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EDITORIAL

LIKE ALMIGHITY GOD,

STATISTICS ARE OMNIPRESENT

We are extremely happy to present this issue (NSV 15, June 2019, No. 1) to our readers. We express our sincere thanks to all our contributors, evaluaters, readers and well wishesh for their continuous and consistent support which has helped us to achieve our goal.

This issue contains **two articles, three research articles, two research notes, two book reveiws, one biography** along with **other items** as usual.

Under the caption of **Management and Statistics, first article** is about an interesting feature associated with market research. It is presented by **A. C. Brahmbhatt.**

Second Article under this caption is presented by **D. S. Dave.** It is about global supply chain management pertaining to India.

First Research Article describes very lucidly about multiple testing procedures. It has been furnished by **Pinakin R. Jani.**

Second Research Article discusses a very crucial and interesting problem regarding simpling and inference under shrunken estimators. It is presented by **U. B. Gothi.**

Third Research Article discusses Human Development status for Rubanised Talukas. It is furnished by **P. H. Thaker.**

First Research Note describes in a lucid manner the problem of confounding in 2^n symmetrical factorial experiements. It is presented by **D. K. Ghosh.**

Second Research Note discusses a very interesting application for Hospital Managment. It is presented by **P. Mariappan, M. B. Thaker** and **Jenifer Christinal.**

Biography of famours Mathematician cum statistician **Pierre De Fermat** is furnished by **H. D. Budhbhatti.**

There are **two book reviews**. Which are presented by **H. M. Dixit** and **M. B. Thaker.**

S. V. News Letter provides some useful informations about new courses as well as some leading seminars and conferences. It is presented by **M. B. Thaker.**

Readers Forum provides some views expressed by readers. It is furnished by **A. M. Patel.**

We are highly indebted to our following referees who have done excellent job of evaluations for the articles/papers submitted for this issue.

(Their names are given one by one in order of their appearance in the journal.)

(1)	**D. S. Dave**	**(2)**	**H. S. Mody**
(3)	**Bhavin Shah**	**(4)**	**M. N. Patel**
(5)	**H. M. Dixit**	**(6)**	**R. G. Bhatt**
(7)	**S. G. Raval**	**(8)**	**Shailesh Teredesai**
(9)	**A. M. Patel**	**(10)**	**Paresh Prajapati.**

We are extremely happy to announce that website of the journal is ready now from July 1, 2019. You can also meet us on **www.sankhayvignan.org.** Kindly give your feedback and suggestions.

We express our sincere thanks to **Shree Ashish Bhatt for Website, Shree Dinesh Darji for DTP work** and **Shree Mehul Shah for printing works**.

Digital copy of this issue will be sent first to all our readers whose email ID are with us. Printed copy will follow soon. Our contributors will be given offprints of their published article alognwith the printed copy and certificate.

Wishing you good health, prospects with seasons' greetings.

Ahmedabad

Date : 30-06-2019

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FROM EDITOR'S DESK

SANKHYA VIGNAN is a peer reviewed refereed Bi-Annually journal that publishes empirical, conceptual and review papers of exceptional quality that contribute to Statistics Theory and enriched Applications of Statistical Techniques in various fields. The objective of the Journal is to disseminate knowledge, which ensures good practice of professional management and its focal point is on research and reflections relevant to academicians and practitioners in the field of **Applied Statistics**.

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(6) To keep all records in perfect order.

(7) Invite and accept suggestions made by the Refrees for Peer Review.

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6. Article should be typed in 12 point - Times New Roman Fonts English with a one and half space and single column with 1 Margin on a Standard A4 size Paper. The Page numbers should be at the center of every page. All headings & sub headings must be in bold letters.
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8. All the tables, charts, graphs, diagrams should be in black and not in colors.
9. Footnotes, italics, and quotation marks should be kept to the minimum.
10. References should be mentioned in APA Referencing Format.

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MANAGEMENT AND STATISTICS
ARTICLE

COOLHUNTING - A SPECIFIC FORM OF MARKETING RESEARCH

A. C. Brahmbhatt[1]

ABSTRACT

This article describes briefly a specific behaviour called coolhunting by individuals which play a very vital role in market research analysis. Many reputed companies these days provide platform for such coolhunting. It is a versatile form for market research.

KEYWORDS

Trend Spotting, FGD, Cool Narcs

The marketing research department of any business unit systematically collects the information regarding the customers' aspirations, expectations, their purchase attitude and behavior, their traits etc. ; analyze them using simpler to sophisticated analytical techniques and draw meaningful inferences that ultimately helps refine the decision making process. It tries maximum to reach out almost all segments of customers. But reaching out some specific portions of youth segments, particularly the 'teen' and 'preteen' segments which are often referred to as 'stubborn demographic' as they do not respond even to the blatant advertising and marketing campaigns targeted at them is difficult. They are defined as the 'cool' people who are endowed with the aesthetic of attitude , behavior, appearance, style etc. They are simply attracted to something which is creative,innovative, non-conventional and awesome like new fashions, new apparels, new clothing styles, new films, new drinks, newer fashionable pairs of shoes, newer mobile apps etc. They demonstrate specific attitude towards such

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objects and that too remains frequently changing.

Some researchers having conducted the scientific surveys tried to describe the characteristics of such 'cool' individuals. They are more often found hedonistic, modern, fashionable, non-traditional, self-confident, humorous, creative and having positive attitude towards brands, products, people etc. They mostly belong to 'teen' or 'preteen' segments.

The coolhunting , the term that was first coined in early 1990 referring to a new kind of marketing researchers –the 'cool hunters' , who make observations and predictions in changes of new or existing 'cool' cultural fads and trends. It is also known as ' trend spotting' –a subset of trend analysis. Initially the coolhunters were particularly focused on fashion and design , but with the advancement of time, they have included other sectors such as technology.

Since fathoming into the unpredictable and inscrutable youth's attitude and behavior is not that easy ,this form of marketing research is considered to be more difficult than other forms of marketing research. The coolhunters have to be extras careful and at times even stealthy in their methods of getting relevant information. The methods like Focus Group Discussions (FGDs) are arranged in which a meaningful interaction and discussion is initiated with youths. The moderator initiates the discussion and invites each one of the group to participate freely .Mostly the issues related to their lifestyle, their culture, their attitudes towards existing fashion trends,their likes-dislikes for them , their predictions about future trends are discussed and the content is recorded.While moderator remains busy conducting the discussion, other 'coolhunters' observe their mannerism and body language. The whole exercise would provide direct insight into the thoughts and feelings of their target demographics .Participants of Focus Group are rewarded in terms of cash or freebies or some other rewards.

The coolhunters also conduct online surveys or participate in chatrooms and webgroups posing as participants and gather information pertaining to trends emerging or declining specially in fashion and technology on the basis of which they make some predictions too. The coolhunters have capability to recognize the innovativeness and creativity in the youths under observation . They are the ones who best comprehend ' The spirit of time' . They have capability of sensing the current and future attitude of these segments towards different brands and products and they also know as to how would it be impacted by social, cultural and demographic factors.

As more and more markets are becoming saturated and differentiated, there are segments of consumers who are stubborn not responding to promotional campaigns of the companies, the role of coolhunters would become more relevant. The coolhunters should have specific skills and knowledge of almost all products and brands in the market. He should be constantly inquiring about 'cool' places, 'cool' products and'cool'people.He should have sharp eye of an observation. He should be well convergent with the changing attitudes of the people in fashion, music, films, books, Tv, Internet , Pop music, mobile apps etc.

Coolhunters will often seek out individuals from within their target demographics who are regarded as leaders or trend setters . They will then hire them as Cool Narcs , who gather information secretly from their peers and report their back to their employees.

Many of the world largest companies have been known to employ in house coolhunting department, Viacom's MTV television network is a classic example of it.

In the digital world now there are virtual platforms for accessing the 'cool' segments and there are no boundaries, no borders to gather intelligence on street fashions and trends.

Coolhunting thus is a versatile form of marketing research that minutely observes the emerging trends and styles usually involving street fashions.

ACKNOWLEDGEMENTS

I thank the referee for reviewing my article.

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GLOBAL SUPPLY CHAIN IN INDIA

DINESH S. DAVE[1]

ABSTRACT

The main objective of supply chain management (SCM) is to make organization more competitive and efficient by enhancing customer satisfaction. Each participating organization in the chain of supply adds the value and increase potential benefit of earning optimal profit. Supply chain management (SCM) continues to gain visibility in today's competitive business environment. Global supply chain management (GSCM) incorporates global organizations in the network of the chain of supply. GSCM is getting more attention as various organizations are linked in a global business network. As the world becomes more flat, the field of GSCM continue to play important role in organizations. GSCM continue to become more complex because international companies being added almost daily in the global supply chain network. This complexity can be addressed effectively with the implementation of emerging technology. The objective of this study is to discuss the importance SCM and GSCM in a global competitive environment. The paper also shades the light on the current economic development in India and how the principles of SCM and GSCM can be applied in India.

KEYWORDS

SCM, GSCM, LOGISTIC

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INTRODUCTION

SCM is the coordination of all activities from raw materials to the finished goods for customer satisfaction. SCM is considered as one of the most important functions in an organization to enhance a firm's competitiveness. A supply chain network incorporates suppliers, producers, distributors, wholesalers, and retailers working together to deliver products of high quality to consumers. SCM continues to gain visibility due to the influence of globalization because numerous organizations engage their business functions with their global partners. Companies are operating in a global supply chain network in today's competitive business environment. Vitasek (2013) explained supply chain management as encompassing the planning and management of all activities involved in sourcing and procurement, conversion, and logistics management activities. Supply chain encompasses three flows: information flow, materials flow, and financial flow. The SCM can also include two additional flows – value flow and risk flow.

GSCM is gaining more visibility due to the effect of globalization as many organizations operate their business functions with their partners globally as a result GSCM is considered as the most critical function in an organization. GSCM is defined as the distribution of goods and services throughout a trans-national companies' global network to maximize profit and minimize waste (Bhatnagar, 2013). GSCM focuses on companies and organizations that are trans-national. According to Thomas and Hult (2003), GSCM has four main areas of concentration: marketing, logistics, supply management, and operations management and successful GSCM requires complying with various international regulations. In GSCM, organizations of one country develop partnership and/or dependency with organizations in other countries to achieve competitive advantages. The objective of this study is to discuss the importance SCM and GSCM in a global competitive environment. The paper also shades the light on the current economic development in India and how the principles of SCM and GSCM can be applied in India.

GLOBAL SUPPLY CHAIN IN INDIA

GSCM is a worldwide network of suppliers, manufacturers, warehouses, distribution centers, and retailers to acquire raw materials, conduct transformation processes, and deliver products to customers. GSCM helps an organization in exploring new markets, enables business growth, and companies can learn about to new technologies from their international partners. GSCM continue to become more complex since international companies being added on a daily basis in the global supply chain network. This complexity can be addressed effectively with the implementation of emerging technology. GSCM provides absolute advantage and comparative advantage. Absolute advantage includes lower cost and/or access to items not available locally. The comparative advantage include differences in the cost of producing products in different countries.

The contributing factors in GSCM include: population size, urbanization, land and resources, technology, and globalized economy. India is the second most populous country, followed by China. India's population is approximately 1.34 billion people as compared to China of 1.39 billion people. It is estimated that the population of India will surpass that of China's in next 5 years. With increasing purchase power and larger middle class, there will be several opportunities for global organizations to market their products in India. Many companies have shifted their production facilities in India to capture the Indian market. GSCM will continue to become more critical function for companies in India to enhance customer satisfaction as well as minimize the manufacturing and logistics costs.

Today, 55% of the world's population lives in urban areas and it is expected to increase to 68% by 2050. Tokyo is the world's largest city, followed by New Delhi and Shanghai. Currently, Cairo, Mumbai, and Beijing all have close population. Delhi is projected to continue growing and to become the most populous city in the world in next decade. As the world continues to urbanize, sustainable development depends increasingly on the successful management of urban growth. India will continue to face challenges in meeting the needs of their growing urban populations, including for housing, transportation, energy systems, infrastructure, employment, and education. The advancement in technology will play an important role when GSCM become more complex due to the addition of partners in the chain of supply. Technology is becoming more affordable, therefore many organizations are encouraged to implement appropriate technology to enhance their chain of supply. Economic globalization is the increasing interdependence of world economies as a result of the growing scale of cross-border trade of commodities and services, flow of international capital and wide and rapid spread of technologies. It is critical that global sourcing decisions be made while accounting for total cost that should include the impact of global sourcing on freight, inventories, lead time, quality, on-time delivery, minimum order quantity, working capital, and stock-outs. Further, according to United Nations, India remains one of the fastest growing economy in 2019 and 2020. India's GDP growth is expected to be 7.4% to 7.6% in 2019-20.

One of the important component in GSCM includes infrastructure. Infrastructure includes transportation, utility, and resources. Recently, there is a focus to build 3,000 km of expressways allowing uninterrupted traffic flow. The proposed expressways will connect Varanasi – Ranchi – Kolkata; Indore – Mumbai; Bengaluru – Pune; Chennai – Trichy).

This Greenfield (i.e., land never been used) road will enhance the road infrastructure to reduce the logistics cost and improve the delivery time. In addition to road infrastructure, India is rapidly growing the utility infrastructure. For example, more electricity is now available in the rural areas and India is emphasizing on solar energy. For example, about 67% of the electricity at Bengaluru's Kempegowda International Airport is generated by solar and about 100% of the electricity at the Cochin Airport is from solar energy. With improved infrastructure and sustainable energy will facilitate faster, multimodal, and better operations in India. The public private partnership (PPP) initiative and introduction to dedicated freight corridor along with rail speed as much as 100 km per hour will enhance the fleet modernization. Improved infrastructure will provide the supply network the ability to handle high volume of material and will provide an opportunity for multimodal logistics by integrating road, air, sea, and rail. With the enhancement GSCM in India, it is projected that India will be more connected with rest of the world.

CONCLUSIONS

GSCM is viewed as the most critical functions from sourcing to the logistics of the finished goods to the final consumers. GSCM continue to gain more visibility because of globalization as many organizations operate their business functions with their partners globally; as a result GSCM is considered as the most critical function in an organization. Some of the variables related to GSCM include GDP, population, FDI, ease of doing business, transportation infrastructure, utility infrastructure, connectivity, and others. According to the World Bank Factbook, WTO, and other organizations, India continues to enhance its GSCM.

