ISSN 2321-0877

SANKHYA VIGNAN
સંખ્યા વિજ્ઞાન

NEW SERIES (NSV 12) JUNE 2016 No. 1

CONTENTS

Published by
GUJARAT STATISTICAL ASSOCIATION

SANKHYA VIGNAN
સંખ્યા વિજ્ઞાન

(e-mail ID : svgsa2015@gmail.com)

Cut Here

THIS IS COMPULSARY FOR YOU

MEMBERSHIP UPDATE PROFORMA

- Please fill up the following proforma as soon as you receive this copy of Sankhya Vignan. (Please ignore if you have already sent this form earlier)
- Please send it immediately by **return of post** at the folliwng address.

Prin. A. M. Patel (Joint Editor)

23, New Sharad Society, Besides Vijaynagar Bus Stop, Naranpura,
Ahmedabad-380013 Phone : (R) 27479287 (M) 9998444161

- **Please note that failure to send this proforma can find you in non-receipt of any further information.**

PROFORMA

(1) Full Name : ______________________

(2) Residential Address (with PINCODE) ______________________

(A) Present : ______________________

(B) Permanent : ______________________

(3) Birth Date : ______________________

(4) Age : ______________________

(5) Your present designation : ______________________

(6) Institution / Organisation Address : ______________________

(7) Work Experience : Months/Years ______________________

(8) Are you a member of GSA ? : Yes / No

If Yes, type of membership LM/OM/IM

(9) Where can we send journal to you ? ______________________

(a) Residence (b) Institution / Office

(10) Your contact details (with STD / Code ISD) ______________________

(a) Phone number (i) Residence __________ (ii) Office __________

(b) Mobile Number (i) __________ (ii) __________

(11) Email address : ______________________

Website : ______________________

Blog : ______________________

Cut Here

Place :

Date : Signature

Note : Please give your feedback. (Write overleaf.)

Cut Here

FEED BACK

Date :

Signature

Note : For any querries you may please contact (1) Dr. B. B. Jani, e-mail : bbjani2012@gmail.com; (2) Dr. J. R. Purohit, drjayesh.purohit@gmail.com, svgsa2015@gmail.com

EDITORIAL

We are very happy to present the issue of S V Journal (NSV 12, June 2016, No. 1) to our readers. This issue contains 2 articles, 5 research articles, 1 technical note, 1 biography, 1 book review and other sections as usual.

Under the caption entitled **Statistics And Management** there are three articles.

First Article presented by **A. C. Brahmbhatt** is a very interesting study on market behaviour and consumer attitude as viewed by statistician's eye.

Second Article which is expressed very lucidly by **Jayesh R. Purohit** concerns about data scientists and their importance in these days.

Third Article is a brief discussion linking TQM philosophy with environment science. This work is carried out by **M. N. Gopalan**.

First Research Article is an interesting study made by **Vijay S. Jariwala** concerning econometric analysis used for a very specific monetary economics problem.

Second Research Article furnished by the **same author** discusses detailed analysis on the above stated problem by means of some specialised tests of significance.

Third Research Article presented by two authors **U. B. Gothi** and **Ankit Bhojak** considers a specialied type of Inventory control model with its application.

Fourth Research Article is furnished by **D. K. Ghosh** which concerns a case study using survival time analysis and its interpretations.

Fifth Research Article describes briefly statistical analysis concerning Onion Crop production, cultivation area and export potential for India. This work is presented by **H. M. Dixit** and **P. M. Parmar.**

On the event of Mahalanobis Day celebration on 29th June, 2016. We are very happy to present technical note on his prestigious work of **Mahalanobis D^2 Statistic**. This work is executed by **Devyani Chatterji**.

A brief biographical SKETCH on a very famous Mathematician (as well as statistician) **James Bernoulli** is presented by **H. D. Budhbhatti**.

A book review is carried out by **H. M. Dixit** on a famous book in econometrics field.

There are very exciting news for all of us, **Prof. D. K. Ghosh** (Ex. Head, Dept. of Statistics, Saurashtra University, Rajkot) has been nominated as **FRSS (Fellow of Royal Statistical Society at London, U.K.)** from 2016. On behalf of the editorial board we all very heartily congratulate him at this occassion. A brief biodata about his achievements has been furnished in the section **Statistics News Letter by K. Muralidharan.**

As usuall, we have **Readers Forum** presenting feedback and views of our readers. This is presented by **A. M. Patel**.

We are highly indebted to our following **REFEREES** who have carried out an excellent job for evaluating the articles submitted for publication in this journal. (The names are taken **one by one** in order of their appearance in this issue.)

(1) Jayesh R. Purohit	**(2) A. C. Brahmbhatt**
(3) Ashwin J. Patel	**(4) Manish B. Thakar**
(5) Himanshu N. Desai	**(6) R. G. Bhatt**
(7) Manish B. Thakar	**(8) S. G. Raval**
(9) U. B. Gothi	**(10) H. M. Dixit**

We shall be seding **DIGITAL COPY** of this issue to all our readers who have registered their email ID with us. We shall also send the **printed copy** of this journal to our readers later on.

We thank our contributors, readers and learned evaluators for their consistent support and help to bring out this issue in time.

Editorial Board proposes to **publish a special issue** on October 20, 2016 **(World Statistics Day).** Articles and papers are invited for this issue. The next December 2016 issue will follow it as usual thus bringing 3 issues in this year.

We wish you pink health and season's greetings.

Ahmedabad
Dated: 29-06-2016. **(Statistics Day)**

STATISTICS HELPING MARKETERS KNOW THE MENTAL MAP OF THE CONSUMERS

A. C. Brahmbhatt*

These are the days when the entire business revolves around the customer. Customer is the king and no one can afford to ignore the customer. The marketers are trying and vying to woo the customers by offering the value added products and services. They all aim at producing the products and services that not only ensure the customer satisfaction but also the customer delight. It is not enough to understand and recognize the demographics of the customers i.e. their age, occupation, education, marital status , income , domicile etc. it is more pertinent to understand their buying behavior, their attitude, their buying intention, their perception , preferences and their psychographics.

The marketers of course, according to their own competency levels make hard efforts to understand both consumer demographics and psychographics , but if they are further equipped with the statistical tools and techniques , that helps them enter the mental map of the customers. Some primary tools do help them in this pursuit of them , but as they go on acquiring more advanced tools , they have an access to the mental map or a perceptual map of the customers.

By adopting descriptive research design and designing a scientific questionnaire that includes ordinally scaled questions they can find out the attributes of their product that attract them the most; e.g .they can ask a ranking question in which the respondents would assign the ranks to the given list of the attributes in the order of its importance to them. They may use the Paired Comparison scale wherein in a square matrix form the different competitive brands are presented and the respondents are asked to mark whether the particular brand is superior (giving code 1) or inferior (giving code 0) according to them.The brand that has maximum total of 1s is the most preferred brand. But the weakness of the scale would be that it compares only

* Ex. chairperson, Ph.D. (Ext.) Programme, IM, Nirma University.
Now Research mentor at Pandit Dindayal Petroleum University, (PDPU), Gandhinagar.
(rcd. and revd. May ’16)

a pair of the brands and neglects the comparison with the other brands in the matrix. Also it suffers from cyclicity i.e. Brand A may turn out to be superior to Brand C , Brand C superior to D and again D superior to Brand A !

