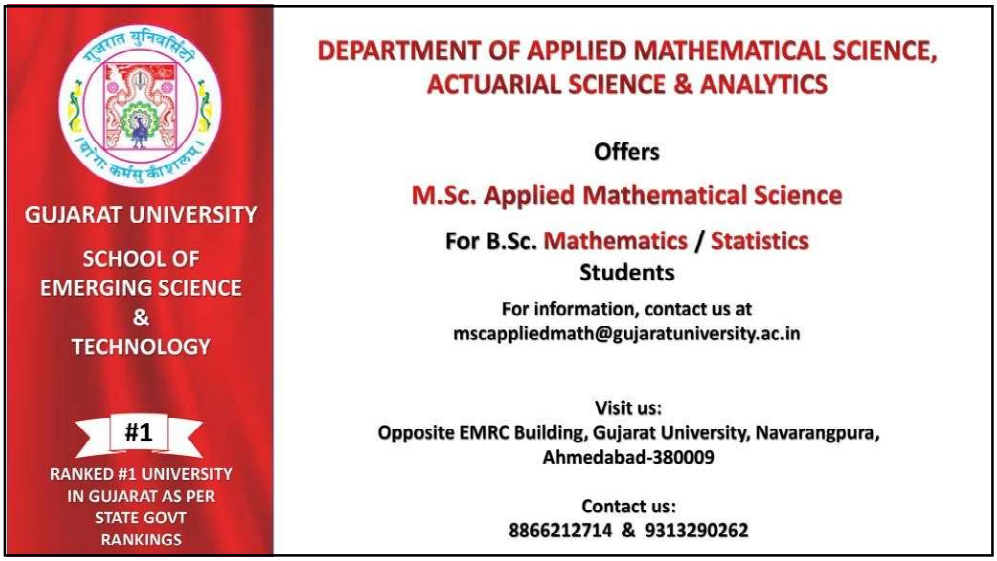
There are three specific news about special courses designed at two places which I come to know at present.

(1) Gujarat University school of emerging science and technology.

(2) Department of Applied Mathematical Science, Acturial Science and Analystical at Gujarat University

(3) M.Sc. Degree course in supply chain management at P.G. and Research Department of Mathematics, Bishop Heber College, (Autonomous) at TRICHY, Tamilnadu. (Brief handouts are presented below for giving apriorie information.

* Head, Statistics Dept., M. G. Science Institute, Ahmedabad.
mbthaker2768@gmail.com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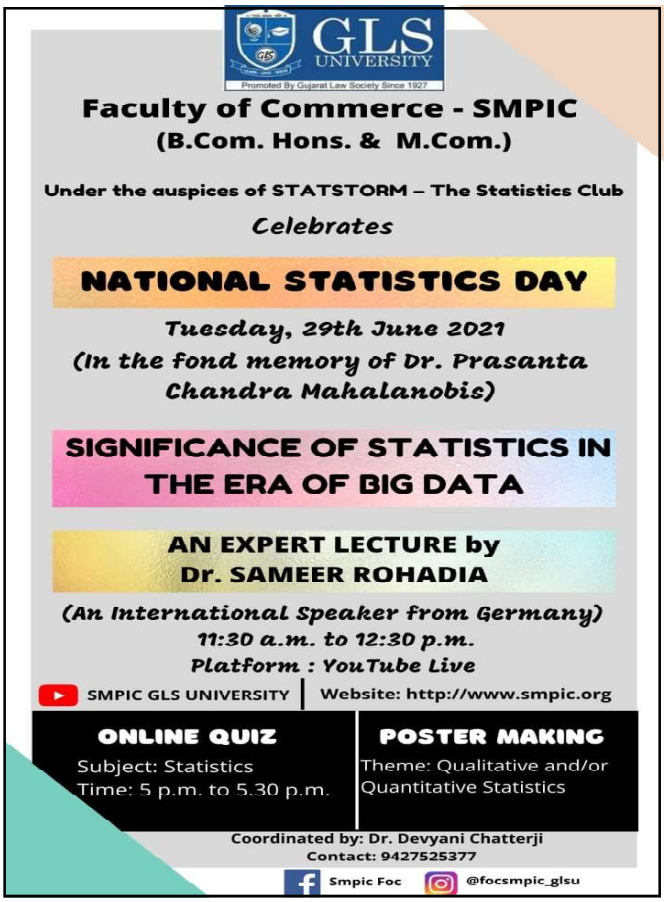
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2 Supply Chain Data Management and Analysis
3 Global Supply Chain Management and Logistics
4 Strategic Procurement and Global Sources
5 R and Python Programming Lab
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* Tie up between Bishop Heber College, Trichy, India & Appalachian State University, Boone, USA
Why Bishop Heber College [BHC]?
Bishop Heber College has beautiful green surroundings, possessing a very good Infrastructure facility. Supply Chain Management Department has very good versatile professors. The best in all India ranking. Securing many university ranks in Bharathidasan University. Many Eminent Scholars are on board. Currently doing an extensive service to the student community through higher education. Presently the first selection for many youngsters is Bishop Heber. Pioneer in introducing new courses always
Why Appalachian State University [ASU]?
The Supply Chain Management program at Appalachian State University realizes the value of professional development and real-world experience. Learn more about how the Walker College of Business, accredited by AACSB since 1976 and consistently ranked as a best business school by The Princeton Review, supports and prepares students throughout their journey.