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RESEARCH ARTICLE

OVERVIEW OF MULTIPLE TESTING PROCEDURE

PINAKIN R.JANI[1]

ABSTRACT

In application of clinical trials, tested hypothesis are often grouped as multiple hierarchically ordered families. To test such structured hypotheses, various gatekeeping strategies have been developed, such as such as series gatekeeping, parallel gatekeeping, tree-structured gatekeeping strategies, etc. However, these gatekeeping strategies are often either non-intuitive or less ûexible when addressing increasingly complex logical relationships among families of hypotheses. In order to overcome the issue, researchers have develop a new family-based graphical approach, which can easily derive and visualize diûerent gatekeeping strategies. In the proposed approach, a directed and weighted graph is used to represent the generated gatekeeping strategy where each node corresponds to a family of hypotheses and two simple updating rules are used for updating the critical value of each family and the transition coeûcient between any two families. Theoretically, the efforts are made by researcher to propose that graphical approach strongly controls the overall family wise error rate at a pre-speciûed level. The overview will cover the case studies and some clinical examples, to demonstrate simplicity and ûexibility of the proposed approach.

KEYWORDS

Multiplicity**,** Bonferroni, Holm, Simes, Hochberg, Dunnet, stepwise Dunnet.

INTRODUCTION

The interest in the problem of multiple comparison began in the 1950s with the work of Tukey and Scheffe, other methods such as closed testing procedures (Marcus et al., 1976) and the Holm-Bonferroni method (1979), later emerge. In 1955, work on the false discovery rate began. In 1966 the first conference on multiple comparisons took place in Israel. Multiple testing refers to any instance that involves the simultaneous testing of more than one hypothesis, since each hypothesis has potential to result in

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to "discovery". The confidence level stated generally applies to each test considered individually but often it is appropriate to have confidence level for whole family of simultaneous tests planned. Failure to compensate for multiple comparisons can have important real-world consequences as illustrated by following examples:

- Suppose the treatment is a new way of teaching writing to students, and the control is the standard way of teaching writing. Students in the two groups can be compared in terms of grammar, spelling, organization, content, and so on. As more attributes are compared, it becomes increasingly likely that the treatment and control groups will appear to differ on at least one attribute due to random sampling error alone.
- Suppose we consider the efficacy of a drug in terms of the reduction of any one of a number of disease symptoms. As more symptoms are considered, it becomes increasingly likely that the drug will appear to be an improvement over existing drugs in terms of at least one symptom.

In both examples, as the number of comparisons increases, it becomes more likely that the groups being compared will appear to differ in terms of at least one attribute. Our confidence that a result will generalize to independent data should generally be weaker if it is observed as part of an analysis that involves multiple comparisons, rather than an analysis that involves only a single comparison.

For example, if one test is performed at the 5% level and the corresponding null hypothesis is true, there is only a 5% chance of incorrectly rejecting the null hypothesis. However, if 100 tests are conducted and all corresponding null hypotheses are true, the expected number of incorrect rejections (also known as false positives or Type I errors) is 5. If the tests are statistically independent from each other, the probability of at least one incorrect rejection is 99.4%.

The multiple comparisons problem also applies to confidence intervals. A single confidence interval with a 95% coverage probability level will contain the population parameter in 95% of experiments. However, if one considers 100 confidence intervals simultaneously, each with 95% coverage probability, the expected number of non-covering intervals is 5. If the intervals are statistically independent from each other, the probability that at least one interval does not contain the population parameter is 99.4%.

Techniques have been developed to prevent the inflation of false positive rates and non-coverage rates that occur with multiple statistical tests.

Type 1 error rate inflation (two hypotheses):

Example we have single null hypotheses at significance level $\alpha = 0.05$.

If we have two null hypotheses and do two independent tests, each at level $\alpha = 0.05$,

- What is the probability of rejecting at least one true null hypothesis?

Pr reject at least one true null $= 1 - \text{Pr reject neither true null}$
$= 1 - 0.95^2$
$= 0.0975 > 0.05$

• The Type I error rate is almost doubled as compared to single null hypothesis at significance level $\alpha = 0.05$.

One possible solution: Test each hypothesis at level $\alpha/2 = 0.025$ (Bonferroni test, see later). Then, Pr reject at least one true null = $0.0494 < 0.05$

Type 1 error rate inflation (More than two hypotheses):

Probability for Type I error increases with larger values of m and α

Example: For $m = 10$ and $\alpha = 0.05$, the probability of at least one Type I error is 40.1%

For large m we almost surely reject incorrectly at least one null hypothesis

There could be different source of multiplicity, multiplicity test problems are very common in clinical trials, example application include the comparison of new treatment with

- Several other treatments
- A control for more than one end point
- A control for more than one population
- A control repeatedly in time …. (Or any combination thereof)

Multiple test problems in clinical trial are very diverse and many different methods are available.

To reduce the degree of multiplicity following can be adhered.

- Addressing a limited number of questions only
- Minimizing number of variables, using composite endpoints, summary statistics,...
- Prioritizing questions

If multiplicity still persists

- Multiplicity adjustment should always be considered
- Regulatory guidance requires a description of the multiplicity adjustment in Phase III study protocols
- If multiplicity is not thought necessary then explain the reason for the same.

COMMON MULTIPLE TEST PROCEDURE (BASIC CONCEPTS)

Assume a "family" of m inferences and parameters of interest are $\theta_1,....,\theta_{m.}$.

Individual null hypotheses $H_1 : \theta_1 = 0,....,H_m : \theta_{m.} = 0$

Example: Comparison of 5ØZÜ treatments with a control therapy then $\theta_i = \mu_i - \mu_0$ are the m treatment effect differences of interest, where

- μ_i denotes the effect for treatment $i = 1,\ldots,m$

- μ_0 denotes the effect for the control therapy

Handling family wise error rate (FEWR), need to extend the usual Type I error rate concept when testing a family of null hypotheses H_1, ... H_m.

A multiple test procedure is said to control the FWER at level α (in the strong sense) if Pr (reject at least one true null) $\leq \alpha$ under any configuration of true/false null hypotheses.

Adjusted p-values extend ordinary (i.e. unadjusted) p-values by adjusting them for a given multiple test procedure, adjusted p-values can be compared directly with the significance level α, while controlling the FWER.

Formally, the adjusted p-value is the smallest significance level at which a given hypothesis is significant as part of the multiple test procedure

Example: Bonferroni method $p_i \leq \alpha / m \Leftrightarrow q_i = \min(mp_i, 1) \leq \alpha$ where p_i is the ordinary and q_i the adjusted p-value for $i = 1, ..., , m.$

In the single step methods, the rejection or non-rejection of a single hypothesis does not depend on the decision on any other hypothesis.

Examples: Bonferroni, Simes, Dunnett etc.

In stepwise methods, the rejection or non-rejection of a particular hypothesis may depend on the decision on other hypotheses.

Examples: Holm, Hochberg, stepdown Dunnett etc.

COMMON MULTIPLE TEST PROCEDURES

Bonferroni Method Overview

Use α / m for all inferences; for $i = 1, ..., , m.$

Reject H_i if $pi \leq \alpha / m$

Example: With m = 3, p-values must be less than 0.05/3 = 0.0167 in order to be "significant"

With adjusted p-values $q_i = \min(mp_i, 1)$

Reject H_i if $qi \leq \alpha / m$

Note that $mp_i > 1$ is possible and we thus need to truncate the adjusted p-values at 1, resulting in the minimum expression.

Both rejection rules above lead to the same test decision

Rationale:

The Bonferroni method follows from the Boole's inequality, where

$\Pr(U_i A_i) \leq \sum i \Pr(A_i)$ where $A_i = \{p_i \leq \alpha / m\}$ denotes the event of rejecting H_i

(i.e.) $\Pr(A_1 \cup A_2) \le \Pr(A_1) + \Pr(A_2)$

For $m = 2$,

Family wise error rate (FWER) = Pr ($p_1 \le \frac{\alpha}{2} or\ p_2 \le \frac{\alpha}{2} \mid H_1, H_2$ are true)

$$= \le \Pr\left(p_1 \le \frac{\alpha}{2} \mid H_1\ is\ true\right) + \Pr\left(p_2 \le \frac{\alpha}{2} \mid H_2\ is\ true\right)$$

$$= \frac{2\alpha}{2} = \alpha$$

The Bonferroni method is a single step procedure, it is rather conservative if:

- The number of hypotheses is large
- The test statistics are strongly positively correlated

The Bonferroni method can be improved:

- Stepwise methods (e.g. Holm procedure, see later)
- Accounting for correlations (e.g. Dunnett test, see later)

While Bonferroni is rarely used in practice, it is the basis for commonly used advanced multiple test procedures.

Holm Procedure Overview:

Simple explanation, assume p-values 0.0121, 0.0142, 0.0191, 0.1986, applying Bonferroni, we use 0.05/4 = 0.0125 and reject H_1, however, having rejected H_1 using 0.05/4, you no longer believe that all four null hypotheses can be true, you now think only H_2, H_3, H_4 can be true, so, test H_2 using 0.053 = 0.0167, rather than 0.05/4 and similarly for H_3 and H_4.

Let $p_{(1)} \le .... \le p_{(m)}$, denote the ordered unadjusted p-values with associated null hypotheses

$H_{(1)}, \ldots, H_{(m)}$

Then we have the following stepwise procedure:

- If $p_{(1)} \le \frac{\alpha}{m}$, reject $H_{(1)}$ and continue; else stop
- If $p_{(2)} \le \frac{\alpha}{(m-1)}$, reject $H_{(2)}$ and continue; else stop
- ……….so on,
- If $p_{(i)} \le \frac{\alpha}{(m-i+1)}$, reject $H_{(i)}$ and continue; else stop
- ……….so on,

• If $p_{(m)} \leq \alpha$, reject $H_{(m)}$

The Holm procedure is a stepwise procedure that is more powerful than the Bonferroni method

• Bonferroni uses the same threshold α/m for all hypotheses

• Holm uses the larger thresholds $\alpha / (m - i + 1)$

Sometimes called "stepdown Bonferroni" procedure, the Holm procedure can be improved by accounting for correlations (e.g. stepdown Dunnett test, see later)

Adjusted p-value Holm procedure:

With $p_{(1)} \leq ..... \leq p_{(m)}$, define adjusted p-values using

• $'q_{(1)} = mp_{(1)}$

• $'q_{(2)} = (m-1)p_{(2)}, if\,(m-1)p_{(2)} > q_{(1)}$

$= q_{(1)}$, otherwise

• ………so on,

• $q'_{(m)} = p_{(m)}$, if $p_{(m)} > q_{(m-1)}$

$= q_{(m-1)}$, otherwise

Formula for adjusted p-values:

$q_{(1)} = \min\{1, mp_{(1)}\}$

$q_{(i)} = \min\{1, \max[(m - i + 1) p_{(i)}, q_{(i-1)}]\}, i = 2,\ldots, m$

<u>Simes Method Overview:</u>

The Simes method tests the global null hypothesis

$H = H_1 \cap H_2 \cap .... \cap H_m : \theta_1 = \theta_2 = ... = \theta_m = 0$, It uses all ordered p-values $p_{(1)}, \ldots, p_{(m)}$, not just $p_{(1)}$,

Reject H if $p_{(1)} \leq \frac{i\alpha}{m}$ for at least one i.

Simes adjusted p-value uses $\min_i mp_{(i)} / i$, which is less than or equal to Bonferroni's $mp_{(1)}$, Simes cannot be used to test the individual hypotheses Hi, type I error rate is at most α under independence or (certain types of) positive dependence of p-values.

Simes method comparison with Bonferroni method (for m = 2)

Bonferroni rejects H, if $p_{(1)} \leq \frac{\alpha}{2}$

Simes rejects H, if $p_{(1)} \le \frac{\alpha}{2}$ or $p_{(2)} \le \alpha$

Under independence of p_1 and p_2,

- Pr (Bonferroni rejects) = $1-\left(1-\frac{\alpha}{2}\right)^2 = \alpha - \left(\frac{\alpha}{2}\right)^2 < \alpha$

- Pr (Simes rejects) = $1-\left(1-\frac{\alpha}{2}\right)^2 = \left(\frac{\alpha}{2}\right)^2 = \alpha$

Simes is more powerful than a global test based on Bonferroni, Simes assumes non-negative correlations between p-values, and Bonferroni does not.

<u>Hochberg Procedure Overview:</u>

The Hochberg procedure is a stepwise version of the Simes method, using the same thresholds as Holm:

- If $p_{(m)} \le \alpha$, reject $H_{(1)},\ldots,H_{(m)}$ and stop; else continue
- If $p_{(m-1)} \le \frac{\alpha}{2}$, reject $H_{(1)},\ldots,H_{(m-1)}$ and stop; else continue
- ……….so on,
- If $p_{(i)} \le \frac{\alpha}{(m-i+1)}$, reject $H_{(1)},\ldots,H_{(i)}$ and stop; else continue
- ……….so on,
- If $p_{(1)} \le \alpha$, , reject $H_{(1)}$

Adjusted p-values are

$q_{(m)} = p_{(m)}$

$_{(i)} = \min[(m-i+1)p_{(i)}, q_{(i+1)}]$, for i = m-1, ...1

The Hochberg procedure is sometimes called "stepup Simes" procedure, it is more powerful than the Holm procedure, both procedures use the same thresholds, but Hochberg starts with the largest p-value, whereas Holm starts with the smallest. It makes the same assumptions as the Simes test (i.e. independence or positive dependence of p-values). The Hochberg procedure can be improved, for example, Hommel procedure based on the closed test procedure.

Dunnett Test Overview:

When comparing several treatments with a control, the Dunnett test can be used. The methods from Bonferroni, Holm, Simes, and Hochberg can also be used in these situations, but only the Dunnett test exploits the correlation between the p-values.

Dunnett test's (Linear model and hypotheses) is as below:

Consider the unbalanced one-way layout

$Y_{ij} = \mu_i + \varepsilon_{ij}$ where,

- Y_{ij} denotes observation j = 1,…,n_i, in group i = 0,1,…,m
- μ_i the effect of treatment group i
- ε_{ij} are independent and identically normally distributed with mean 0 and variance σ^2,

i.e. $\varepsilon_{ij} \sim N(0, \sigma^2)$

The ANOVA F-test tests the global null $H : \mu_0 = ... = \mu_m$

Here, we are interested in comparing m treatments with the control treatment i = 0, i.e. testing the m null hypotheses $H_i : \theta_i = \mu_i - \mu_0 \leq 0 =$ 1,…,m.