In both the above scales the comparison is made only on one dimension say they are compared on price dimension or a quality dimension , that means both the scales are unidimensional. In reality when the customer compares the competitive brands , he compares on several dimensions –it could be not only price or quality , it could be packaging, size, after sales service , availability, self esteem associated with it etc. The earlier two scales false short of doing it. There is **Multidimensional Scaling Method(MDS)** that generates perceptual maps or mental maps of different dimensions facilitating the comparison. It shows us as to how the judgement formation process takes place in the minds of the customers while comparing the competitive brands , it could de soaps, shampoos, toothpastes etc.

The operational part of MDS generates a matrix showing different scale values assigned by the respondents to the brands on 7-point Similarity-Dissimilarity scale . The softwares like **ALSCAL** or **MDSCAL** would convert the matrix into the Distance matrix using inter-point distance formula and generating Perceptual Maps . The brands perceived to be very similar are positioned by plotting the points closed by on the map, the brands that are perceived to be dissimilar are as points plotted wide apart from each other. The number of dimensions of the perceptual maps are decided by the Stress Index –lower the stress index ,better the fit.

The marketers by the help of MDS enters into the perceptual map or a mental map of the customers and comes to know which brands are in the direct competition(the very closed by positioned brands on the map) on what dimension, also which is a stand alone brand with no threat of competition. One additional benefit the marketers derive from MDS and it is of positioning a new brand in the market. The quadrant of a perceptual map which is fully empty ,directs the marketers to fill up that gap by their new brand. How wonderful it is! ; simply sitting in your cozy ,air cooled room and viewing a gap on perceptual map on your laptop, you do the positioning of your new brand!!

ACKNOWLEDGEMENTS

I sincerely thank Dr. B. B. Jani for his invitation to present this article under this caption.

I also thank the refere (anonymous) for reviewing my article.

IMPORTANSCE OF DATA SCIENTISTS (They earn more than Engineers, CAs)

Jayesh R. Purohit*

1 What is Data Science?

Data science is an interdisciplinary field about processes and systems to extract knowledgeor insights from datain various forms, either structured or unstructured,which is a continuation of some of the data analysis fields; such as statistics, data mining and predictive analytics,similar to**Knowledge Discovery in Databases** (KDD).

2 Overview

Data science employs techniques and theories drawn from many fields within the broad areas of mathematics, statistics, operations research, information scienceand computer science, including signal processingprobability models, machine learning, statistical learning, data mining, database, data engineering, pattern recognition and learning, visualization, predictive analytics, uncertainty modelling, data warehousing, data compression, computer programming, artificial intelligenceand high performance computing. Methods that scale to big dataare of particular interest in data science, although the discipline is not generally considered to be restricted to such big data and big data solutions are often focused on organizing and pre-processing the data instead of analysis. The development of machine learninghas enhanced the growth and importance of data science.

Data Science affects academic and applied research in many domains, including machine translation, speech recognition, robotics, search engines, digital economy but also the biological sciences, medical informatics,health care, social sciences and the humanities. It heavily influences economics,business and finance. **From the Business perspective, Data Science is an integral part of competitive intelligence, a newly emerging field that encompasses a number of activities, such as data mining and data analysis**

3 Data Scientist

Data Scientists use their data and analytical ability to find and interpret rich data sources; manage large amounts of data despite hardware, software and bandwidth

*Mantra Consultants, Ahmedabad. (rcd. May’16 / rvd. June’16)

constraints; merge data sources; ensure consistency of datasets; create visualizations to aid in understanding data; build mathematical models using the data and present and communicate the data insights/findings.

They are often expected to produce answers in days rather than months, work by exploratory analysis and rapid iteration and to produce and present results with dashboards (displays of current values) rather than papers/reports, as statisticians normally do.

Data Scientist has become a popular occupation with **Harvard Business Review** dubbing it **The Sexiest Job of the 21st Century** and McKinsey & Company projecting a global excess demand of 1.50 million new data scientists. Universities are offering masters courses in data science.Shorter private boot camps are also offering data science certificates including student-paid programs like General Assemblyto employer-paid programs like The Data Incubator.

4 Data Science Process

The term **Data Science** (originally used interchangeably with **Datalogy**) has existed for over thirty years and was used initially as a **substitute for computer science**by **Peter Naur** in 1960. In 1974, Naur published *Concise Survey of Computer Methods*, which freely used the term data science in its survey of the contemporary data processing methods that are used in a wide range of applications. In 1996, members of the International Federation of Classification Societies (IFCS) met in Kobe for their biennial conference. Here, for the first time, the term data science is included in the title of the conference **(Data Science, classification, and related methods**).

In November 1997, **C.F. Jeff Wu** gave the inaugural lecture entitled **Statistics = Data Science?**for his appointment to the **H. C. Carver Professorship at the University of Michigan**. In this lecture, he characterized statistical work as a trilogy of data collection, data modelling and analysis, and decision making. In his conclusion, he initiated the modern, non-computer science, usage of the term **data Science** and advocated that **Statistics be renamed Data Science and Statisticians as Data Scientists**. Later, he presented his lecture entitled "**Statistics = Data Science**" as the first of his 1998 **P.C. Mahalanobis Memorial Lectures**. These lectures **honour Prasanta Chandra Mahalanobis, an Indian Scientist and Statistician and founder of the Indian Statistical Institute**

In 2001, **William S. Cleveland** introduced data science as an independent discipline, extending the field of statistics to incorporate "advances in computing with data" in his article **"Data Science: An Action Plan for Expanding the Technical Areas of**

the Field of Statistics",which was published in Volume 69, No. 1, of the April 2001 edition of the **International Statistical Review** / Revue Internationale de Statistique. In his report, Cleveland establishes six technical areas which he believed to encompass the field of data science: multidisciplinary investigations, models and methods for data, computing with data, pedagogy, tool evaluation, and theory.

In April 2002, the **International Council for Science: Committee on Data for Science and Technology (CODATA)** started the *Data Science Journal,* a publication focused on issues such as the description of data systems, their publication on the internet, applications and legal issues. Shortly thereafter, in January 2003, Columbia University began publishing *The Journal of Data Science*,which provided a platform for all data workers to present their views and exchange ideas. The journal was largely devoted to the application of statistical methods and quantitative research.

In 2005, The National Science Board published "Long-lived Digital Data Collections: Enabling Research and Education in the 21st Century" defining data scientists as "the information and computer scientists, database and software and programmers, disciplinary experts, curators and expert annotators, librarians, archivists, and others, who are crucial to the successful management of a digital data collection" whose primary activity is to "conduct creative inquiry and analysis."

In 2013, the IEEE Task Force on Data Science and Advanced Analytics was launched, and the first international conference: IEEE International Conference on Data Science and Advanced Analytics was launched in 2014. In 2015, the International Journal on Data Science and Analytics was launched by Springer to publish original work on data science and big data analytics.

In 2008, DJ Patiland Jeff Hammerbacherused the term "data scientist" to define their jobs at LinkedIn and Facebook, respectively. In India too, companies are scrambling to hire Data Scientists, sending salaries soaring.

According to TeamLease, a staffing solution company, Data Scientists with around Five Years' experience are earning over Rs 60 to 75 lakh per annum as compared to Rs 10 to 15 lakh for CAs and Rs 7 to 10 lakh for engineers with the same experience level.

Very specifically, Data Scientists are the guys who know how to look at the data a company generates and derive the all-important insights it needs to garner more business and enhance customer experience in this age of social media.