Dr. DINESH S DAVE
Professor & Director of Supply Chain Management
Appalachian State University, Boone, USA.
daveds@appstate.edu

Speciality of this Twining Program Pupils are extremely privileged to have faculties from both the Institution.

- Dr. D. Paul Dhayabaran - Principal
- Dr. Perumal Mariappan - BHC
- Dr. Dinesh S Dave - USA
- Dr. M. Antony Raj - BHC
- Dr. B.K.Mohanty - IIM Lucknow
- Dr. R. Bhargavi - Entrepreneur
- Dr. J. Kavikumar - Malaysia
- Dr. C. Muguntharajan - KSR

For large organizations, the ability to incorporate performance trade-offs across multiple functional areas, both internal and external, is critical. Meanwhile, small and mid-sized businesses must be able to deliver a unique value proposition in an increasingly competitive marketplace. As a result, supply chain management professionals are growing in importance across business types, industry sectors, and global regions.

For progressive business leaders, the need for supply chain management knowledge and skills has never been greater

High demand — and high pay — for supply chain managers focused on market intel : Dive Brief

CAREERS

- Supply Chain Manager
- Inventory Manager
- Supplier Quality Product Manager
- Strategic Sourcing Manager
- Logistics Planner
- New Product Procurement Planner
- Supplier Management Analyst
- Inventory Planner
- Procurement Manager of Specialist
- Logistics Analyst
- Supply Chain Consultant
- Wholesale and Retail Buyer

Number of Industries in Tamilnadu City wise Breakup

CITY	COUNT	CITY	COUNT
Chennai	16406	Madurai	852
Coimbatore	4986	Erode	639
Tirupur	3406	Others	4187
Karur	1825	Total	32301

What is Supply Chain Management?

Supply chain management is not just a process for reducing costs and achieving greater operational efficiencies within an organization. Although these are important considerations, modern supply chain management involves the strategic alignment of end-to-end business processes to achieve market and economic value, as well as competitive advantage.

Supply chain management is achieved by an organization proactively adopting initiatives to move suppliers and customers into collaborative relationships for mutual gain. Business professionals who want to make a significant impact on the efficiency, effectiveness or profitability of their organizations should consider learning more about supply chain management and its value for companies operating in a competitive global marketplace.

3 Reasons a Career in Logistics and Supply Chain Management is the Smart Choice for Your Future

- Good Pay and High Demand Come with a Career in Logistics and Supply Chain Management
- Graduates of Logistics Courses will Rarely be Bored on the Job
- A Career in Logistics Could include Opportunities for Travel

Supply Chain News

High Demand for Supply Chain Professionals in the Manufacturing and Retail Industries

Because of advancements in digital technology and an increased focus on supply chain risk and globalization, supply chains are expanding in scope and the need for top talent has increased with it, and companies are having to pay more for the right people.