Consider the m pairwise t-tests

$t_i = \mu_i' - \mu_0' / \sigma'$ sqrt $\left(\frac{1}{n_i} + \frac{1}{n_0}\right)$, i = 1,…,$m$.

Where μ_i' and σ' are the ordinary least squares of μ_i and σ, respectively

Note that $t_i \sim t_v$ under H_i, where t_v denotes the univariate t-distribution with $v = \sum i \ n_i - m - 1$ degrees of freedom, furthermore, $(t_1,...,t_m)$ follows the m-variate t-distribution with v degrees of freedom and correlations $\rho_{ij} = sqrt\left(\frac{n_i}{n_j + n_0}\right)$, where i, j = 1, …., m

For the m individual null hypotheses,

Reject H_i if $t_i \geq c_m, 1-\alpha$, the quantile $c_m, 1-\alpha$, is computed such that

$P\left[\left(t_{(1)},.........., t_{(m)}\right) \leq \left(c_{m,1-\alpha},.............., c_{m,1-\alpha}\right)\right] = P\left(\max_i t_i \leq c_{m,1-\alpha}\right) = 1 - \alpha$ follows the m-variate t-distribution with v degrees of freedom and correlations ρ_{ij} for $i, j = 1, \ldots,$ m= 1,…,5ØZÜ.

In other words, $c_{m,1-\alpha}$ is the $1-\alpha$ quantile of the distribution of the maximum of m t-distributed random variables.

Single step test, which is better than Bonferroni as it exploits the known correlations between test statistics. Adjusted p-values can be calculated numerically based on the multivariate t - distribution. The Dunnett test shown here can be extended to any linear and generalized linear model and this test can be improved by extending it to a stepwise procedure, similar to the Holm procedure. Other well-known parametric tests follow the same principle, for example, the Tukey test compares all treatment groups against each other, also using a multivariate t - distribution.

Stepwise Dunnett Test Overview:

Let $t_{(1)} \geq \ldots\ldots\ldots\ldots \geq t_{(m)}$ denote the ordered test statistics with associated null hypotheses $H_{(1)}, \ldots, H_{(m)}$.

Then we have the following stepwise procedure:

- If $t_{(1)} \geq c_{m,1-\alpha}$, reject $H_{(1)}$ and continue; else stop
- If $t_{(2)} \geq c_{m-1,1-\alpha}$, reject $H_{(2)}$ and continue; else stop
- ………so on,
- If $t_{(i)} \geq c_{m-i+1,1-\alpha}$, reject $H_{(i)}$ and continue; else stop
- ………so on,
- If $t_{(m)} \geq c_{m-1,1-\alpha}$, reject $H_{(m)}$

where, $c_{m-i+1,1-\alpha}$ denotes the 1- α quantile of the distribution of the maximum of $m - i + 1$, t-distributed random variables and is computed from the corresponding multivariate t-distribution.

For the stepwise Dunnett test, the quantiles change as hypotheses are rejected

• For example, if $H_{(1)}$ is rejected, then the quantile $c_{m-1,1-\alpha}$ is computed from a (m-1) -variate t-distribution

The stepwise Dunnett test is better than the single step Dunnett test

• It can be shown that $c_{m,1-\alpha} \geq c_{m-1,1-\alpha} \geq \ldots\ldots\ldots\ldots\ldots \geq c_{1,1-\alpha}$, where $c_{1,1-\alpha} = t_{v,1-\alpha}$ is the quantile from the univariate t-distribution with v degrees of freedom

• The Dunnett test $c_{m,1-\alpha}$ for all comparisons

The stepwise Dunnett test is better than the Holm procedure as it exploits the known correlations between test statistics

• The stepwise version shown here is sometimes called "stepdown Dunnett" test
• A "stepup Dunnett" test also exists, similar to Hochberg

Overall summary

Single step methods are less powerful than stepwise methods and not often used in practice, accounting for correlations leads to more powerful procedures, but correlations are not always known, Simes-based methods are more powerful than Bonferroni-based methods, but control the FWER only under certain dependence structures. In practice, we select the procedure that is not only powerful from a statistical perspective, but also appropriate from clinical perspective. There are hierarchical test procedure, closed test procedure and graphical approach to deal with multiple comparison, the application keeps changing on the objective and regulatory requirements.

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SHRUNKEN ESTIMATORS FOR MEAN AND VARIANCE IN THE PRESENCE OF LARGE OR SMALL TRUE OBSERVATIOS

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ABSTRACT

In this paper I have defined shrunken estimator $\hat{\mu}$ of mean, when sample is taken from the exponential distribution truncated from right and bias, mean square error and relative efficiency of $\hat{\mu}$ are obtained. We have also defined the shrunken estimator of variance $\hat{\sigma}^2$ for the normal distribution truncated from left as well as right and studied its properties. The derived results are illustrated by means of an example.

1. INTRODUCTION

Generally, the sample mean $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ of a random sample $y_1, y_2, ..., y_n$ of size n are used to estimate mean μ and variance σ^2 of the population. If a sample contains one or more observations from the right tail of the distribution, which are true large sample observations, then in this case Searls (1966) has defined an estimator

$$\bar{y}_t = \frac{1}{n}\left[\sum_{j=1}^{r} y_j + (n-r)t\right] \quad r = 0,1,2,...,n;\ y_j \le t \tag{1.1}$$

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of the mean μ by replacing all sample values larger than the pre-determined cutoff point t by the value of t itself; where r is the number of observations less than or equal to t. He has proved that

$$E(\bar{y}_t) = p\mu_t + qt \tag{1.2}$$

$$V(\bar{y}_t) = \frac{1}{n}\left[p(\sigma_t^2 + \mu_t^2) + qt^2 - (p\mu_t + qt)^2\right] \tag{1.3}$$

and $$MSE(\bar{y}_t) = \frac{1}{n}\left[p(\sigma_t^2 + \mu_t^2) + qt^2 - (p\mu_t + qt)^2\right] + q^2(\mu_t' - t)^2 \tag{1.4}$$

where $p = P[y_j \le t]$ and $q = 1 - p$ (1.5)

μ_t and σ_t^2 are the mean and variance of the right truncated distribution and μ_t' is the mean of the left truncated distribution.

Sometimes, a sample may contain one or more observations from the right tail or left tail of the distribution which are true large or small sample observations. In this case following the Searls' (1966) technique, Gothi (2018) has defined the estimator $\bar{y}_t$ for mean μ and estimator S_t^2 for variance σ^2 as

$$\bar{y}_t = \frac{1}{n}\left[\sum_{j=1}^{r_2} y_j + r_1 t_1 + r_3 t_2\right] \tag{1.6}$$

$$S_t^2 = \frac{1}{n-1}\left[\sum_{j=1}^{r_2} y_j^2 + r_1 t_1^2 + r_3 t_2^2 - n\bar{y}_t^2\right]; \qquad \left(t_1 \le y_j \le t_2\right) \tag{1.7}$$

where r_1 is the number of observations with values smaller than the pre-determined cutoff point t_1, r_3 is the number of observations larger than the pre-determined cutoff point t_2 and r_2 is the number of observations which are greater than or equal to t_1 and less than or equal to t_2. He proved that

$$E\left(S_t^2\right) = (\delta - 1)\sigma^2 \tag{1.8}$$

$$MSE\left(S_t^2\right)=\left[\frac{\mu_4^*}{n}+\frac{3-n}{n(n-1)}\sigma_*^4\right]+\left(\sigma_*^2-\sigma^2\right)^2 \qquad (1.9)$$

where $\sigma_*^2 = p_1t_1^2 + p_2(\sigma_t^2+\mu_t^2)+p_3t_2^2-(p_2\mu_t+p_1t_1^2+p_3t_2)^2$

$$\delta=\frac{\sigma_*^2}{\sigma^2}$$

$$\mu_4^* = (p_1t_1^4+p_2\alpha_{4,t}+p_3t_2^2)-4(p_1t_1^3+p_2\alpha_{3,t}+p_3t_2^2)\text{z}$$
$$+6\{p_1t_1^2+p_2(\sigma_t^2+\mu_t^2)+p_3t_2^2\}z^2-3z^4$$

$$\beta_2^*=\frac{\mu_4^*}{\sigma_*^4}$$

Let us define

$z = p_1t_1 + p_2\mu_t + p_3t_2$

$\mu_t = E(y_j \mid t_1 \le y_j \le t_2) = E(y_j \mid r_2)$

$\sigma_t^2 = Var(y_j \mid t_1 \le y_j \le t_2) = Var(y_j \mid r_2)$

$\alpha_{3,t} = E(y_j^3 \mid t_1 \le y_j \le t_2) = E(y_j^3 \mid r_2)$

$\alpha_{4,t} = E(y_j^4 \mid t_1 \le y_j \le t_2) = E(y_j^4 \mid r_2)$

$p_1 = P(y_j < t_1)$, $p_2 = P(t_1 \le y_j \le t_2)$ and $p_3 = P(y_j > t_2) = 1-(p_1+p_2)$

When some prior information regarding mean μ in the form of guess value say μ_0 is available to the experimenter, Thompson (1968) has proposed a shrunken estimator of mean μ defined as

$\hat{\mu}_s = l\bar{x} + (1-l)\mu_0 \quad (0 \le l \le 1)$

where $\bar{x}$ is the usual unbiased estimator of mean μ and l is the constant specified by the experimenter according to his belief in μ_0. l near to zero implies a strong belief in μ_0. Applying the same technique, in this paper we define the shrunken estimator of mean, when sample is taken from the exponential distribution truncated from right and its properties are studied.

Also Pandey and Singh (1997) have proposed a shrunken estimator for the population variance σ^2 for a normal distribution, and it is defined as

$$\hat{\sigma}_s^2 = kS^2 + (1-k)\sigma_0^2$$

In this paper, I have also defined the shrunken estimator of variance for the normal distribution truncated from left as well as right and studied its properties.

2. The Estimator For Mean

Let $y_1, y_2, \ldots\ldots, y_n$ be a random sample from the exponential population with p.d.f.

$$f(y \mid \mu) = \frac{1}{\mu} e^{-\frac{y}{\mu}}; \quad y > 0, \quad \mu > 0 \tag{2.1}$$

We want to estimate mean μ. Suppose that the guess estimate μ_0 of mean μ is available and last (n-r) observations which are larger than t are replaced by the value of t itself. We then define the shrunken estimator

$$\hat{\mu} = k\overline{y}_t + (1-k)\mu_0 \tag{2.2}$$

where $$\overline{y}_t = \frac{1}{n}\left\{\sum_{j=1}^{r} y_j + (n-r)t\right\} \tag{2.3}$$

3. Bias and MSE of $\hat{\mu}$

The p.d.f. of the right truncated distribution is given by

$$g(y) = \frac{1}{p} \cdot \frac{1}{\mu} e^{-\frac{y}{\mu}}; \quad 0 < y \le t, \quad \mu > 0 \tag{3.1}$$

Following results are used to derive the expressions for bias and MSE of $\hat{\mu}$.

$$p = P(y \le t) = 1 - e^{-t/\mu} \text{ and } \quad q = 1 - p = e^{-t/\mu} \tag{3.2}$$

$$E(\overline{y}_t) = p\mu_t + qt = \int_0^t y g(y) dy + qt = p\mu \tag{3.3}$$

$$p(\sigma_t^2+\mu_t^2)+qt^2=2\mu^2-2q\mu^2-2q\mu t \qquad (3.4)$$

$$\mu_t'=\frac{1}{q}\int_t^{\infty} yg(y)dy=t+\mu \qquad (3.5)$$

$$Bias(\overline{y}_t)=-q\mu \qquad (3.6)$$

$$V(\overline{y}_t)=\frac{1}{n}(2p\mu^2-2q\mu t-p^2\mu^2) \qquad (3.7)$$

$$MSE\left(\overline{y}_t\right)=\frac{1}{n}\left(2p-2q\frac{t}{\mu}-p^2+nq^2\right)\mu^2 \qquad (3.8)$$

Using (3.2) to (3.8), we get

$$\begin{aligned} Bias(\hat{\mu}) &= E[\hat{\mu}-\mu] \\ &= E[k(\overline{y}_t-\mu)+(1-k)(\mu_0-\mu)] \\ &= -kq\mu+(1-k)(\mu_0-\mu) \end{aligned} \qquad (3.9)$$

$$MSE(\hat{\mu})=E\left[k(\overline{y}_t-\mu)+(1-k)(\mu_0-\mu)\right]^2$$

$$=k^2MSE(\overline{y}_t)+(1-k)^2(\mu_0-\mu)^2+2k(1-k)(\mu_0-\mu)(-q\mu)$$

$$=\frac{\mu^2}{n}\begin{bmatrix} k^2\left(2p-2q.\frac{t}{\mu}-p^2+nq^2\right)+n(1-k)^2\left(\frac{\mu_0}{\mu}-1\right)^2 \\ -2nk(1-k)q\left(\frac{\mu_0}{\mu}-1\right) \end{bmatrix} \qquad (3.10)$$

It is easy to see that the value of k, for which $MSE(\hat{\mu})$ is minimum, is given by

$$k = \frac{n\left(\frac{\mu_0}{\mu}-1\right)\left(q+\frac{\mu_0}{\mu}-1\right)}{2p-2q.\frac{t}{\mu}-p^2+n\left(q+\frac{\mu_0}{\mu}-1\right)^2} \qquad .(3.11)$$

4. Relative Efficiency of

The relative efficiency of $\hat{\mu}$ with respect to the usual estimator $\overline{y}$ is given by

$$REF(\hat{\mu},\overline{y}) = \frac{V(\overline{y})}{MSE(\hat{\mu})} = \frac{1}{M_2} = R_1 \; (say) \qquad (4.1)$$

Similarly, the relative efficiency of with respect to the estimator $\overline{y_t}$ is given by

$$REF(\hat{\mu},\overline{y_t}) = \frac{MSE(\overline{y_t})}{MSE(\hat{\mu})} = \frac{M_1}{M_2} = R_2 \quad (say) \qquad (4.2)$$

where $M_1 = 2p - 2q\frac{t}{\mu} - p^2 + nq^2$ (4.3)

and $M_2 = k^2\left(2p-2q\frac{t}{\mu}-p^2+nq^2\right)+n(1-k)^2\left(\frac{\mu_0}{\mu}-1\right)^2-2nk(1-k)q\left(\frac{\mu_0}{\mu}-1\right)$ (4.4)

5. Estimator for Variance

Let $y_1, y_2, \ldots\ldots, y_n$ be a random sample of size n from a normal population with mean zero and variance σ^2. Suppose we want to estimate σ^2 when some guess estimate σ_0^2 is available and the first r_1 observations, which are smaller than the pre-determined cutoff point t_1, are replaced by t_1 and last r_3 observations, which are larger than cutoff point t_2, are replaced by t_2 itself. We propose the shrunken estimator of σ^2 as

$$\hat{\sigma}^2 = kS_t^2 + (1-k)\sigma_0^2 \qquad (0 \le k \le 1) \tag{5.1}$$

where $$S_t^2 = \frac{1}{n-1}\left[\sum_{j=1}^{r_2} y_j^2 + r_1 t_1^2 + r_3 t_2^2 - n\bar{y}_t^2\right] \tag{5.2}$$

$$\bar{y}_t = \frac{1}{n}\left[\sum_{j=1}^{r_2} y_j + r_1 t_1 + r_3 t_2\right] \tag{5.3}$$

k is a constant specified by the experimenter according to his belief about σ_0^2.