With companies across industries striving to bring their Research & Analysis (R&A) departments up to speed, the demand for qualified Data Scientists is rising. India will face a demand – supply gap of 200,000 analytics professionals over the next three years. Even in the US, only 40 out of 100 positions for analytics professionals can be filled, said Rituparna Chakroborty, co-founder & senior VP of the TeamLease Services. In the US, Data Scientists get upwards of $200,000 per annum.

Data analytics professionals are primarily Mathematicians, Statisticians, Data Base / Data warehouse engineers, Data Miners and IT professionals with data warehousing skills. **A Data Scientist is a hybrid of many of the above listed skills and therefore, a rare breed.** To meet their talent requirements, some companies have come up with unique programmes.

WNS, which is in the business process management industry, has created a fully funded post-graduate MBA programme in business analytics customised around its needs. All student completing the programme would be absorbed by WNS. Keshav Mrugesh, Group CEO of WNS said: "Business models are changing across the world. A lot more data is available and companies want to provide focussed, prescriptive solutions. If analytics is going to play such a big part, we need to make sure we have the right kind of talent and skills inside the company to feed those programmes."

WNS currently draws 13.00 % of its revenue from Research & Analysis and believes there is further growth potential. In a competitive world, companies like WNS want to leverage data using interactive models, algorithms, automation and then create prediction based analytics to serve clients better.

Two years ago you wouldn't be talking about Data Scientists, Social architects or anthropologists working in a company, pointed out Prithvi Shergill, CHRO at HCL Technologies. But now a lot of people arevery keen to align their long term carriers around analytics. However, curriculums have not evolved fast enough and this is not restricted to India alone. US – Based thought leader and HR consultant, Jason Averbook, said: In addition to Mathematics and Statistics, there is also a requirement for people with Marketing Communications.

5 Acknowledgements

I thank the referee for reviewing and revising this article

TQM AND ENVIRONMENTAL SCIENCE

M. N. Gopalan*

ABSTRACT

This article briefly discusses the hazards affecting environment in day to day life. The concept of TQM when systemised properly can be made advantageous to deal with the situations.

1. INTRODUCTION

The philosophy of total quality managemnt (TQM) canbe applied to every field of human activity. The main objective of TQM philosophy is to ensure a high degree of performance of a system by filtering the factors detrimental to the performance of the system.

In what follows, the performance of a system governing the environment is discussed. Environment is full of natural and man made impurities detrimental to the very existence of life on earth.

Environment gets polluted due to hazardous elements present in air and water. One additional source of pollution of environment is sound. Sound produced beyond a critical limit could be extremely hazardous to living beings. Environmental Science presents various strategies to protect environment from hazardous factors and ensure that life is worth living.

These strategies are nothing but those adopted by TQM experts in controlling the quality of a production process.

2. GRASHAM'S LAW AND ENVIRONMENT

Grasham's law states that 'black money drives white money out of circulation'. This statement is very much relevent in the field of Environmental Science. Environment has to be free from the dangerous influences of poisonous elements for survial of life. These elements could be from various sources, many of which are man-made.

In this write-up, an effort is made to study the influences of three hazardous

* Ex. Professor, I.I.T., Mumbai. Also retired Emeritous Professor at University of Mysore, Mysore.

elements, namely,

1. Poisonous gases affecting the atmosphere
2. Chemical fertilizers affecting water
3. Sound of high magnitude created by man-made systems, affecting the health of operators nearby these systems.

2.1 ENVIRONMENTAL HAZARD DUE TO POISONOUS GASES

Some of the poisonous gases are

1. Carbon-di-oxide
2. Carbon monoxide
3. Sulphur-dio-xide
4. Methane

Carbon-di-oxide is let out to atmosphere by all living creatures when they breathe. Carbon monoxide is emitted by diesel and petrol-driven vehicles. Of these two gases, carbon monoxide is more harmful to life. Carbon-di-oxide is useful to plants and trees. They use carbon-di-oxide to produce chlorophyl and let out oxygen. Oxygen is used by all living beings for breathing. Chlorophyl is used by plants and trees for their growth. Such is the wonderful relationship between the plants, trees, animals and human beings. Though carbon-di-oxide is useful to living beings in the way explained earlier, much of the gas produced by burning fossil fuels to run vehicles escapes to atmosphere and causes the "Green House Effect". This is the major problem confronting the environmental scientist. Steps have to be taken to arrest the emission of carbon monoxide is injurious to health when it is inhaled. It should also be curbed from escaping to atmosphere. It is only through research and development efforts, one can overcome the hazards of these gases.

Sulphur-di-oxide is emitted by chemical and other industries which use Sulphur as an ingredient to products manufactured by them. The gas produced during the manufacture of sulphur-based products reaches the atmosphere and reaches the earth again in the from of acid-rain.

Methane is generated as a bi-product by various sources on earth. It is generated by various types of wastes of organic origin. Methane is useful as fuel for domestic and other purposes.

The atmosphere could be compared to a huge lake full of pure water. The atmosphere gets polluted with the inflow of impurities in the form of poisonous gases.

In a way, pure water can be compared to white money whereas, poisonous gases can be compared to black money. Grasham's law is thus applicable to the atmosphere around us. Once the atmosphere gets highly polluted, life seizes to exist on earth. It is, therefore, imperative to adopt stringent measures suggested by TQM experts to control pollution of the atmosphere to enable life to exist on earth.

2.2 ENVIRONMENTAL HAZARD DUE TO CHEMICAL FERTILISERS

Chemical fertilizers are, of late, used in large quantities in agricultural fields to grow more crops. Chemical fertilizers have proved to be a curse to the environment in more than one way. The soil loses its natural fertility. The water used in the fields gets mixed with the chemical fertilizers used and flows into the river nearby thus polluting the river water. The polluted water becomes unfit for living beings as, it leads to various types of health hazards.

The river water can be compared to white money, as long as it is free from pollution. The hazardous chemicals let out to river nearby from the fields can be viewed as black money. Grasham's law is thus applicable to water, a vital source for existence of life on earth. It is the duty of everyone concerned to strictly adopt the philosophy of TQM to control the contamination of water by taking precautionary steps at the right place and at the right time.

2.3 ENVIRONMENTAL HAZARD DUE TO NOISE

Noise is dangerous to one's health if, its intensity exceeds a critical limit.

Research and development efforts should be undertaken to reduce noise pollution. Noise affects one's ears. It is extremely harmful especially to patients suffering from heart problems and also to new-born babies as well as birds and animals also.

Noise can be viewed as black money. Even a bit of noise can cause ripples to disturb the surroundings. It can severely affect the brain of a living creature. The damage caused by noise could be too severe and beyond repair. Unless noise is checked in time, it could result in dangerous consequences to society in the form of health hazards. It is indeed possible to achieve the desired goal if and only if one adopts the basic philosophy of TQM from the very early stage itself.

3. CONCLUDING REMARKS

Man has started polluting the basic elements needed for survival of life on Earth. With more and more industrialization, the entire atmosphere would be further polluted with the result that, it becomes impossible for living being to live on Earth. Dr. Sir

S. Radhakrishnan, one of the world-known thinkers has righty said that 'man has learnt how to swim in waters like fish, fly in the sky like birds but, he is yet to learn to live on earth like an ordinary human being'. While technology is essential for man to live in comfort, it should not be a threat to survival of life in comfort, it should not be a threat to survival of life on Earth. Technology should be eco-friendly. Of late, man has developed technology that could wipe out the entire life on Earth. What a tragedy'. Man has been trying for years to establish peace on Earth by developing sophistaced weapons of mass destruction. How strangel. How unfortunate it is ! Man should seek ways and means for the establishment of peace on Earth by living in harmony with nature. The only way open to mankind to establish peace on Earth is to treat every living being as Mother Nature's children with love and affection. Man has no right to upset balance of nature. He should live as a child of nature.