Supply Chain Management is in Growing Demand

In the 21st Century, supply chain management professionals are expected to possess the knowledge and capabilities to support the improved efficiency, effectiveness and profitability of modern businesses.

ELIGIBILITY
Any Under Graduate Students with Mathematics / Statistics as Major or Allied Level

RESERVATION
Tamil Nadu : 15 Seats
Other States : 10 Seats

SELECTION MODE
Written Test (Objective Type)
Personal Interview

READERS FORUM

Ashvin J. Patel*

* **Achyut Patel (Rajkot)**

Welcome to new format of SV Journal in Digital form, we find inspiring papers in the issues of SV. Best wishes to the whole team.

* **Paresh M. Prajapati (Ahmedabad)**

My feeling is that more applied research papaers should be included. Section on statistics and managements should be continued further. Efforts by SV team are really praiseworthy. Please keep this spirit further. Best luck.

* **K. Muralidharan (Vadodara)**

Appreciable works presented in short times. Progress link of the issue has been a continous chain. It is a good and healthy sign of the academic work. Keep the spirit to march ahead.

* **Bipin Mehta (Ahmedabad)**

I did not like the idea to stop printing of the issues and send only digital copy. To read a book in one hand and a cup of tea in the other hand is a pleasure we have experienced life time. However I appreciate the efforts of SV team.

* **Bhavin Shah (Indore)**

Digital copy of SV issue is a good venture. Make it more and more expanding and interesting. You may include research project studies, querries, classroom notes, suggestion box to make issues more lively and interesting. Best Luck.

* **Pinakin Jani (Industry)**

There is a short coming of not having many articles connected with industrial applications. Management and statistics section was really good which always gave new ideas. Please continue the same. Congrats to the SV team.

* Head, Statistics Dept., R. H. Patel Arts & Commerce College, Vadaj, Ahmedabad.

Gujarat Statistical Association

Established : 1969

[Registered under Public Trust Act of 1950 (Bombay)]

R. No. E2502 A'bad - 1974

The objective of the association is primarily to promote statistical ideas in pure and applied fields in the form of study, teaching and research in statistics. The membership of GSA consists of Life / institutional / ordinary members.

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P C MAHALANOBIS

Statistics Day - 29th June 2021

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EDITORIAL ADVISARY BOARD

RESEARCH TEAM

HERALD CRAMÉR*

Harald Cramér was born in **Stockholm, Sweden** on 25 September 1893. He entered the **University of Stockholm** as an undergraduate in 1912, where he studied **mathematics** and **chemistry**. He began to focus solely on **mathematics**. He eventually began his work on his doctoral studies in mathematics at the University of Stockholm led to a Ph.D. in 1917.

Following his Ph.D., he served as an Assistant Professor of Mathematics at Stockholm University from 1917 to 1929. He laso made some important statistical contributions. In the last 120s, Cramer became interested in the field of probability. In 1929, Cramer was appointed to a newly created chair in Stockholm University, becoming the **first Swedish Professor of Actuarial Mathematics and Mathematical Statistics.** Cramér retained this position up until 1958. Cramer remained an active contributor to his professional career for an additional 20 years. Following his retirement in 1961, he became extremely active in research, which had been slowed due to his Chancellorship. During the years from 1961 to 1983, Cramer traveled throughout the United States and Europe to continue his research, making significant stops at Berkeley, Princeton, and at the Research Triangle Institute of North Carolina.

His academic career spanned over seven decades, from 1913 to 1982.

Known for Cramér-Rao bound, Cramér's conjecture, Cramér's decomposition theorem, Cramér's theorem (large deviations), Cramér-World theorem, Cramér-von Mises criterion, Cramér function of large deviations theory, Ruin Theory, Cramér's V Cramérs inequality.

Awards Guy Medal (Gold, 1972)

***Brief biographicasl sketch is given inside the journal.**

Printed Matter **BOOK-POST**
(Journal of GSA, Ahmedabad)

To,

From : **Dr. B. B. Jani**
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