6. Bias and MSE of $\hat{\sigma}^2$

The p.d.f. of the normal distribution with mean zero and variance σ^2 truncated at cutoff point t_1 on the left and at t_2 on the right is

$$g(y) = \frac{1}{p_2} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\frac{y^2}{\sigma^2}}; \; t_1 \le y \le t_2, \; \sigma > 0 \tag{6.1}$$

In the derivation of the expressions for Bias and MSE of the estimator $\hat{\sigma}^2$, we shall use the following results.

$$p_1 = P(y < t_1) = \Phi(t_1') \tag{6.2}$$

$$p_2 = P(t_1 \le y \le t_2) = \Phi(t_2') - \Phi(t_1') \qquad 6.3)$$

$$p_3 = P(y > t_2) = \Phi(-t_2') \tag{6.4}$$

where $t_1' = \dfrac{t_1}{\sigma}$, $t_2' = \dfrac{t_2}{\sigma}$ and $\Phi(.)$ is the distribution function of a standard normal variate.

$$Bias(S_t^2) = (\delta - 1)\sigma^2 = B' - A'^2 \quad .(6.5)$$

$$\sigma_*^2 = \sigma^2 (B' - A'^2) \quad (6.6)$$

$$\mu_4^* = \sigma^4 \{D' - 4C'A' + 6B'A'^2 - 3A'^4\}$$

$$\beta_2^* = \frac{\mu_4^*}{\sigma_*^4} \quad (6.7)$$

$$p_2\mu_t + p_1 t_1 + p_3 t_2 = \sigma A' \quad (6.8)$$

$$p_2(\sigma_t^2 + \mu_t^2) + p_1 t_1^2 + p_3 t_2^2 = \sigma^2 B' \quad (6.9)$$

$$p_2\alpha_{3,t} + p_1 t_1^3 + p_3 t_2^3 = \sigma^3 C' \quad (6.10)$$

$$p_2\alpha_{4,t} + p_1 t_1^4 + p_3 t_2^4 = \sigma^4 D' \quad (6.11)$$

$$A' = \{\phi(t_1') - \phi(t_2')\} + p_1 t_1' + p_3 t_2' \quad (6.12)$$

$$B' = \{t_1'\phi(t_1') - t_2'\phi(t_2')\} + p_2 + p_1 t_1'^2 + p_3 t_2'^2 \quad (6.13)$$

$$C' = \{t_1'^2 \phi(t_1') - t_2'^2 \phi(t_2')\} + 2\{\phi(t_1') - \phi(t_2')\} + p_1 t_1'^3 + p_3 t_2'^3 \quad (6.14)$$

$$D' = \{t_1'^3 \phi(t_1') - t_2'^3 \phi(t_2')\} + 3\{t_1'\phi(t_1') - t_2'\phi(t_2')\} + 3p_2 + p_1 t_1'^4 + p_3 t_2'^4 \quad (6.15)$$

$$MSE(S_t^2) = \sigma^4 . M_1' \quad (6.16)$$

where $M_1' = \left[\frac{\beta_2^*}{n} + \frac{3-n}{n(n-1)}\right]\delta^2 + (\delta - 1)^2$

$$Bias(\hat{\sigma}^2) = E[\hat{\sigma}^2 - \sigma^2] = E\left[k(S_t^2 - \sigma^2) + (1-k)(\sigma_0^2 - \sigma^2)\right]$$

$$= \sigma^2\left[k(\delta - 1) + (1-k)\left(\frac{\sigma_0^2}{\sigma^2} - 1\right)\right] \quad (6.17)$$

$$MSE(\hat{\sigma}^2) = E[\hat{\sigma}^2 - \sigma^2]^2$$

$$= k^2 MSE(S_t^2) + (1-k)^2(\sigma_0^2 - \sigma^2)^2 + 2k(1-k)(\sigma_0^2 - \sigma^2)(\delta - 1)\sigma^2$$

$$= \sigma^4 . M_2{}' \qquad (6.18)$$

where

$$M_2{}' = k^2 M_1{}' + (1-k)^2 \left(\frac{\sigma_0^2}{\sigma^2} - 1\right)^2 + 2k(1-k)(\delta - 1)\left(\frac{\sigma_0^2}{\sigma^2} - 1\right) \qquad (6.19)$$

It may be noted that the value of k, for which $MSE(\hat{\sigma}^2)$ can be minimum is

$$k = \frac{\left\{\left(\frac{\sigma_0^2}{\sigma^2} - 1\right)^2 - (\delta - 1)\right\}}{T + \left(\frac{\sigma_0^2}{\sigma^2} - \delta\right)^2}$$

.....(6.20)

where $T = \left[\frac{\beta_2^*}{n} + \frac{3-n}{n(n-1)}\right]\delta^2$

7. Relative Efficiency of $\hat{\sigma}^2$

The relative efficiency of $\hat{\sigma}^2$ with respect to the usual unbiased estimator s^2 is given by

$$REF(\hat{\sigma}^2, S^2) = \frac{Var(S^2)}{MSE(\hat{\sigma}^2)} = \frac{2}{(n-1)M_2'} = R_1' \quad \text{(say)} \tag{7.1}$$

Similarly, the relative efficiency of $\hat{\sigma}^2$ with respect to the estimator S_t^2 is given by

$$REF(\hat{\sigma}^2, S_t^2) = \frac{MSE(S_t^2)}{MSE(\hat{\sigma}^2)} = \frac{M_1'}{M_2'} = R_2' \qquad \text{(say)} \tag{7.2}$$

8. Concluding Remarks

a) For the estimator $\hat{\mu}$

For application purpose refer to Table 8.1 and 8.2 on page numbers 10 and 11.

Here, Table 8.1 and 8.2 exhibit the efficiency of $\hat{\mu}$ relative to $\overline{y}$ which is denoted by R_1 and relative to $\overline{y_t}$ which is denoted by R_2. Note that R_1 and R_2 depend on the parameters n, $\frac{t}{\mu}$, k and $\frac{\mu_0}{\mu}$. The values of R_1 and R_2 are given by these two tables for $n=5$ and $n=10$, $\frac{t}{\mu} = 0.5, 1, 1.5$ and 2, $k = 0.3, 0.5, 0.7, 0.9$ and $\frac{\mu_0}{\mu} = 0.8, 0.9, 1.0, 1.1$ and 1.2.

From the Tables 8.1 and 8.2, we observe that if the other parameters remain unchanged, R_1 and R_2 decrease as k increases. We also observe that R_1 decreases as n increases. But for R_2, however, there is no regular behavior

It is also observed that for fixed values of k, n and $\frac{\mu_0}{\mu}$, $R_1 < R_2$ when $\frac{t}{\mu} < 1$ and $R_1 > R_2$ when $\frac{t}{\mu} \geq 1$.

Also we find that $REF\ (R_1\ and\ R_2)$ is greater than unity for small values of k; and for large values of k, REF is smaller. It is also observed that R_2 is greater than one for any set of values of all the parameters and this shows that the proposed estimator $\hat{\mu}$ is always better than the estimator $\overline{y_t}$. For $k < 0.5$, R_1 is always greater than one and so $\hat{\mu}$ is always better than the conventional estimator $\overline{y}$. But for $k \geq 0.5$, R_1 is sometimes less than one also.

b) For the estimator $\hat{\sigma}^2$

Please refer to Tables 8.3 to 8.6 on page number 12 and 13

Tables 8.3 to 8.6 give the efficiency of $\hat{\sigma}^2$ relative to S^2 which is denoted by R_1' and relative to S_t^2 which is denoted by R_2'. It may be noted that the expressions of R_1' and R_2' are the functions of the parameters t_1', t_2', k, n and $\delta' = \frac{\sigma_0^2}{\sigma^2}$.

From the tables 8.3 to 8.6, we observe that, if the values of the other parameters remain unchanged, R_1' and R_2' both increase as k and n decrease.

Also, it is observed that when k is small, R_1' and R_2' increase as δ' increases, and they become maximum for $\delta' = 1$; and when k is large (i.e. $k > 0.5$), R_1' and R_2' increase with the increase in the values of δ' and they attain the maxima for $\delta' > 1$ and thereafter they go on decreasing. It can also be seen that $R_1' > R_2'$ for any set of values of all the parameters.

We also observe that for all the values of $t_1' \leq -2.5$ and $t_2' \geq 2.75$, our proposed estimator $\hat{\sigma}^2$ is better than s^2 and S_t^2.

From tables 8.3 and 8.4, we observe that

(i) If k is small $(i.e.\ k < 0.5)$ and $\delta' < 1$, R_1' and R_2' increase.

(ii) If $k < 0.5$ and $\delta' \geq 1$, R_1' and R_2' decrease.

(iii) If $k \geq 0.5$ and $\delta' \leq 1$, R_1' decreases and R_2' increases and

(iv) If $k \geq 0.5$ and $\delta' > 1$, R_1' and R_2' decrease as t_1' decreases.

Also from tables 8.3 and 8.5, we observe that

(i) If $k < 0.5$ and $\delta' < 1$, R_1' and R_2' decrease.

(ii) If $k < 0.5$ and $\delta' \geq 1$, R_1' and R_2' increase.

(iii) If $k \geq 0.5$ and $\delta' \leq 1$, R_1' increases and R_2' decreases and

(iv) If $k \geq 0.5$ and $\delta' > 1$, R_1' and R_2' increase as t_2' decreases.

In short, our estimator is better than the conventional estimators if σ_0^2 is closed to σ^2 and n and k are small.

Relative Efficiency of the Estimator of Mean

Table 8.1

$n = 5$							
k	$\frac{t}{\mu}$		μ_0 / μ				
			0.8	0.9	1.0	1.1	1.2
0.3	0.5	R_1	1.921	3.128	5.958	15.390	90.042
		R_2	3.582	5.833	11.111	28.702	167.930
	1.0	R_1	3.077	5.739	13.793	50.639	62.527
		R_2	2.479	4.623	11.111	40.794	50.371
	1.5	R_1	4.178	8.401	20.974	39.493	19.244
		R_2	2.213	4.450	11.111	20.922	10.195
	2.0	R_1	4.933	9.921	20.889	22.752	11.232
		R_2	2.624	5.278	11.111	12.102	5.975
0.5	0.5	R_1	1.220	1.586	2.145	3.057	4.695
		R_2	2.276	2.959	4.000	5.701	8.757
	1.0	R_1	2.291	3.269	4.965	8.202	14.825
		R_2	1.851	2.634	4.000	6.607	11.942
	1.5	R_1	3.401	3.269	4.965	11.216	14.109
		R_2	1.802	2.634	4.000	5.942	7.475
	2.0	R_1	3.990	5.577	7.520	8.957	8.672
		R_2	2.122	2.966	4.000	4.764	4.613
0.7	0.5	R_1	0.843	0.956	1.094	1.264	1.477
		R_2	1.572	1.784	2.041	2.358	2.755
	1.0	R_1	1.763	2.099	2.533	3.106	3.873
		R_2	1.420	1.691	2.041	2.502	3.120
	1.5	R_1	2.693	3.216	3.852	4.504	5.439

Relative Efficiency of the Estimator of Mean

Table 8.2

$n = 10$							
k	$\frac{t}{\mu}$		μ_0 / μ				
			0.8	0.9	1.0	1.1	1.2
0.3	0.5	R_1	0.963	1.570	2.999	7.834	50.229
		R_2	3.566	5.814	11.111	29.019	186.07
	1.0	R_1	1.566	2.968	7.496	35.850	49.057
		R_2	2.322	4.400	11.111	53.139	72.715
	1.5	R_1	2.205	4.699	14.269	39.420	12.714
		R_2	1.717	3.659	11.111	30.696	9.900
	2.0	R_1	2.734	6.175	17.821	20.715	7.224
		R_2	1.705	3.850	11.111	12.916	4.504
0.5	0.5	R_1	0.125	0.797	1.080	1.544	2.383
		R_2	2.269	2.953	4.000	5.718	8.829
	1.0	R_1	1.193	1.726	2.699	4.725	9.739
		R_2	1.768	2.558	4.000	7.004	14.435
	1.5	R_1	1.931	3.019	5.137	9.250	13.978
		R_2	1.504	2.351	4.000	7.203	10.884
	2.0	R_1	2.556	4.024	6.415	8.833	8.296
		R_2	1.594	2.509	4.000	5.508	5.173
0.7	0.5	R_1	0.424	0.481	0.551	0.637	0.745
		R_2	1.569	1.782	2.041	2.360	2.761
	1.0	R_1	0.933	1.124	1.377	1.722	2.206
		R_2	1.384	1.666	2.041	2.552	3.270
	1.5	R_1	1.653	2.065	2.621	3.369	4.345