Need arises to find sustainable ways for a happier tomorrow. **'Go Green'** policy should be adopted to deal with environmental hazzards affecting our lives. **On world environmental day (6th June)** we can resolve about plantation of one house, one plant/tree programme which may solve our problems. If this is not done, there will be tremendous effect on our daily livelihood. Switching to energy efficient equipments can reduce our overall energy consumption reasonably which in turn can partially solve our problems of environments.

ACKNOWLEDGEMENT

I thank Director, Prasarangas University of Mysore for his consent to publish this article, I also thank the referee for his review and comments made on the earlier draft of this paper.

SELECTED REFERENCES

[1] **E. Belagwak Samy (1984)** *Realiability engineering* (Tata McGrow Hills Publishers, New Delhi)

[2] **M. N. Gopalan (2013).** *Higher system performance through total quality management* (Prasaranga University of Mysore, Manasgangotri, Mysore)

[3] **M. N. Gopalan (2014),** *TQM in Education* (Sankhya Vignan NSV 10, June 1)

[4] **Montgomery D.C. (1991)** I*ntroduction to statistical quality control* (John Wiley, New York)

[5] **P. G. Gupta (1999)** *Statistical Quality Control* (Khanna Publishers, Delhi)

[6] **S. C. Sharma (1995)** *Inspection, Quality Control and Reliability* (Khanna Publishers, Delhi

RESEARCH ARTICLE

AN ECONOMETRIC ANALYSIS OF MONTHLY DEMAND FOR MONEY IN INDIA

Vijay S. Jariwala[1]

ABSTRACT

Demand for money function in India has been experimented with various specification and estimation issues.The main focus of this exercise is to re-evaluate the performance of the money demand specifications in the context of identifying suitable determinants.The major concern of this effort is to find out the appropriate channel of monetary transmission mechanism in the economy.In contrast with the prior studies, this study has applied multiple regression, unit root and cointegration tests on host of various money supply aggregates for monthly time series from April, 1990-91 to July, 2013-14 along with output, incomeand interest rates as money demand function variables.The statistical insignificance of the interest rate variable implies that interest rate cannot be a suitable variable for monetary management variable in India.

Keywords: *Money Demand,Multiple Rgression,Cointegration*

1. INTRODUCTION

Money is considered as next to the two great inventions of the world, viz. the fire and the wheel because of the widespread utilization of it. Broadly, speaking money serves four major functions- medium of exchange, store of value, unit of account and source of deferred payments. Money serves as a medium of exchange and as a standard unit in which prices of commodities and debts are expressed. The world of economic transactions has become broader with the use of money.Money cannot satisfy consumption need directly like goods and services that possess utility. Money is utilized to acquire such goods and services. Money is a means not an end. Then the question arises, "Why do people demand money? Why do people hold money?" Money is demanded because of the functions performed by it. Empirical research in this area centre around a few interesting issues. They include: (i) empirical definition of money, (ii) choice of appropriate scale and opportunity cost variables, (iii) role of adjustment and expected lags, (iv) estimation of functional forms and (v) temporal stability (Laidler,

[1] Assistant Professor, Department of Economics, M K Bhavnagar University, Bhavnagar-364002
Email- viju_19782004@yahoo.com (rcd. April '16 / frvd. May'16/srvd. June'16)
(This paper was presented at 51st Annual Conference of The Indian Econometric Society (TIES) at Patiala University, Punjab during 12-14 December, 2014.)

1969).Thispaper covers traditional multiple regression analysis along with stationarity and cointegration tests.

2. REVIEW OF LITERATURE

Voluminous literature is now available for analyzing money demand functions. Previous works were mainly confined to industrialized countries.Almost all the Central banks have realized that for the stronger monetary base and monetary policy, stable money demand function is essential. During the last ten years or so advancement made in time series analysis of money demand models are mostly based on empirical models built previously.The various earlier studies on money demand can be classified on the basis of the inclusion of scale variables like income and consumption. On the other hand, opportunity cost variables like interest rate and inflation have been found in these studies [Fase (1994), Sriram (2001), Knell and Stix (2005)].

Demand for money function in India has been extensively studied with various specification and estimation issues. M.S.Trivedi (1980, 1983, 1984) has studied money demand function in India by applying partial adjustment models. R.K. Sampath and ZakirHussain have analyzed various conventional models for the period of 1960 to 1975 by using Log-linear functional forms.Moosa(1992) was first to study stationarity and cointegration relationship among the variables of money demand function in India.He found cointegration relationship among all the money supply aggregates with output and interest rates.Bhattacharya (1995)found cointegrating relationship among the variables only when M1 is utilized and concluded that the long-term interest rates are more sensitive to money demand. Takeshi Inoue and Shigeyuki Hamori (2008) have analyzed India's money demand functionusing M1 and M2 money supply and detected cointegrating vector among real money balances, interest rates and output.As is the case with the prior studies referred above that India's money demand function is stable.Views therefore differ regarding which money supply definition should be utilized while framing monetary policy.

3. METHODOLOGY AND MODEL SPECIFICATION

To formulate demand for money function, the traditional quantity theory of money (QTM) demand expressed as MV = Py[1] has been considered. As suggested by Friedman (1987) including various determinants of money demand, asimple money demand function can be formulated as $(M/P)^d = f\ (y, r, E, S)$, where M is money stock, P is general price level, y is real income, r is interest rate, E is exchange rate, S is stock price. The exchange rate and stocks price are included as additional determinants of demand for money. The rationality is that foreign exchange and stocks constitutes a part of portfolio of economic agents. Depreciation in exchange rate may result in further depreciation of the currency, which will force individuals to hold money as foreign currency to avoid possible losses. Net effect of stock price could be either positive or negative. Thus, the demand for real money balances as a function of real income, interest rate, real stock market capitalization, real exchange rate can be specified as[2]

$(M/P)^d_t = \beta_0 + \beta_1(Y/p)_t + \beta_2 r_t + \beta_3 E_t + \beta_4 S_t + \varepsilon_{t-}$ .... (1)

Where, M is nominal money supply at time t, P is the price level (WPI), Y is nominal income, r_t is the opportunity cost variable , such as interest rates and Et is the real exchange rate, St is the real stock price at time t. In the equation, $(M/P)^d_t$ represents the real money balance and (Y/p)t is the real income at time t. Theoretically, demand for money is directly related with real income and indirectly with interest rates. However, the sign of the exchange rate andstock price is uncertain. Therefore, by convention, the values of the coefficient of income (β_1) should be positive and interest rate (β_2) is negative but for exchange rate (β_3) and stock market capitalization (β_4) it could be either negative or positive. It calls for empirical estimation of money demand functions and test of stability.

Equation (1) can be estimated by multiple linear regression models, although regression model does not explain dynamic relationship among the specified variables. The relationship could also be spurious. The equation can be expressed as a vector of variables, where each variable might be dynamically interrelated. Their long run and short-run equilibrium relationship can be examined using cointegration and error correction model respectively. Technically speaking the money demand equation can be considered as a cointegrating equation.