Relative Efficiency of the Estimator of Variance

$$[n=5,\ t_1'=-2.5,\ t_2'=3.0]$$

Table 8.3

k		δ'				
		0.8	0.9	1.0	1.1	1.2
0.1	R_1'	12.683	35.819	106.50	42.990	14.382
	R_2'	11.909	33.632	100.00	40.365	13.504
0.3	R_1'	7.431	10.026	11.834	11.252	8.862
	R_2'	6.977	9.414	11.111	10.565	8.321
0.5	R_1'	3.736	4.062	4.260	4.287	4.136
	R_2'	3.508	3.814	4.000	4.025	3.883
0.7	R_1'	2.091	2.140	2.714	2.191	2.191
	R_2'	1.964	2.009	2.041	2.057	2.057
0.9	R_1'	1.306	1.311	1.315	1.319	1.322
	R_2'	1.226	1.231	1.235	1.238	1.241

Relative Efficiency of the Estimator of Variance

$$[n=5,\ t_1'=-2.75,\ t_2'=3.0]$$

Table 8.4

k		δ'				
		0.8	0.9	1.0	1.1	1.2
0.1	R_1'	12.912	36.616	104.43	41.251	14.024
	R_2'	12.365	35.064	100.00	39.502	13.429
0.3	R_1'	7.545	10.044	11.603	10.822	8.459
	R_2'	7.225	9.618	11.111	10.363	8.100
0.5	R_1'	3.732	4.022	4.177	4.164	3.986
	R_2'	3.574	3.851	4.000	3.988	3.817
0.7	R_1'	2.068	2.107	2.131	2.139	2.131
	R_2'	1.980	2.018	2.041	2.049	2.041
0.9	R_1'	1.283	1.286	1.239	1.292	1.293
	R_2'	1.228	1.232	1.235	1.237	1.238

Relative Efficiency of the Estimator of Variance

$$[n = 5,\ t_1' = -2.5,\ t_2' = 2.75]$$

Table 8.5

k		δ'				
		0.8	0.9	1.0	1.1	1.2
0.1	R_1'	12.547	35.344	107.70	44.099	14.605
	R_2'	11.649	32.816	100.00	40.945	13.561
0.3	R_1'	7.360	10.008	11.967	11.519	9.119
	R_2'	6.834	9.293	11.111	10.696	8.467
0.5	R_1'	3.735	4.083	4.308	4.361	4.228
	R_2'	3.468	3.791	4.000	4.049	3.926
0.7	R_1'	2.104	2.156	2.198	2.221	2.227
	R_2'	1.954	2.004	2.041	2.062	2.068
0.9	R_1'	1.318	1.324	1.330	1.334	1.338
	R_2'	1.224	1.230	1.235	1.239	1.243

Relative Efficiency of the Estimator of Variance

$$[n = 10,\ t_1' = -2.5,\ t_2' = 2.75]$$

Table 8.6

k		δ'				
		0.8	0.9	1.0	1.1	1.2
0.1	R_1'	5.954	19.125	105.04	25.220	7.009
	R_2'	5.663	13.206	100.000	24.009	6.672
0.3	R_1'	4.917	8.165	11.672	10.755	6.926
	R_2'	4.681	7.773	11.111	10.238	6.593
0.5	R_1'	3.144	3.748	4.202	4.316	4.034
	R_2'	2.993	3.568	4.000	4.109	3.840
0.7	R_1'	1.953	2.060	2.144	2.194	2.207
	R_2'	1.859	1.961	2.041	2.089	2.101
0.9	R_1'	1.273	1.286	1.297	1.307	1.132
	R_2'	1.212	1.224	1.235	1.244	1.252

9. ACKNOWLEDGEMENT

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RESEARCH ARTICLE

MEASURING HUMAN DEVELOPMENT (HD) STATUS FOR RURAL DENSED POPULATED (RUBANIZED) TALUKAS

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ABSTRACT

With a view to quantifying Human Development Aspect, Human Development Index (HDI) was developed by Mahebub-ul-Haq. It is observed that the components used in calculating HDI at Country/State level can not be used for deriving this measure below State (i.e. at region/district/taluka) level, owing to data limitations. Hence, in this paper, an attempt is designed to suggest methodology to derive HDI for Rubanized Talukas as a workable solution. Before describing this methodology, an attempt is also made to answer elaborately the question Why the present HDI formulae can not be used to study HD Status below State level?

KEY WORDS

Rubanized Talukas, HD Status, HDI, Agrarian Economy.

1. INTRODUCTION

In a very simple and lucid language, Human Development (HD) can be defined as the aims to increase the capabilities of people which enable them to access larger opportunities in life. With reference to India, HD indicates promoting basic capabilities among those, who lack them. Mahbub ul Haq has said that the objective of development is to create an enabling environment for people to enjoy long, healthy and creative lives.

Now the questions arises What do we mean by the capabilities? How can

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we quantify them? What are the most relevant parameters associated with the capabilities ? How to measure them? Last but not the least, whether the data on such parameters are available in long term basis, without changing their definition and method of collecting them?

Amartya Sen has clearly said that Human Capital and Human Capability aspects should be used in understanding the process of Economic and Social Development, which concentrates on the agency of human beings - through skill, knowledge and effort. The Human Capability focuses on the ability of human beings to lead lives they have reason to value and to enhance the substantive choices they have. These two parameters are concerned with the role of human beings and in particular with the actual abilities that they achieve and acquire. He argued that to study the development of any country or region, only economic growth should not be emphasized but along with this growth, Social Sector Development should also be visualized.

2. OBJECTIVES

This paper aims

A) to examine the reasons, Why the three components normally used in constructing HDI can not be used to study Human Development Status below State level?

B) to suggest the methodology to derive HDI for the Rubanized Talukas as a workable solution to have relative comparison of HD Status of these Talukas.

PART- A

In this part, the explanation about the question why the present measure of HDI can not be used for studying HD Status below State level.

SOME MEASUREMENT OF HUMAN DEVELOPMENT:

United Nations Development Programme (UNDP) measures Human Development through follow indices.

(1) Human Development Index (HDI):

It is a summary measure of human development that captures the average achievement in a country or a geographic entity in three basic dimensions of Human Development given below.

- A long and healthy life as measured by the life expectancy at birth (LEB)
- Knowledge, as measured by the adult literacy rate (with two-thirds weight) and combined with primary, secondary and tertiary gross enrollment.
- A decent standard of living as measured by per capita (PPP US$) Gross Domestic Product (GDP).

HDI reduces all three above mentioned basic indicators by measuring achievement in each indicator as the relative distance from the desirable goal.

(2) Gender – Related Development Index (GDI):

GDI measures achievement in the same basic capabilities as HDI, but takes note of inequality in achievement between men and women. In short, GDI is HDI adjusted for gender inequality.

(3) Gender Empowerment Measure (GEM):

GEM measures whether women and men are able to actively participate in economic and political life and in decision making, while GDI focuses on expansion of capabilities, GEM is concerned with the use of those capabilities to take advantage of the opportunities in life. GEM captures gender inequality in three following areas.

A) Political participation and decision making power.

B) Economic participation and decision making power.

C) Power over economic resources as measured by women's and men's estimated earned income.

(4) Human Poverty Index (HPI):

In 1997, UNDP introduced HPI. In 1998, UNDP spilt HPI into HPI-1 for developing countries like India, and HPI-2 for Organization for Economic Co-

Operation and Development (OECD) countries. Further, HPI measures deprivation, while HPI-1 measures dimensions of human development captures in HDI.

3. SELECTION OF PARAMETERS FOR HDI:

(A) Healthy Life:

HDI measures this component in terms of Life Expectation at Birth (LEB). The critical aspects of health such as the incidence of disability, short term and long term (chronic) morbidity, which reflect the equality of health enjoyed by people are not included in HDI.

(B) Standard of Living:

In HDI, this component is measured by per capita income. As proxy of income, the consumer expenditure data is also used for measuring standard of living for the following reasons.

(I) It smoothens down Income Fluctuations.

(II) It includes non-monetized transactions, which are significant in rural areas in developing countries like India.

(III) It covers the implications of Non-System of National Accounts transactions for instance common property resources in villages.

(IV) Under reporting income data in developing countries.

(V) It may capture individual's command over resources more accurately.

(C) Access to Knowledge:

HDI includes adult literacy rate and combined enrollment ratio for measuring access to knowledge. These data are not strictly comparable across the different population censuses in India because of non availability of adult literacy rate for first three consecutive censuses after independence of India.

4. Some Studies in Social Sector Development Status:

Raval and Vaghela (2018) have summarized some studies on Social Sector Development. These studies are done by Amartya Sen (1989), Newman and Thomson (1989), UNDP (1990), Mark Gillivray (1991), Michael Hopkins (1991), Das Gupta and Martin Weale (1992), Srinivasan and Verma (1993), S. P. Pal and D. K. Pant (!993), Mahbub ul Haq (1995), UNDP (1995), Inderjeet Singh and Reena Singh (2001), Achin Chakraborthy (2002), UNDP (2002) and Paul Streeten (2003). They have sparingly utilized indices like HDI, GNP, GDI and GEM to study various aspects of Social Sector Development at large and Human Development in particular. Because, this issue is out of coverage of the present paper, these studies are not discussed here at length.

Indira and Darshini (2004) have also studied elaborately the HD Status of Gujarat vis-à-vis other States. They have utilized various measures like Income rank, Education rank, Health rank, Participation rank, Gender Development Measure and Gender Equality Index.

Amartya Sen (1989) had suggested that the regional balanced socio-economic development appears to be a very good proposition for uplifting the Status of HD. If one studies this Status for Rubanized talukas, this proposition becomes more relevant. If one considers Gujarat State, it is observed that nearly one-fifth of Gross Domestic Product (we call it State Income) is shared by Agriculture sector (including Animal Husbandry).

It is well accepted argument that in order to uplift HD Status of people, the regional imbalanced Socio-Economic Development prevailing around the people under study must be reduced to the extent possible. It is also visualized that this imbalanced development is not only observed between rural and urban areas of the same region, but also between rural areas of different regions.

The probable reasons for this imbalanced development may be A) scanty and less rain fall, B) inadequate irrigation facility C) less self employment opportunities, D) insufficient road connectivity, E) non- availabity of power supply either for 24 hours or for major part of day and night and F) in–adequate infrastructure facility. In this given situation, it does not appear to be logical to compare HD relative status of rural population of different talukas of entire State.

It is absolutely true that during recent period, sincere planning efforts are made by State and Central Governments to smoothen down this imbalance by providing road connectivity, providing electric facility for the major part of the day and night, by giving various incentives like increasing minimum support price for almost all Agriculture Products, by providing fertilizers and other agriculture inputs at subsidized price, by allowing farmers to sell their products individually without going to the market etc. But, these planning efforts will take another five years or so to achieve balanced Socio-Economic Development at peak in entire rural India.

5. HDI MEASURE PRESENTLY IN USE:

In order to have relative comparison of HD between countries or between states, the following three components are generally used in computing Human Development Index (HDI).

A) A long and healthy life as measure by the Life Expectancy at Birth (LEB).

B) Knowledge as measured by the adult literacy rate (with two – thirds weight) and combined with primary, secondary and tertiary gross enrollment.

C) A decent standard of living as measured by per capita (PPP US $) Gross Domestic Product (GDP).

It is crystallized clear that if one desires to uplift Human Development Status, it is most essential to trace out the hurdles creating problems at bottom level for upliftment of HD status. But, because of certain limitations of basic data, above stated three parameters can not be used for deriving HDI below State level. Component wise these limitations are spell out below.

A) Long and Healthy…….. (LEB):

The life expectancy at birth measures the average number of years a person is expected to live under prevailing mortality conditions. A life tables states the probabilities of survival of a hypothetical group or cohort at different ages, which gradually diminish due to deaths. There are several methods for constructing life tables. At present, the life tables are generated using mortality package for life table estimation (MORTPAK 4). It is a United Nation's Software package for mortality measurements.

The sex wise estimates of Life Expectancy prepared for Country/State with rural–urban break up are using data base of Sample Registration System (SRS). Since SRS is based on Thin Sample (For instance, 8875 sample units (75 lakh population) for India and 478 sample units (4 lakh populations) for Gujarat, the estimation procedure is not allowing the users to generate the estimate of expectancy of life at birth below State level (i.e. either at Region/District/Taluka level).

C) Knowledge as measured...…..Gross Enrollment:

There are two points in respect of using this component for HDI construction.

i) The sex wise literacy rate through population census is available even at urban block and village level at an interval of ten years (i.e. during next population census). NSS data on education level by sex with rural- urban break up can be conveniently used to bridge this time gap. But these data are not collected at regular interval of time and secondly such data are based on small sample (hardly covering 0.5 percent state population), hence estimation procedure does not allow users to generate the literacy rate below State level.

ii) So far as primary, secondary and tertiary gross enrolment is concerned, the effect of enrolment of the students getting admission in more than one institutions for better choice of education is not considered, which affects the cleanness of net enrolment figure.

B) A decent ....GDP Per Capita (PPP US $)

As a representative of per capita income component per capita Gross Domestic Product (GDP) is suggested. Unfortunately per capita GDP estimates at District and Taluka level for each State of India are not yet prepared. Other issue, in regard to GDP is that District and Taluka economy is open economy, while State or National economy is of close nature. This fact creates problems in generating reliable GDP eastimates below State level.

Another important point in using per capita Gross Domestic Product is that GDP calculation does not considered the value of services used in producing products (i.e. depreciation), hence, per capita GDP may not be realistic representative of per capita income. If one uses Net Domestic Product (which considers effect of depreciation) instead of Gross Domestic Product in the calculation of per capita income, then the another limitation is the absence of appropriate method of generating depreciation estimate for each economic activity.

Indira and Darshini have utilized per capita monthly consumer expenditure for domestic consumption of household from NSS round in studying HDI of Gujarat. Here, it is suggested that:

A) Monthly consumer expenditure after only food items should be considered or should be weighted more instead of monthly expenditure of all domestic items, because the data of consumer expenditure on each non-food item are not reported from all sample households as each non food item is not being consumed by all sample households.

B) As NSS sample hardly covers 0.5 percent population, it is not advisable to generate this estimate below State/Region level and hence, it can not be used as component of HDI for District and Taluka level estimates.

6. PART – B

In the previous part, the elaborate description is presented to answer the question "Why the earlier mentioned three components used as proxy of Health, Education and Income for constructing HDI at country/state level can not be used in that form for preparing HDI either at Region/District/Taluka level?

In this part, an attempt is designed to suggest the methodology for constructing HDI for Rubanized (Thickly Rural Populated) Talukas under the valid assumption that the rural economy is normally found to be agrarian (including animal husbandry).

COMPONENTS SELECTED:

With a view to visualizing Human Development Status for rural populated talukas, the following 16 parameters classified under three well known components can be selected. Out of 16 parameters, 8 are pertaining to Income and infrastructure, 3 are of Health and 5 are of Education, with explicit weightage of 8:3:5.

These parameters are narrated below.

(A) Income and Infrastructure :

The eight parameters are suggested for this component.

(1) Productivity of Total Food Grains.

(2) Productivity of Vegetables (average of three years).

(3) Productivity of Fruits (average of three years).

(4) Productivity of Spices and Condiments (average of three years).

(5) Irrigation Intensity.

(6) Milk production increase (in MT) per lakh population (during five years)

(7) Percentage increase in Total poultry (during five years).

(8) Percentage increase in Surface Pucca Roads per lakh population (during five years).

(B) Health:

The following three parameters are suggested under Health Component.

1) Percentage of Institutional Deliveries in total number of Registered Deliveries during last five years.

2) Number of Pregnant/Lactic women served per projected lakh number of eligible females (20-44 the period of fertile age group).

3) Percentage of number of malnourished children (Grade–III and IV) in total number of children weighted.

(C) Education:

Education covers five parameters given below.

1) Drop Out Rate for Primary Education (Std. I to VIII or Std. I to VII as

case may be).

2) Number of Anganwadis in operation per lakh projected rural population.

3) Percentage of children enrolled in Anganwadis per lakh projected rural children population (0-6 years).

4) Number of Mid-Day-Meal beneficiaries per lakh projected population (06-14 years).

5) Percentage point increase in higher secondary examination results over last five years).

7. METHODOLOGY:

Visualizing the nature of 16 parameters suggested above, all rurbanized talukas' values are arranged either in ascending order or descending order of magnitude of parameter in such way that most developed taluka in respective parameter comes at first place. After arranging all talukas in this way for each parameter, most developed taluka in respect of HD status occupies at first place and least developed taluka stands at last among the given set of Rubanized Talukas under study of HD status.

Limitations :

1) The HDI suggested in this paper gives relative comparison of HD status at taluka level.

2) It is important to mention that the taluka, which occupies last rank in HDI can not be considered as un-developed or under development in Human Development Comparative view but should be considered as least developed taluka at that point of time among the set of talukas.

3) The HDI measure suggested in this paper is workable solution. One can improve by incorporating more parameters or discarding few parameters. However, in order to have valid conclusion about HD Status of all talukas of the State, it is recommended that the same set of parameters for all three components should be considered.

8. ACKNOWLEDGEMENT:

I express my sincere thanks to unknown referee for going through earlier version of this paper and offering valuable comments.

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RESEARCH NOTE

ALTERNATE METHOD FOR IDENTIFYING CONFOUNDED INTERACTIONS IN 2^N SYMMETRICAL FACTORIAL EXPERIMENTS

D. K. Ghosh[(1)]

ABSTRACT

In this article we have discussed an alternate method for identifying the confounded interactions from a 2^n Symmetrical factorial experiments.

KEY WORDS

Conounding, Identification, Generalised confounded Interactions.

1. INTRODUCTION

Das and Giri (1973) discussed the method for identifying confounded interactions from symmetrical factorial experiments. Further Ghosh and Bagui(1998) carried out another method of identification of confounded interactions from a symmetrical factorial experiments using another blocks(other than key block) of the replication. In this investigation we have developed alternate method of identification of confounded interactions from 2^n symmetrical factorial experiments using the key block itself. The beauty and advantages of this method is that one can find out independent confounded interactions as well as generalized confounded interactions from the same key block.

2. METHOD OF IDENTIFICATION

2.1 Identification of confounded interactions for 2^n confounded factorial experiments confounded in to a block of sizes 2^{n-1}.

Let us consider a 2^n confounded factorial experiments confounded in to a

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block of sizes 2^{n-1} per replication. Next consider the key block so naturally the other block will be termed as alternate block, because in this case there will be two blocks per replication only. From the key block we understand a block in which there is one treatment combination whose levels of all the factors are at low level, that is, all the levels of the factors are zero. This treatment combination is also called control treatment combination. Now we select those treatment combination(s) whose levels of all the factors except one factor is at low level and are absent from the key block. Take the product of all such single letter treatment combinations with reduced mod 2. Geometrical representation of this treatment combination is the confounded interaction. In this case there will be only one confounded interaction as the experiment contains two blocks per replication.

2.1.2 Example. Consider a 2^4 confounded factorial experiments confounded in to a block of size 8. Treatment combinations in the key blocks are (I, a, bd, cd, abd, acd, abc, bc). Identify the confounded interaction?

In this example single letter treatment combination is "a" only present in the key block so the single letter treatment combination which are absent in the key block are b, c, d. Hence the confounded interaction is bcd.

2.2. Identification of confounded interactions for 2^n confounded factorial experiments confounded in to a block of sizes 2^r.

Consider the key block. If all the treatment combinations are in the form of level zero and one then its ok, otherwise, convert it in to level zero and one. In this case number of independent confounded interactions are (n-r) and total number of confounded interactions (including generalized confounded interactions) are $(2^{n-r} - 1)$, that is, number of generalized confounded interactions are $[(2^{n-r} - 1) - (n - r)]$. Out of 2^r treatment combinations which have n columns and r rows, leave first and (n-r-1) columns and then select unit matrix of size r from the remaining columns. Check if the first column has all element as 1 then the combination of first column and r column factor gives one independent confounded interaction. Because unit matrix of size r has one element 1 in each row and each column and first column has all element as 1 so each row has now even number of one. Similarly check the (n – r – 1) columns with r columns if each row has even number of one then combinations of those are confounded interactions. We can identify remaining confounded interactions using the combinations of first column, last (n – r -1) columns and r columns of unit matrix together or alone.

2.2.1 Example. Consider a 2^5 confounded factorial experiments confounded in to a block of size 8. Treatment combinations in the key block are (00000, 10011, 01010, 00101, 11001, 10110, 01111, 11100). Identify the confounded interaction?

Here n – r = 2. So leaving first and last column, select the unit matrix of size 3 from key block as block size is 2^3. This unit matrix is following:

A	B	C	D	E
1	1	0	0	1
0	0	1	0	1
1	0	0	1	1

Here one can check that last column has all element as 1and hence interaction BCDE is confounded. Because under columns B, C, D and E each row has even number of 1. Next consider column A and r columns B, C, D. If we delete column C then we can find rows under Columns A, B, D has either even number of 1 or zero number of 1 and hence interaction ABD is confounded. Generalized confounded interaction is BCDE X ABD = ACE. In this example there is n – r = 5 – 3 = 2 independent confounded interactions BCDE, ABD and [(2^{5-3} - 1) – (5 – 3) = 1 generalized confounded interaction ACE.

Next example will show that one can also identify independent as well generalized confounded interactions together from the columns of r unit matrix and extra columns directly.

2.2.2. Example. Consider a 2^6 confounded factorial experiments confounded in to a block of size 8. Treatment combinations in the key blocks are (000000, 100110, 010101, 001011, 110011, 101101, 011110, 111000,). Identify the confounded interaction?

Here $n - r = 3$. So leaving first and last two columns, select the unit matrix of size 3 from key block as block size is 2^3. This unit matrix is following:

A	B	C	D	E	F
1	1	0	0	1	1
0	0	1	0	1	1
1	0	0	1	1	0

Here one can check that column E has all elements as 1 and columns of B, C, D form a unit matrix and hence interaction BCDE is confounded. Because under columns B, C, D and E each row has even number of 1. Again consider column F and the columns of unit matrix B, C, D. If we delete the column D and take columns B, C, F, we can find that under this column each row has either even number of 1 or zero number of 1 so interaction BCF is confounded. Again consider column A and the columns of unit matrix B, C, D. If we delete the column C and take columns A, B, D, we can find that under this columns each row has either even number of 1 or zero number of 1 so interaction ABD is confounded. Again consider columns E, F and the columns of unit matrix B, C, D. If we delete the columns B, C and take columns D only, we can find the under this columns each row has even number of 1 so interaction DEF is confounded. Again consider columns A, E and the columns of unit matrix B, C, D. If we delete the column B, D and take columns A, C, E, we can find the under this columns each row has even number of 1 so interaction ACE is confounded. Next consider column A, E, F and r columns B, C, D. If we delete columns C, D then we can find rows under Columns A, B, E, F has even number of 1 and hence interaction ABEF is confounded. Again consider columns A, F and the columns of unit matrix B, C, D. If we delete the column B and take columns A, C, D, F, we can find the under this columns each row has even number of 1 so interaction ACDF is confounded.

In this example there is $n - r = 6 - 3 = 3$ independent confounded interactions and $[(2^{6-3} - 1) - (6 - 3) = 4$ generalized confounded interactions. That is, total number of confounded interactions are $(2^{6-3} - 1) = 7$. Advantages of this method is that we found seven confounded interactions BCDE, BCF, ABD, DEF, ACE, ABEF and ACDF, which includes independent as well as generalized confounded interactions, are identified directly. Therefore there is no need to find generalized confounded interactions separately.

We illustrate one more example where treatment combinations are in the form of alphabetic letters.

2.2.3 Example. Consider a 2^5 confounded factorial experiments confounded in to a block of size 4. Treatment combinations in the key block are (I, ace, bcd, abde). Identify the confounded interactions?

Here treatment combinations are given in the form of alphabetic so write its geometric representation in the form of level 0 and 1 which are given below:

(00000, 10101, 01110, 11011).

Here n – r = 5 – 2 = 3. So leaving first and last two columns, select the unit matrix of size 2 from key block as block size is 2^2. This unit matrix of size 2 is given below:

A	B	C	D	E
1	1	0	1	1
1	0	1	0	1

Here one can check that column A has all elements as 1 and columns of B, C form a unit matrix and hence interaction ABC is confounded. Because under columns A, B and C each row has even number of 1. Again consider column E and the columns of unit matrix B, C which forms the unit matrix. We can see that under column E all the elements are 1 and hence interaction BCE is confounded Because under columns B, C and E each row has even number of 1. Again consider columns A, D, E and the columns of unit matrix B, C. If we delete the columns C and take columns B only, we can find that under this columns each row has even number of 1 so interaction ABDE is confounded. This way we identified three independent confounded interactions. Four generalized confounded interactions are obtained either from usual method or from columns of unit matrix and extra first and last two columns.

We can see that under columns C, D, E, each row has even number of 1 so CDE is confounded. Again under columns A, E, each row has even number of 1 so AE is confounded. Again under columns B, D, each row has either even number of 1 or zero number of 1, so BD is confounded. Again under columns A, C, D, each row has even number of 1 so ACD is confounded.

Remarks: If the key block contains only four treatment combinations then it is very easy to identify one confounded interaction whatever be the number of blocks per replication.

One can easily identify one confounded interaction only by taking the product of any two treatment combinations with reduced mod 2 present in the key block. If the outcome is the third treatment combination itself then that resulting treatment combination is confounded. To prove this result considers the example 2.2.3. Here Treatment combinations in the key blocks are (I, ace, bcd, abde). we can check that product of any two treatment combinations with reduced mod 2 is third treatment combination itself. For an example product of 'ace' and 'bcd' with reduced mod 2 is 'abde', so interaction ABDE is confounded in this example. However, if we take the product of 'ace' and 'abde' then outcome is 'bcd' but the interaction BCD is not confounded because only one interaction is confounded. Moreover the remaining confounded interactions are identified using example 2.2.3.

3. ACKNOWLEDGEMENTS

Author is very much thankful to the students of Gujarat State who approached me recently few days back to find easy method for identifying the confounded interactions as these questions are frequently asked in the competitive examination. Because of them this problem is solved. Author is also grateful to referee and Dr. B. B. Jani for their valuable suggestions.

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RESEARCH NOTE

APPLICATION OF OPERATIONS RESEARCH MODEL IN HOSPITAL MANAGEMENT

P. Mariappan[(1)], M. B. Thaker[(2)], Jenifer Christinal[(3)]

ABSTRACT

Operations Research models are very much used to make decision in all walks of life. It is such a wonderful tool for decision making. In this article application to Hospital Management is discussed with an application.

1. INTRODUCTION

In order to understand what OR is, one must know its history and evolution. It is broadly concurred that the field began during World War II. Many strategic and technical problems linked with military effort were highly complicated. In answer to these complex problems, groups of scientists with varied educational backgrounds assembled as special units inside the armed forces.

Since the scientists are talented men, pressure sues to war time necessity and the synergism generated from the interactions of different studies, they were unusually successful in improving the strength of complex military operations. By 1941, each of the three wings of the British Armed Forces was utilizing such scientist teams. Referable to the quick success of the idea, other allied nations took the same attack and coordinated their own squads. Because the problems attributed to these groups were in the nature of military operations, their study was called Operational Research in U.K. and in other nations called Operational Research. Even though, the Americans put their effort at a later date, produced many mathematical techniques for analyzing military problems.

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Subsequently the World War II, many soldiers belongs to the military OR groups turned their attention to the possibilities of applying similar approaches to day to day problems in the company. Initially in a large scale profit making organizations like petroleum companies, etc. have introduced this OR techniques. Then slowly the researchers designed the model to suit even the small industries. Now, one can apply this OR technique even for the day problems of any size and any category. Today OR is having more applications in Engineering, Business Management, Agriculture, Transporting system and so on.

It is really interesting to observe that the modern perception of Operations Research as a body of well-built models and techniques. A modification of this kind is to be required in any emerging area of scientific research.