The estimation of Johansen –Juselius cointegration equation is based on Vector Auto Regression (VAR) model estimated through maximum likelihood estimation procedure. Johansen (1988, 1991), Johansen-Juselius (1990, 1992) methodology is design to determine the number of cointegrating vector in the VAR system. The methods specify two test statistics in order to test the number of cointegrating vectors. Those are λ max (the maximum Eigen value statistics) and λ trace statistics. The first step of testing cointegration is to tests whether the series are stationary or not i.e. I (1) or I (0). Then apply the cointegration for non-stationary series i.e the series at levels if the variables are I (1). Augmented Dickey-Fuller (ADF), Phillips-Peron (PP), Bayesian and KPSS unit roots tests both with and without trend for the logs of real money balances, output, income, and interest rates (Dickey and Fuller 1979) are carried out. Each variable has been tested and found to have unit-root, whereas the first difference of each variable found without a unit-root. Thus, it can be summarized that each

[1] According to the standard textbooks, real income determines the demand for money in the classical sense as money demanded for transaction purpose, whereas for Keynesian it is real income and interest rate as people demand money for transaction, precautionary and speculative purpose. On the other hand, Friedman has given a list factors affecting money demand such as stock, bonds, etc. which generated wealth. Similarly, exchange rate also determines the demand for money because of substitution as well as wealth effect.

[2] According to Rangarajan(1984), it is possible to build into such a formulation the lagged impact of the factors that influence the money holding.

variable is a nonstationary variable with a unit root.

The cointegration test of Johansen-Juselius (1990) possibly avoids the problem by allowing feedback relationship and provides the long-run equilibrium relationship among variables. The error correction mechanism, Granger causality and host of other stability analysis are the other specialized tests which can be carried out fora separate analysis.

This study comprises of Monthly data over the period of April 1990 to July 2013 a data set of 280 observations for various variables. The data source for the Industrial Production Index (IIP) and the Whole sale Price Index (WPI) as well as for Consumer Price Index (CPI) has been from Economic Survey 2012-13. While the money supply aggregates are collected from various issues of the RBI Bulletin and deflated by WPI. Apart from it, the weighted average of call money rates is taken as the interest rate and considered from various issues of RBI Bulletin. Due to the unavailability of appropriate Income measure for monthly data, various scale variables have been experimented. However, IIP is not the correct measure for Income; it has been considered as the scale variable in the money demand equation. Moreover, a host of other variables like market capitalization measured in BSE-Stock Market deflated with WPI, Average annual price of 10g. Gold price deflated with WPI are taken to capture the wealth effect of the money demand as being a store of value. Gold is generally hedge against the inflation, therefore to capture the effects, I estimated Real Average Gold price in the money demand equation by deflating Gold price with Average WPI.

4.1 REGRESSION RESULTS

Money demand equation 1 can be estimated through multiple regression model. I have estimated eight different regression equations for four different alternative combinations of monetary aggregates using two types of opportunity cost variables separately. The regression model is estimated with Newy-West HAC standard error and covariance with lag truncation to avoid the possibility of unknown heteroscedasticity and autocorrelation problem. The regression results are reported in table 1.3 to 1.6 in appendix. The regression results are robust due to high R^2, significant F statistics, no autocorrelation and no heteroscedasticity problem. The limitation is that multiple regression model does not explain dynamic relatioship among the variables. The cointegration techniques e.g. Johansen-Juselius (1990) applied here can overcome such problem.

Four alternative specifications each for M1 and M3 have been estimated and presented in table 1.2 in appendix. The first four equations are experimented with both monetary aggregates along with stock market capitalisation and opportunity cost variables CR and CI. The second set of four equations is experimented with log of gold price variable (LGP) with both of opportunity cost variables. The estimated t-ratios are given in the parentheses below to the respective coefficients. The results show that by using the industrial production index (IIP) as a proxy for income variable, found to be more than unity in all specifications except one withM1 which is less than one. The call money rate as a proxy for opportunity cost of holding money found surprisingly with a positive elasticity in the M1 equations but revealed expected negative sign with being statistically significant for the M3 functions. The coefficients are significant either at 1% or 5% level for all the specifications. It is further observed that the stock market capitalisation is positively related to monetary aggregates but not statistically significant except in one case. The exchange rate variables turned out to be significant with appropriate negative sign in all eight specifications. All the estimated equations are found with high $\overline{R}^2$ and low D-W statistics.

4.2 RESULTS OF STATIONARY SERIES

If any linear combination of two or more non-stationary series is stationary then the series is said to be cointegrated. The application of cointegration needs prior checking of stationary properties. Augmented Dicky Fuller (ADF), Phillips-Perron (PP,1988) and Kwiatkowsk, Phillips, Schmidt, and Shin (KPSS,1992) unit root tests both with trend and without trend for the whole sample period hasbeen estimated. The results are presented in table 1.7 & 1.8. ADF and PP testsaddress the issue of possible serial correlation in the regression model and tests the hypothesis. While KPSS test is a confirmatory test. The results are reported in the table. The models are estimated including constant (C) and with constant & trend (C & T) term in the regression equations separately. For ADF test, p-value is given in the parentheses. For PP and KPSS tests the brackets represent the bandwidth of Newey- West using Bartlett kernel. For PP tests p values are in the parenthesis.

The PP tests assume the null hypothesis of unit root against the alternative of stationary. On the other hand KPSS is a confirmatory test, which assumes the null hypothesis of stationarity against the alternative of non-stationarity. For all variables at level the null hypothesis is accepted. However, for the variables at first difference, we reject the null hypothesis of unit root at 1% significance level for ADF, PP and KPSS test of alternative model specification. Thus variables are stationary at first difference and non-stationary at level. Hence, we can apply cointegration tests at level data.

4.3 COINTEGRATION RESULTS

The next step is to apply the multivariate cointegration test of Johansen (1988, 1991) and Johansen-Juselius (1990, 1992), estimated through maximum likelihood estimation procedure. Two tests statistics such as λ trace and λ maximum Eigen value is used to determine the number of cointegration vector. For n variable cases if at least one (r=1) cointegrating vector is present, it is sufficient to conclude that the variables are cointegrated. The number of cointegrating vector is estimated through Vector Auto Regression (VAR) model for which it is necessary to specify the number of lag length in the autoregressive process. We have started with one lag and maximum of eight is taken in the process. The lag length of five is chosen based on Akaike Information

Criteria, Schwarz Bayesian Criteria and log likelihood ratio tests, which is theoretically and practically justified. The robustness of the models have also been checked using ARCH, LM, JB, Heteroscedasticity test.

Once the optimal lag length is determined then the next step is to apply cointegration test. The obtained results are reported in table 1.9 to 1.12. Panel A specify the cointegration equation with constant term, whereas panel B specify model with the linear deterministic trend term. In both the cases, we have estimated cointegrating equations with two different opportunity cost variables. In panel A, irrespective of money demand specification with any interest rates, the null hypothesis of zero cointegrating vector (r=0) is rejected against the alternative of at least one cointegrating vector at 1% significance level. The same result is repeated in panel B also. For both the models, we found minimum one cointegrating vector. Further, testing more number of cointegrating vectors, we might obtain different results, as shown in the table. This is evident from both trace and eigen value statistics. For example, the null hypothesis of r=1 cointegrating vector is rejected and alternative of three cointegrating vector is accepted for LRM1 money demand function with constant term. Similarly for LRM3 money demand function with the trend, the null hypothesis of r=1 cointegratingvector is rejected at the 5 % significant level and alternative of r> 2 is accepted. The result strongly supports the presence of one cointegrating vector for both the demand functions. Therefore, we can conclude that cointegration exists between variables and hence in the long-run they are related.

5. IMPLICATIONS OF DATA RESULTS

From the data estimations, which covers not only recent period but also having more observations,provide good indications for Indian money market and stock market. The following implications may be drawn.First, the demand for real cash balances is positively and significantly related to the real stock market capitalisation reflecting the presence of wealth or transaction effect. These findings further confirm the results obtained from other studies. Secondly , the high income elasticity again confirms the monetarist view in the case of India, that money holding is a luxury. Thirdly, the significance of the real effective exchange rate indicates the presence of absorption reducing effect of exchange rate depreciation through a rise in wealth on account of the net debtors position in terms of ownership claims on the rest of the world. This result is in contrast to the results obtained from the other studies. This implies that the effect is only a temporary one, short run phenomenon, which may peters out in the long run. As majority of model specifications satisfy the cointegration in VAR, error correction mechnism, Granger causality and stabilitiy tests can be performed for other analysis.

6. ACKNOWLEDGMENT

This research endeavour is based on my Ph.D. thesis entitled Demand for Money in India- An Econometric Analysis. I would like to thank (anonymous) referees for their insight and comments. It has helped me to improve and reshape this manuscript in a meaningful way.

7. APPENDIX

Monthly Data Analysis from April, 1990-91 to July, 2013-14

TABLE 1.1 - Description of Variables

Variables	Definitions
RM1	Real Narrow Money- Real Narrow money is comprised of currency with the public, demand deposits with banks and other deposits with RBI
RM3	Real Broad Money – which includes M1 and time deposits with banks
IIP	Index number of Industrial Production
CR	Weighted Average of Call/Notice Money Rates
CI	Average Monthly Inflation Rate based on Consumer Price Index
RBSEMCAP	Real market capitalisation (BSE- Stock Market)
REER	Real effective exchange rate (36 country trade weighted)
GP	Average monthly price of 10g Gold at Mumbai Bullion Market
LRM1	Log value of Real Narrow Money
LRM3	Log value of Real Broad Money
LRBSMA	Log value of Real market capitalisation (BSE-Stock Market)
LREER	Log value of Real effective exchange rate (36 country trade weighted)
LGP	Log value of Average monthly price of 10g Gold at Mumbai Bullion Market

Table – 1.2 Money Demand Specifications

Sr. No.	Dependent Variable	Independent Variables
1.	LRM1	LIIP, LREER, LRBSMA, CR
2.	LRM3	LIIP, LREER, LRBSMA, CR
3.	LRM1	LIIP, LREER, LRBSMA, CI
4.	LRM3	LIIP, LREER, LRBSMA, CI
5.	LRM1	LIIP, LREER, LGP, CR
6.	LRM3	LIIP, LREER, LGP, CR
7.	LRM1	LIIP, LREER, LGP, CI
8.	LRM3	LIIP, LREER, LGP, CI

OLS Estimates of Demand for Money Function

Table – 1.3 Model Specifications 1 & 2

Dependent Variables	Independent Variables				
	CONSTANT	LIIP	LREER	LRBSMA	CR
LRM1	4.8979* (15.343) (0.000)	1.0045* (17.076) (0.0000)	-0.27858* (-3.1435) (0.0019)	0.0505 (1.4933) (0.1365)	-0.00068* (-4.3634) (0.0000)
	R^2 = 0.981 Adj. R^2 = 0.981 D.W. = 0.966 F = 3686.791 P = 0.00000				
LRM3	5.3836* (10.212) (0.0000)	1.4003* (13.2099) (0.0000)	-0.3579** (-2.3744) (0.0183)	-0.0152 (-0.2678) (0.7890)	-0.01040* (-4.4371) (0.00000)*
	R^2 = 0.973 Adj. R^2 = 0.973 D.W. = 0.7255 F = 2540.593 P = 0.00000				

(*, ** Denotes 5%, and 1% significant level respectively.)

Table – 1.4 Model Specifications 3 & 4

Dependent Variables	Independent Variables				
	CONSTANT	LIIP	LREER	LRBSMA	CI
LRM1	4.8731* (13.8633) (0.0000)	0.9871* (15.936) (0.0000)	-0.3039* (-3.3844) (0.0008)	0.07645** (2.3820) (0.0179)	-0.00787* (-3.2963) (0.0011)
	R^2 = 0.980 Adj. R^2 = 0.98 D.W. = 0.81490 F = 3549.066 P = 0.00000				
LRM3	5.3168* (9.7798) (0.0000)	1.3753* (12.554) (0.0000)	-0.3941* (-2.62378) (0.0092)	0.0218 (0.4028) (0.6874)	-0.001159* (-3.1217) (0.0020)
	R^2 = 0.972 Adj. R^2 = 0.972 D.W. = 0.5975 F = 2423.53 P = 0.00000				

(*, ** Denotes 5%, and 1% significant level respectively.)

Table – 1.5 Model Specifications 5 & 6

Dependent Variables	Independent Variables				
	CONSTANT	LIIP	LREER	LGP	CR
LRM1	4.505* (15.546) (0.0000)	1.0715* (25.140) (0.0000)	-0.1834* (-3.1203) (0.0020)	0.0174 (0.6537) (0.5138)	-0.0070* (-4.9436) (0.0000)
	R^2 = 0.980 Adj. R^2 = 0.980 D.W. = 1.015 F = 3536.006 P = 0.00000				
LRM3	5.1438* (8.4389) (0.0000)	1.3181* (14.7325) (0.0000)	-0.3320** (-2.5055) (0.0128)	0.0391 (0.6623) (0.5083)	-0.0106* (-4.3904) (0.0000)
	R^2 = 0.973 Adj. R^2 = 0.973 D.W. = 0.6718 F = 2559.860 P = 0.00000				

(*, ** Denotes 5%, and 1% significant level respectively.)

Table – 1.6 Model Specifications 7 & 8

Dependent Variables	Independent Variables				
	CONSTANT	LIIP	LREER	LGP	CI
LRM1	4.0198* (10.5789) (0.0000)	1.0453* (18.833) (0.0000)	-0.1264 (-1.5419) (0.1242)	0.0567 (1.5635) (0.1191)	-0.0082* (-3.1569) (0.0018)
	R^2 = 0.979 Adj. R^2 = 0.979 D.W. = 0.8373 F = 3346.993 P = 0.00000				
LRM3	4.4363* (7.4476) (0.0000)	1.2543* (13.8975) (0.0000)	-0.2617** (-1.9225) (0.0556)	0.1188** (2.0244) (0.0439)	-0.01576* (-2.7098) (0.0072)
	R^2 = 0.974 Adj. R^2 = 0.973 D.W. = 0.5527 F = 2577.901 P = 0.00000				

(***, **** Denotes 5%, and 1% significant level respectively.)