2. SCOPE OF OPERATIONS RESEARCH

It has got wide scope, more specifically in developing nations, Mathematicians, Statisticians, Economists, Management experts, etc., use the OR Techniques to take the best decision based on its applications. It can be applied in Agriculture, any type of Organization, Military operations, Planning, Transportation and Hospital etc.,

3. PHASE OF OPERATIONS RESEARCH METHODOLOGY

S1 : Formulate the problem

S2 : Observe the system Mathematical model

S3 : Construct the Mathematical model

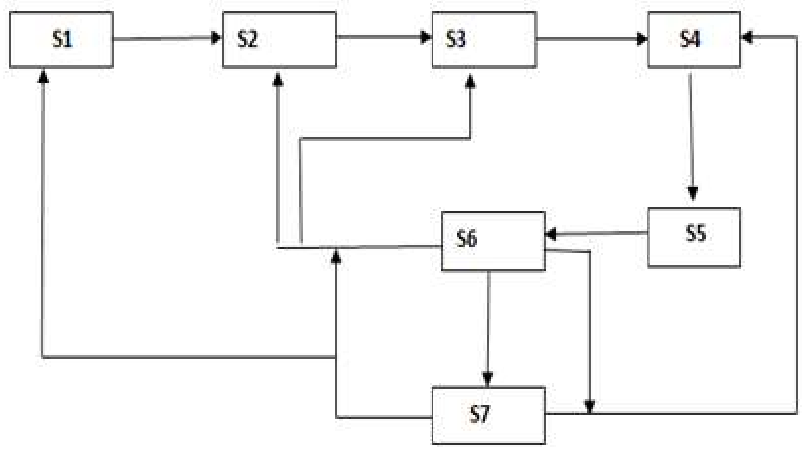
S4 : Verify the model and use the model for prediction

S5 : Choose the best alternative

S6 : Implement and evaluate the recommendations

The O.R phases can be depicted as follows:

OR Methodology



4. GENERAL MODEL OF THE LINEAR PROGRAMMING PROBLEM

Maximize /Minimize $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to the constraints

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1r}x_r + \cdots + a_{1n}x_n\{\leq, =, \geq\}b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2r}x_r + \cdots + a_{2n}x_n\{\leq, =, \geq\}b_2$

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$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mr}x_r + \cdots + a_{mn}x_n\{\leq, =, \geq\}b_m$

$x_1, x_2 \ldots x_n \geq 0$

The above problem can be expressed using mathematical symbols as follows:

Maximize /Minimize $Z = \sum_{j=1}^{n} c_j x_j$

Subject to

$\sum_{j=1}^{n} a_{ij}\, x_j\{\leq, =, \geq\}b_i$

$(for\ i = 1,2, \ldots m);\ x_j \geq 0 (for\ j = 1,2, \ldots n)$

Note: Where c_j's; b_i's and a_{ij}'s are constants for all [j = 1 to n] and [*i*=1 to m]. The decision variables are referred by x_j ; for all [j=1 to n].

3. APPLICATIOBN IN THE HOSPITAL MANAGEMNT

SHIFT SCHEDULING PROBLEM

A local hospital administration wishes to determine a work schedule for registered nurses (RNs). The union contract specifies that nurses work a normal day of six hours. The administrator has determined a daily work schedule in which the day is divided into eight three-hour shifts.

The following table indicates the estimated minimum requirement for nurses per shift. The Nurses will start work each day at the beginning of one of these shifts and end

work at the end of the following shift.

Shirt	Period							
	12-3 am	3-6 am	6-9 am	9-12 am	12-3 pm	3-6 pm	6-9 pm	9-12 pm
Minimum No. of RNs Required	30	20	40	50	60	50	40	40

The administration must specify the number of nurses that is to begin work each shift, so that the required number of nurses is available for each three hour period.

Objective : To optimize the number of nurses to be recruited

Variables : Let xj be the number of nurses to be recruited in the jth shift, j=1,2,..,8

Constraints : In each shifts the minimum number of nurses required should to taken care.

Tabulate all the information

Shifts	Period							
	12-3a.m.	3-6a.m.	6-9a.m.	9-12a.m.	12-3p.m.	3-6p.m.	6-9p.m.	9-12p.m.
Minimum number of required RNs	30	20	40	50	60	50	40	40

MATHEMATICAL MODELING

The objective is to minimize the total number of nurses employed NP. Let Xi stand for the number of nurses report up to work at the beginning of period j.

Min z $=\sum_{j=1}^{8} x_j$

Consider the staffing requirement of atleast 30 nurses for period 1. Nurses on duty fot this period would include those starting work at the beginning of this shift plus those who began work during the previous shirt. Thus the constraints for period 1 is $x_1 + x_8 \geq 30$

similarly,

$x_1 + x_2 \geq 20$ $\quad x_4 + x_5 \geq 60$

$x_2 + x_2 \geq 40$ $\quad x_5 + x_6 \geq 50$

$x_3 + x_4 \geq 50$ $\quad x_6 + x_7 \geq 40$

$x_7 + x_8 \geq 40$

Optimum solution : Min Z=170, x_1=30, x_2=0, x_3=40, x_4=10, x_5=50, x_6=0, x_7=40 and x_8=0

Conclusion:

The LPP is solved with the help of TORA package.

The final solution communicates that the Hospital authorities should employ thrity nurses in shift-1, forty nurses in shift-3, ten nurses in shift-4, fifty nurses in shift-5 and forty nurses in shift-7 to optimize the number. The total number of nurses required to execute the complete show needs 170 nurses.

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We thank the referee for review of this paper which has helped us to revise it.

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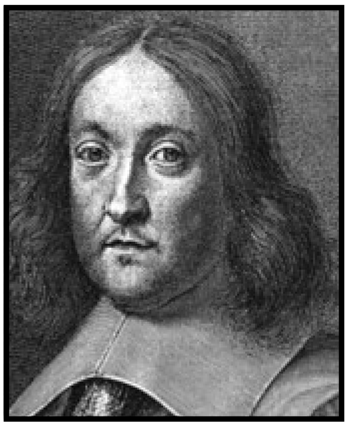
BIOGRAPHY

PIERRE DE FERMAT

H. D. Budhbhatti[1]



Together with **Rene Descartes, Fermat** was considered to be one of the leading mathematicians of the first half of the 17^{th} century. He was born in Beaumont-de-lomagne, France on 6^{th} Decemeber 1607 and he died after living for 57 years on 12 January 1665, at Castres, France.

He was from Gascony, where his father, Dominique Fermat, was a wealthy leather merchant, and served three one-year terms as one of the four consuls of Beaumont-de-Lomagne. His mother was Claire de Long. Pierre had one brother and two sisters and was almost certainly brought up in the town of his birth.

He attended the University of Orléans from 1623 and received a bachelor in civil law in 1626, before moving to Bordeaux. In Bordeaux he began his first serious mathematical researches, and in 1629 he gave a copy of his restoration of Apollonius's *De Locis Planis* to one of the mathematicians there. Certainly in Bordeaux he was in contact with Beaugrand and during this time he produced important work on maxima and minima which he gave to Étienne d'Espagnet who clearly shared mathematical interests with Fermat. There he became much influenced by the work of François Viète.

In 1630, he bought the office of a councillor at the Parlement de Toulouse,

(1) Adapted from wikipedia (the free encyclopedia) and other related source. We express our sincere thanks and gratitude for this assistance.

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(rcd. June'19 / rvd. July '19)

one of the High Courts of Judicature in France, and was sworn in by the Grand Chambre in May 1631. He held this office for the rest of his life. Fermat thereby became entitled to change his name from Pierre Fermat to Pierre de Fermat. Fluent in six languages (French, Latin, Occitan, classical Greek, Italian and Spanish), Fermat was praised for his written verse in several languages and his advice was eagerly sought regarding the emendation of Greek texts.

(Monument to Fermat in Beaumont-de-Lomagne)
(Bust in the Salle Henri-Martin in Capitole de Toulouse)

He communicated most of his work in letters to friends, often with little or no proof of his theorems. In some of these letters to his friends he explored many of the fundamental ideas of calculus before Newton or Leibniz. Fermat was a trained lawyer making mathematics more of a hobby than a profession. Nevertheless, he made important contributions to analytical geometry, probability, number theory and calculus. Secrecy was common in European mathematical circles at the time. This naturally led to priority disputes with contemporaries such as Descartes and Wallis.

Work

Fermat's pioneering work in analytic geometry (*Methodus ad disquirendam maximam et minimam et de tangentibus linearum curvarum*) was circulated in manuscript form in 1636 (based on results achieved in 1629), predating the publication of Descartes' famous *La géométrie* (1637), which exploited the work. This manuscript was published posthumously in 1679 in *Varia opera mathematica*, as *Ad Locos Planos et Solidos Isagoge* (*Introduction to Plane and Solid Loci*).

Fermat developed a method (adequality) for determining maxima, minima, and tangents to various curves that was equivalent to differential calculus. In these works, Fermat obtained a technique for finding the centers of gravity of various plane and solid figures, which led to his further work in quadrature.

Fermat was the first person known to have evaluated the integral of **general power functions.** With his method, he was able to reduce this evaluation to the sum of **geometric series.** The resulting formula was helpful to Newton, and then Leibniz, when they independently developed the **fundamental theorem of**

calculus.

In number theory, Fermat studied **Pell's equation, perfect numbers, amicable numbers** and what would later become **Fermat numbers**. It was while researching perfect numbers that he discovered **Fermat's little theorem**. He invented a factorization method—**Fermat's factorization method**—as well as the proof technique of infinite descent, which he used to prove **Fermat's right triangle theorem** which includes as a corollary Fermat's Last Theorem for the case n = 4. Fermat developed the t**wo-square theorem**, and the **polygonal number theorem,** which states that each number is a sum of **three triangular numbers, four square numbers,** five pentagonal numbers, and so on.

Although Fermat claimed to have proven all his arithmetic theorems, few records of his proofs have survived. Many mathematicians, including **Gauss,** doubted several of his claims, especially given the difficulty of some of the problems and the limited mathematical methods available to Fermat. His famous **Last Theorem** was first discovered by his son in the margin in his father's copy of an edition of **Diophantus**, and included the statement that the margin was too small to include the proof. It seems that he had not written to **Marin Mersenne** about it. It was first proven in 1994, by **Sir Andrew Wiles,** using techniques unavailable to Fermat.

Although he carefully studied and drew inspiration from Diophantus, Fermat began a different tradition. Diophantus was content to find a single solution to his equations, even if it were an undesired fractional one. Fermat was interested only in integer solutions to his **Diophantine equations,** and he looked for all possible general solutions. He often proved that certain equations had no solution, which usually baffled his contemporaries.

Through their correspondence in 1654, **Fermat and Blaise Pascal** helped lay the foundation for the theory of probability. From this brief but productive collaboration on the **problem of points, they are now regarded as joint founders of probability theory.** Fermat is credited with carrying out the first ever rigorous probability calculation. In it, he was asked by a professional gamblerwhy if he bet on rolling at least one six in four throws of a die he won in the long term, whereas betting on throwing at least one double-six in 24 throws of two dice resulted in his losing. Fermat showed mathematically why this was the case.

The first **variational principle in physics** was articulated by Euclid in his ***Catoptrica.*** It says that, for the path of light reflecting from a mirror, the angle of **incidence equals the angle of reflection.** Hero of Alexandria later showed that this path gave the shortest length and the least time. Fermat refined and generalized this to "light travels between two given points along the path of shortest *time*" now known as the ***principle of least time.*** For this, Fermat is recognized as a key figure in the historical development of the fundamental principle of least action in physics. The terms **Fermat's principle and *Fermat functional* were** named in recognition of this role.

According to **Peter L. Bernstein,** in his book *Against the Gods*, Fermat "was a mathematician of rare power. He was an independent inventor of analytic geometry, he contributed to the early development of calculus, he did research on the weight of the earth, and he worked on light refraction and optics. In the course of what turned out to be an extended correspondence with Pascal, he made a significant contribution to the theory of probability. But Fermat's crowning achievement was in the theory of numbers."

Regarding Fermat's work in analysis, **Isaac Newton** wrote that his own early ideas about calculus came directly from **"Fermat's way of drawing tangents."**

Of Fermat's number theoretic work, the 20th-century mathematician André Weil wrote that: "what we possess of his methods for dealing with curves of genus 1 is remarkably coherent; it is still the foundation for the modern theory of such curves. It naturally falls into two parts; the first one ... may conveniently be termed a method of ascent, in contrast with the descent which is rightly regarded as Fermat's own." Regarding Fermat's use of ascent, Weil continued: "The novelty consisted in the vastly extended use which Fermat made of it, giving him at least a partial equivalent of what we would obtain by the systematic use of the **group theoretical properties of the rational points on a standard cubic."** With his gift for number relations and his ability to find proofs for many of his theorems, **Fermat essentially created the modern theory of numbers.**

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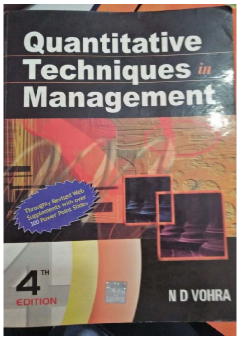
BOOK REVIEW



QUANTITATIVE TECHNIQUES IN MANAGEMENTS

-N. D. Vohra

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Pages : 1063

In present scenario it is being difficult for any business entity to maintain its profit margin, market share, customerbase and its presence in the corporate world. Therfore promoters and especially managers are supposed to be vigilant and dynamic.

A good manager is identified through his/her managerial skills and decesion making. If they are not conversant with theoretical knowledge and application of various decision making techniques they may not be surviving in the corporate world or may have to remain as unwanted without any recognition.

In the light of this, to address the above mentioned expectations from any successful manager, this textbook is published. It is found to be one of the most widely adopted books in the area of quantitative analysis. Though many other books relating to the same area are also accesible to users, this book by the learned author is found to be very popular among the teachers, students, managers, practitioners and research workers.

There are 3 specific reasons for the popularity of this textbook.

(1) Language and subject content is presented in a lucid manner.

(2) The most appealing feature of this book is that it covers almost all corners of operations research / quantitative analysis.

(3) Various topics are developed without restoring to frightening mathematical formulae. However enough care is also taken in order to maintain padagogical foundation.

The entirebook is divided into 19 chapters ranging from decision making and quantitative techniques to forecasting models.

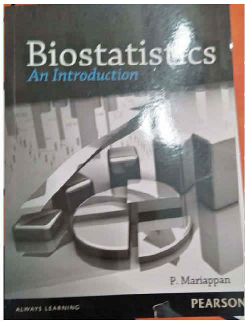
Each chapter contains with chapter overview, sufficient theoretical discussion, well mix of various illustrations with sufficiently good number of excercises. The last edition (Eighth reprint of 2013) has thoroughly revised web supplements with over 300 power point slides, very rich Bibliographical contents are additional qualitative aspects of this book.

Especially the chapters on Dynamic Programming, Simulation, Investment Analysis and Forecasting are definitely praisewise.

This book satisfies all the basic needs for students, teachers, research workers and practitioners.

DR. H. M. DIXIT
Head Dept of Statistics,
Arts, Commerce and Science College, PILVAI (NG)

Ahmedabad
10-06-2019



BIOSTATISTICS - AN INTODUCTION

-P. MARIAPPAN

Publisher : Pearson, Chennai, Delhi
Edition : 2013
Pages : 464

Biostatistics now a days is a very important and essential branch for solving research problems in life sciences. Unless the user understands its application and purpose, it is of no use to apply the statistical tools and techniques used in Biostatistics. The research worker must know enough about the basic principles of data analysis and has to be certain that all available information is used effectively to solve a given problem. This text emphasizes statistical applications, statistical model pubilding approach and hunting for the manual solution method.

Author of this book, Dr. P. Mariappan is Head and Associate Professor in the department of Mathematics at Bishop Heber College in Tiruchirapalli, Tamilnadu. The author has been working for teaching and research areas such as Biostatistics, Decision sciences, Data analysis, operations management, numerical methods, optimisation techniques as well as computer programming. He has received many prestigious awards. He has also received the best teacher award in 2004 by the association of central for Indian Intellectuals, India.

This text consists of 13 chapters which are arranged in a logical sequence based upon the principle of gradation. In particular chapter 12 on sampling theory and chapter 13 on testing of hypothesis are presented in an extremely distinguished and efficient manner.

e.g. In particular page 337 and page 344 represent flow chart presentations for the relevant tests in hypothesis testing, which is unique.

Emphasis is placed on the use of statistical softwares wherever necessary. This text has two main distinctive features.

(1) Separate section is devoted to examples and excercises relating to university examination question papers.

(2) Book has distinctive design which emphasizes on the self taught learning method.

This textbook can definitely be useful for the students, teachers and research workers in Biology, Botany, Environmental Science and all other life science courses. It will provide a comprehensive reference for the users of applied statistics.

Date : 29/06/2019

DR. MANISH B. THAKER
Head, Department of Statistics.
M. G. Science Institute,
Gujarat University, Ahmedabad - 380009

M. B. Thaker*

Gujarat University, Mathematics department, School of Sciences is starting three new five year full time M. Sc. integrated courses

1. **M.Sc. in Actuarial Science**
2. **M.Sc. in Artificial Intelligence & Machine Learning**
3. **M.Sc. in Data Science**

1. About the Actuarial Profession:

An actuary is a business professional who deals with the financial impact of risk and uncertainty. They provide expert assessments of financial security systems, with a focus on their complexity, their mathematics, and their mechanisms. They mathematically and statistically evaluate the likelihood of events and quantify the contingent outcomes in order to minimize losses associated with uncertain undesirable events.

Every area of business is subject to risks, so an actuarial career offers many employment options, including banking, insurance, healthcare, pensions, investment and also non-financial areas.

PROGRAM OBJECTIVES

This program lays the foundation of the core Mathematical, Statistical, Financial and Economic principles which will enable the participants to have a solid foundation and training in the actuarial sciences. This course will also be helpful to students who want to pursue careers in Actuarial Science/Financial Risk Management/Financial Engineering. Classroom training in this course will also help students to write examinations of Institute and Faculty of Actuaries (IoFA, U.K.), or the Institute of Actuaries, India.

* Head Department of Statistics, M. G. Science Institute, Navrangpura, Ahmedabad-380009. email : mbthaker2768@gmail.com

CAREER OPPORTUNITY

Many students who pursue a degree in Actuarial Sciences become professional actuaries. Working with private or public-sector employers, actuaries assess financial risks as diverse as natural disaster, insurance premiums and the impact of climate change.

Careers in the actuarial field include:

- Actuary
- Risk analyst or consultant
- Insurance underwriter

In addition to the specialized careers listed above, Actuarial Science students graduate with statistics and financial mathematics expertise that can lead to career opportunities in the following fields:

- Finance
- Management
- Statistician (in any of the diverse fields where statisticians are employed, ranging from marketing to pharmacology to engineering

Actuaries primarily work in the insurance industry. They are the ones who are consulted on what insurance premiums should look like based on the history of the policy holder. They have a wholesome picture of mortality and disability rates based on demography, location, and other related factors that establish the probabilities of undesirable events. They determine how much does a vehicle owner, a homeowner, a business owner, have to pay against insuring their property, or safeguarding their liabilities, for the future.

They also work with corporations, assisting their growth and decision-making processes by analyzing their financial risks.

They are equipped with the skill to determine future pension schemes, gratuity, benefit plans, and other social security policies, both for the public and the private sector.

A lot of actuaries function as consultants, working independently and providing their invaluable expertize to the government or private businesses.

2. ABOUT THE ARTIFICIAL INTELLIGENCE (AI) & MACHINE LEARNING (ML) PROFESSION:

With the increasing need for intelligent and accurate decision making, there is an exponential growth in the adoption of AI and ML technologies. The Artificial Intelligence industry will be worth $1.2 trillion in 2018, with customer experience solutions creating the most business value. Artificial intelligence describes a machine that is capable of imitating and performing intelligent human behavior. Some of these tasks could include problem-solving and decision-making or specific activities requiring acute perception, recognition, or translation abilities. Thus there is a wide scope in the field of AI-ML. Today's world focuses on computer based accurate decision making process, where AI & ML is key to such solution.

PROGRAM OBJECTIVES:

Artificial Intelligence & Machine learning methods based in different fields, including neural networks, signal processing, control, and data mining, in order to present a unified treatment of machine learning problems and solutions. This program will focus on real time problem solving using AI & ML methods. AI-ML is the theory and development of computer systems to be able to perform tasks normally requiring human intelligence, such as visual perception, speech recognition, and neural network, decision-making. Machine Learning (ML) makes it possible for machines to learn from experience, adjust to new inputs and perform human-like tasks. Classroom training in this course will also help students to be industry ready with better career perspectives.

CAREER OPPORTUNITIES

In recent years, careers in artificial intelligence (AI) have grown exponentially to meet the demands of digitally transformed industries. While there are plenty of jobs in artificial intelligence, there's a significant shortage of top tech talent with the necessary skills.

According to the job site Indeed, the demand for AI skills has more than doubled over the past three years. Typical jobs include AI based software development, Data Scientist, Machine Learning engineer, automation engineer and AI research scholar.

The role of a machine learning engineer is at the heart of AI projects. However, it's also necessary to be an AI programmer and demonstrate a thorough understanding of multiple programming languages.

Machine learning engineers should also be able to apply predictive models and leverage natural language processing when working with enormous datasets.

To get hired, it will help if candidates are highly experienced with agile development practices and familiar with leading software development tools.

Preference is often given to technology professionals with strong mathematical skills. Most AI & ML careers also require candidates to be experts in machine learning, deep learning, and neural networks, with strong computer programming skills, analytical skills, and experience with cloud applications.

3. ABOUT THE DATA SCIENCE PROFESSION:

In the growing world of big data, it is important to have an effective data science strategy to help make informed business decisions. A Data Scientist is a high-ranking professional with the training and curiosity to make discoveries in the world of Big Data. Therefore it comes as no surprise that Data Scientists are coveted professionals in the Big Data Analytics and IT industry. The Data Science career opportunities for skilled professionals in IT world will be having huge demand for Data Scientists in start-ups as well as well-established companies. This discipline's applications enable Data Scientists, Predictive Modelers, and other analytics professionals to analyze growing volumes of structured transaction data. It leads to smarter business moves, more efficient operations, higher profits and more satisfied customers. Big Data Analytics examines large amounts of data to uncover hidden patterns, correlations and other insights. The most emerging area in today's technology sector including Big Data Analytics helps organization harness their data and use it to identify new opportunities.

Program objectives:

This program lays the foundation of the core data science, data management, advanced database operations. M.Sc. Data Science program aims to impart training to enrolled students with regard to existing and evolving techniques and theories related to information retrieval, big data analysis, data mining, data warehousing and data visualization, data security, etc. Classroom training in this course will also help students to be industry ready with better career perspectives.

CAREER OPPORTUNITIES

In a world where 2.5 quintillion bytes of data is produced every day, a professional who can organize this humongous data to provide business solutions is much sought after. A Data Scientist, according to Harvard Business Review, "is a high-ranking professional with the training and curiosity to make discoveries in the world of Big Data". Therefore it comes as no surprise that Data Scientists are coveted professionals in the Big Data Analytics and IT industry.

Shortage of skilled professionals in a world which is increasingly turning to data for decision making has also led to the huge demand for Data Scientists in start-ups as well as well-established companies. A corporate consulting firm study states that by 2020, the world will face a shortage of about 8,00,000 professionals with deep analytical skills. With the Big Data wave showing no signs of slowing down, there's a rush among global companies to hire Data Scientists to tame their business-critical Big Data.

Not only are Data Scientists responsible for business analytics, they are also involved in building data products and software platforms, along with developing visualizations and machine learning algorithms. Some of the prominent Data Scientist job titles are: Data Scientist, Data Architect, Data Administrator, Data Analyst, Business Analyst, Data/Analytics Manager, Business Intelligence Manager.

MINIMUM ELIGIBILITY FOR ALL THREE COURSES ABOVE (WHO CAN APPLY):

Students with a minimum of 10+2 years of formal education in Science or Commerce with Mathematics/Statistics/Accountancy/Physics/Chemistry/Biology at the 10+2 level.

SOME FORTH COMING PROGRAMS

		Conference Title	Conference Dates	Place of Conference	Conference websight
1	INDSTATS 2019	2019 International Indian Statistical Association Conference, Innovations in Data and Statistical Sciences	26^{th} -30^{th} December-2019	Victor Menezes Convention Center, IIT Bombay, Mumbai, India	IISA2019@intindstat.org
2	ICORDS 2019	International Conference on Operations Research and Decision Sciences 2019	December 28-30, 2019	Indian Institute of Management Visakhapatnam	http://www.iimv.ac.in/icords
3	NCMSA'19	National Conference on Mathematics, Statistics & Applications	July 19-20, 2019	Division of Mathematics, Vignan's Foundation for Science, Technology & Research, Guntur, Andhra Pradesh, India	http://www.vignan.ac.in/ncmsa19
4	DSFES-2019	First Workshop on Data Science for Future Energy Systems	17-Dec-19	Hyderabad, India	https://easychair.org/cfp/dsfes-2019
5	ICAMCS2019	International Conference on Applied Mathematics and Computational Sciences 2019	October 17-19, 2019	DIT University, Dehradun, India	http://icamcs.org
6	ICGAMS-2K19	8th International Conference Cum 24th National Conference of Gwalior Academy of Mathematical Sciences	December 13-16, 2019	VIT Bhopal University, Bhopal, India	
7	FORBS 2019	Second International Conference on "Frontiers of Operations Research & Business Studies (FORBS 2019)"	December 27-28, 2019	Calcutta Business School, Kolkata, India	http://www.calcuttabusinessschool.org/FORBS2019
8	ORSI 2019	52nd Annual convention of the Operational Research Society of India & International Conference: Practice of Management Science & Analytics	December, 15-18, 2019	Indian Institute of Management, Ahmedabad	https://conference.iima.ac.in/orsi2019/

READERS FORUM

A. M. PATEL*

* **Paresh Prajapati (Ahmedabad)**
  It is nice to here news about the website of the journal. It is at present in the preliminary stage but step by step it can be developed further. My best wishes for he team.
* **R. N. Saptarshi (Vadodara)**
  Congrats to the team for good, inspring and spirited team work. Congrats also for the website. Please keep it up.
* **R. K. Joshi (Ahmedabad)**
  My hearty congratulations for indeed nice team work.
  In the issues of the journal, the articles by A. C. Brahmbhatt are really praiseworthy and inspiring. Best wishes to the team.
* **P. Mariappan (Tiruchirupally)**
  I had the opprtunity to come to Ahmedabd for academic work entrusted to me on Statistics Day Celebration Programme at M. G. Science Institute on 29th June '19. My three days passed with intimate friends like Dr. Manish Thaker and Dr. B. B. Jani with exclent memorable events. I knew about the journal, other activities and website.
  I am deeply impressed by it.
  It is an excellent team work.
  I congratulate the entire team for their painstaking efforts to execute the academic activities. My best wishes to all.
* **Ravi Gor (Ahmedabad)**
  Congrats for journal and website. Thanks for publishing about other three new courses in S. V. Letter. best wishes.
* **S. G. Raval (Ahmedabad)**
  Since long, I am assoicated with this journal and the team working for it. I get inspirations from them. My best wishes to all of them who work for this journal.

* Rtd. Principal, H. K. Commerce College, Ahmedabad
and Ex. Secretary, Gujarat Vidyasabha and Brahmachariwadi Trust, Ahmedabad.

Gujarat Statistical Association

Established : 1969

[Registered under Public Trust Act of 1950 (Bombay)]

R. No. E2502 A'bad-1974

The objective of the association is primarily to promote statistical ideas in pure and applied fields in the form of study, teaching and research in statistics.

The membership of GSA consists of Life / Institutional / ordinary members.

Membership	Fees	
	In India	Abroad
Insitutional Member (for 3 years)	₹ 2,000/-	US $ 500
Life Member	₹ 1,500/-	US $ 300
Ordinary Member (p.a.)	₹ 500/-	US $ 50

Annual Subscription for Gujarat Stastistical Review / Sankhya Vignan for individuals, Institutions and library is ₹ 500 (in India), US $ 50 (Abroad)

SANKHYA VIGNAN

સંખ્યા વિજ્ઞાન

E-mail : svgsa2015@gmail.com

http://www.sankhyavignan.org

PIERRE DE FERMAT*

Pierre de Fermat was born in Beaumont-de-Lomagne, France on 6 December, 1607. He lived for 57 years and passed away on 12 January, 1665 at Castres, France. He studied at the University of Orleans from 1623 and graduated in Civil Law in 1626. He became french lawyer at the Parliament of Toulouse. Besides his occupation in law, he was deeply interested also in mathematics which made him universally famous.

He was a trained lawyer making mathematics more of a hobby than a profession. He was fluent in six languages and also was famous for his written verse in several languages.

He is very well known for his contributions to **number theory, analytic geometry, probability theory, Folium of Decarts' Fermat's Last theorem, Adequality, Fermat's difference quotient method** etc. Fermat was a mathematician of rare power and together with **Rene Descartes**, he was considered as one of the two leading mathematicians of the first half of the 17th Century.

***(Brief Biographical sketch is given inside the journal)**

This page is specially donated by Prof. Shailesh Teredesai (Ex. Head), Statistics Dept., S. M. Patel Insitute of Commerce, GLS, Ahmedabad-380 009.

Printed Matter **BOOK-POST**

(Journal of GSA, Ahmedabad)

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