Table – 1.7 Unit Root Test at Level

Variables	ADF Test at Level		PP Test at Level		KPSS Test at Level	
	C	C & T	C	C & T	C	C & T
LRM1	-0.6271 (0.8610)	--3.8703* (0.0145)	-0.7687(26) (0.8258)	-4.3296(05) (0.0033)*	-1.9654(14)	0.14678(13)
LRM3	-0.4736 (0.8928)	-2.275 (0.4454)	-0.0537(03) (0.9513)	-2.41978(03) (0.3684)	1.9642(14)	0.1240(14)
LIIP	-0.9328 (0.7768)	-2.3063 (0.4286)	-0.2649(21) (0.9267)	-7.3635(07) (0.000)*	1.9317(14)	0.1164(12)
LRBSMA	-2.2789 (0.1796)	-2.8144 (0.1933)	-2.236(03) (0.1914)	-3.0115(04) (0.1309)	1.7861(14)	0.1167(13)
LREER	-2.1671 (0.2190)	-2.0067 (0.5947)	-2.1803(05) (0.2141)	-2.0423(05) (0.5751)	0.7584(14)	0.2228(14)
LGP	1.3356 (0.9988)	-0.73360 (0.9689)	1.2630(01) (0.9985)	-0.7715(02) (0.9659)	1.6150(14)	0.4626(14)
CI	-3.283** (0.0166)	-3.9049** (0.0700)	-2.8239(03) (0.0562)**	-2.8195(03) (0.1915)	0.3960(13)	0.3374(13)
CR	-3.5978* (0.0064)	-7.8699* (0.000)	-6.6743(07) (0.0000)*	-8.1655(07) (0.000)*	1.0520(13)	0.2083(12)

(***, **** Denotes 5%, and 1% significant level respectively.)

Table – 1.8 Unit Root Test at First Difference

Variables	ADF Test at First di?erence		PP Test at First di?erence		KPSS Test at First di?erence	
	C	C & T	C		C	C & T
LRM1	-13.0878* (0.0000)	-13.0712* (0.0000)	-28.219(21) (0.000)*	-28.316(21) (0.000)*	0.1363(27)*	0.11315(27)*
LRM3	-3.3837* (0.0029)	-3.7699* (0.0196)	-18.6032 (0.000) *	-18.5733(02) (0.000)*	0.08810(03)*	0.08546(03)*
LIIP	-3.6602* (0.0052)	-3.6739** (0.0258)	-34.165(29) (0.0001)*	-34.0634(29) (0.000)*	0.1203(25)*	0.11018(25)*
LRBSMA	-14.879* (0.000)	-14.935* (0.000)	-14.9073(02) (0.000)*	-14.9386(01) (0.000)*	0.1719(04)*	0.0599(03)*
LREER	-17.1567 (0.000)*	-17.176* (0.000)	-17.1583(04) (0.000)*	-17.1801(04) (0.000)*	0.1253(04)*	0.038004(03) *
LGP	-15.351* (0.000)	-15.5113* (0.000)	-15.335(02) (0.000)*	-15.5132(01) (0.0000)*	0.5016(03)**	0.10147(01)*
CI	-8.0488* (0.0000)	-8.0887* (0.0000)	-11.888(07) (0.0000)	-11.863(07) (0.0000)	0.0563(0)*	0.0423(0)*
CR	-15.3488* (0.000)	-15.322* (0.000)	-66.078(217) (0.0001)*	-115.133(250) (0.0001)*	0.48943(272)*	0.4570(252)

(***, **** Denotes 5%, and 1% significant level respectively.)

Table – 1.9 Johansen – JuseliusCointergation Test for Model 1 & 2

	Hypothesis: Trace Stat.	Hypothesis Max Stat.	Panel A (with Constant)				Panel B (with Trend)			
			λ Trace Stat.	P Value	λ Max Stat.	P Value	λ Trace Stat.	P Value	λ Max Stat.	P Value
LMR1, LIIP, LRBSMA, CR, LREER	H_0: r=0, H_1 ; r >0	H_0: r=0, H_1 ; r =1	148.9*	0.000	73.011*	0.000	163.04*	0.000	74.44*	0.000
	- $_0$: r=1, H_1 ; r >1	H_0: r=1, H_1 ; r =2	75.588*	0.000	52.9*	0.000	89.60*	0.0001	53.688*	0.000
	H_0: r=2, H_1 ; r >2	H_0: r=2, H_1 ; r =3	22.59	0.2665	15.153	0.2783	35.913	0.2094	20.650	0.2080
LRM3, LIIP, LRBSMA, CR, LREER	H_0: r=0, H_1 ; r >0	H_0: r=0, H_1 ; r =1	147.87*	0.000	71.812*	0.000	156.53*	0.000	72.959*	0.000
	H_0: r=1, H_1 ; r >1	H_0: r=1, H_1 ; r =2	76.06*	0.000	54.970	0.000	83.578*	0.0005	55.23*	0.000
	H_0: r=2, H_1 ; r >2	H_0: r=2, H_1 ; r =3	21.091	0.3520	12.544	0.4949	28.347	0.6012	12.65	0.8293

(***, **** Denotes 5%, and 1% significant level respectively.)

Table – 1.10 Johansen – JuseliusCointergation Test for Model 3 & 4

Coint. Equation	Hypothesis: Trace Stat.	Hypothesis Max Stat.	Panel A (with Constant)				Panel B (with Trend)			
			λ Trace Stat.	P Value	λ Max Stat.	P Value	λ Trace Stat.	P Value	λ Max Stat.	P Value
LMR1, LIIP,	H_0: r=0, H_1 ; r >0	H_0: r=0, H_1 ; r =1	102.47*	0.000	53.789*	0.0001	117.54*	0.0001	54.749*	0.0003
LRBSMA, CI,	H_0: r=1, H_1 ; r >1	H_0: r=1, H_1 ; r =2	48.685**	0.0417	30.467**	0.0207	62.797**	0.0614	30.709**	0.0735
LREER	H_0: r=2, H_1 ; r >2	H_0: r=2, H_1 ; r =3	18.218	0.5500	11.454	0.6020	32.08	0.3837	18.693	0.3263
LRM3, LIIP,	H_0: r=0, H_1 ; r >0	H_0: r=0, H_1 ; r =1	102.567*	0.000	56.395*	0.000	111.69*	0.0004	57.328*	0.000
LRBSMA, CI,	H_0: r=1, H_1 ; r >1	H_0: r=1, H_1 ; r =2	46.170**	0.0714	28.417**	0.0390	54.367	0.2425	28.779	0.1212
LREER	H_0: r=2, H_1 ; r >2	H_0: r=2, H_1 ; r =3	17.752	0.5844	10.836	0.6637	25.58	0.7595	11.099	0.9217

(***, **** Denotes 5%, and 1% significant level respectively.)

Table – 1.11 Johansen – JuseliusCointergation Test for Model 5 & 6

Coint. Equation	Hypothesis: Trace Stat.	Hypothesis Max Stat.	Panel A (with Constant)				Panel B (with Trend)			
			λ Trace Stat.	P Value	λ Max Stat.	P Value	λ Trace Stat.	P Value	λ Max Stat.	P Value
LMR1, LIIP,	H_0: r=0, H_1 ; r >0	H_0: r=0, H_1 ; r =1	130.75*	0.000	76.63*	0.000	145.79*	0.000	76.99*	0.000
LGP, CR,	H_0: r=1, H_1 ; r >1	H_0: r=1, H_1 ; r =2	54.12**	0.0115	42.00*	0.0004	68.800*	0.0182	42.66*	0.0018
LREER	H_0: r=2, H_1 ; r >2	H_0: r=2, H_1 ; r =3	12.117	0.9277	7.084	0.9505	26.133	0.7301	16.787	0.4758
LRM3, LIIP,	H_0: r=0, H_1 ; r >0	H_0: r=0, H_1 ; r =1	120.27*	0.000	74.388*	0.000	136.28*	0.000	75.37*	0.000
LGP, CR,	H_0: r=1, H_1 ; r >1	H_0: r=1, H_1 ; r =2	45.88**	0.0757	32.68**	0.0101	60.91**	0.0866	33.49**	0.0337
LREER	H_0: r=2, H_1 ; r >2	H_0: r=2, H_1 ; r =3	13.20	0.8822	7.8722	0.9113	27.41	0.6569	15.30	0.6068

(***, **** Denotes 5%, and 1% significant level respectively.)

Table – 1.12 Johansen – JuseliusCointergation Test for Model 7 & 8

Coint. Equation	Hypothesis: Trace Stat.	Hypothesis Max Stat.	Panel A (with Constant)				Panel B (with Trend)			
			λ Trace Stat.	P Value	λ Max Stat.	P Value	λ Trace Stat.	P Value	λ Max Stat.	P Value
LMR1, LIIP,	H_0: r=0, H_1 ; r >0	H_0: r=0, H_1 ; r =1	81.6877*	0.0042	48.24*	0.000	103.019*	0.0032	48.93*	0.0022
LGP, CI,	H_0: r=1, H_1 ; r >1	H_0: r=1, H_1 ; r =2	33.44	0.5323	21.065	0.2723	54.088	0.2519	30.04**	0.0876
LREER	H_0: r=2, H_1 ; r >2	H_0: r=2, H_1 ; r =3	12.37	0.9179	7.3080	0.9408	24.04	0.8351	14.345	0.6926
LRM3, LIIP,	H_0: r=0, H_1 ; r >0	H_0: r=0, H_1 ; r =1	75.234*	0.0173	41.526*	0.0051	94.974**	0.0167	46.73*	0.0044
LGP, CI,	H_0: r=1, H_1 ; r >1	H_0: r=1, H_1 ; r =2	33.707	0.5179	20.551	0.3042	48.24	0.4943	21.69	0.5174
LREER	H_0: r=2, H_1 ; r >2	H_0: r=2, H_1 ; r =3	13.156	0.8844	7.7700	0.9172	26.549	0.7069	14.626	0.6678

(***, **** Denotes 5%, and 1% significant level respectively.)

8. SELECTED REFERENCES

[1]. **Arestis, Philip** (1988a), 'The Demand for Money in Small Developing Economies: An Application of the Error Correction Mechanism', in Contemporary Issues in *Money and Banking: Essays in Honor of Stephen Frowen*, ed. by Philip Arestis, Macmillan, 1988.

[2]. **Afexntion, C. Panayotis** (1977), 'Money Supply and the Demand for Money: Some Issues', *The Indian Economic Journal*, Vol.25, Jan-March 1977-78, pp. 144.

[3]. **Bhattacharya, B. B.** (1974), 'Demand and Supply of Money in a Developing Economy: A Structural Analysis for India', *Review of Economics and Statistics*, Vol.56, No.-4, Nov. 1974, pp. 502.

[4]. **Dickey D. A. and Wayne A. Fuller** (1979), 'Distribution of the Estimators for Autoregressive Time Series With and a Unit Root', *Journal of the American Statistical Association*, Vol. 74, No. 366, June, pp. 427-431.

[5]. **Fase, M.M.G., and C.C. A. Winder** (1994), 'Money Demand within EMU: An Analysis with the Divisia Measure' *Quarterly Bulletin*, De Nederlandsche Bank, Sept. 1994, pp. 25-55.

[6]. **Friedman, M.**(1987), 'Quantity Theory of Money', *The New Palgrave: A Dictionary of Economics*, Vol.4, p.15.

[7]. **Inoue, Takeshi, and Hamori Shigeyuki** (2008), 'An Empirical analysis of the money demand function in India', *IDE Discussion Papers 166*, Institute of Developing Economies, Japan External Trade Organization (JETRO).

[8]. **Johansen, S.** (1988), 'Statistical Analysis of Cointegration Vectors', *Journal of Economic Dynamics and Control*, No. 12, pp. 231-254.

[9]. **Johansen, S. and Katarina Juselius** (1990), 'Maximum likelihood estimation and inference on Cointegration- with applications to the demand for money', *Qxford Bulletin of Economics and Statistics*, Vol.52, No.2, pp. 169-210.

[10]. **Knell, M. and Stix, H.** (2006), 'The Income Elasticity of Money Demand: A Meta-Analysis of Empirical Results', *Journal of Economic Surveys*, 19: 513–533.

[11]. **Kwiatkowski, D.**, **Phillips, P.C.B., Schmidt, P., and Shin, Y.** (1992), 'Testing the null hypothesis of stationarity against the alternative of a unit root', *Journal of Econometrics*, Vol. 54, No.1, pp. 159-178.

[12]. **Moosa, A. Imad**(1992), 'The Demand for Money in India: A Cointegration Approach', *The Indian Economic Journal*, Vol.40, July-Sept. 1992, pp. 101.

[13]. **Phillips, P.C.B. and Perron, P.** (1988), 'Testing for a Unit Root in Time Series Regression', *Biometrika*, Vol. 75, No. 2, pp. 335-346.

[14]. **Rangarajan, C. and Anoop Singh** (1984), 'Reserve Money: Concepts and Policy Implications for India', *Reserve Bank of India Occasional Papers,* June.

[15]. **Sampath, R. K. and ZakirHussain**(1981), 'Demand for Money in India', *The Indian Economic Journal,* Vol.29, No. 1, July-Sept. 1981, pp. 17.

[16]. **Sriram, S. S.** (1999), 'Survey of Literature on Demand for Money: Theoretical and Empirical Work with Special Reference to Error- Correction Model', *IMF Working Paper*, WP/99/64, 1999.

[17]. **Trivedi, M. S**. (1980), 'Inflationary Expectations and Demand for Money in India (1951-1975)', *The Indian Economic Journal,* Vol.28, July-Sept. 1980, pp. 62.

[18]. **Trivedi, M. S**. (1983), 'Inflationary Expectations and Demand for Money Per Capita Real Balances: Evidence from the Indian Economy', *ArthaVignana,* Vol.25, No. 1, 1983, pp. 21.

[19]. **Trivedi, M. S**. (1984), 'Price Expectations, Price Uncertainty and Demand for Money in India: Some Evidence', From *Liberalism and LDCS*, 1984 pp. 226.

RESEARCH ARTICLE

SOME SPECIALIZED TESTS OF SIGNIFICANE FOR MONTHLY MONEY DEMAND IN INDIA

Vijay S. Jariwala[1]

ABSTRACT

A very large number of empirical studies have been conducted in many countries to evaluate the determinants and stability of the demand for money.The analysis of this paper deals with error correction, causality and stability issues in money demand estimation. In this study various money supply aggregates for monthly time series from April, 1990-91 to July, 2013-14 along with output, incomeand interest rates as money demand function variables have been experimented. Bidirectional and unidirectional Granger Causalities have been found among few of the variables.It is inferred that money demand systems are unstable and need to have error corrections to restore equilibrium therein.The demand for real cash balance is positively and significantly related to the movement of real stock price.The ineffectiveness of the interest rate shows the lack of response mechanism through the changes in the interest rate.Long term asset's market has emerged as a possible channel of monetary transmission which supports the hypothesis that money is required for investment purposes.

Keywords: *Money Demand, Error Correction Mechanism, Granger Causality*

1. INTRODUCTION

In most countries, price stability has become the primary objective of monetary policy. In 2006, US Fed chairman Ben S. Bernanke said: "Price stability plays a dual role in modern central banking: It is both an end and a means of monetary policy. Central bankers, economists, and other knowledgeable observers around the world agree that price stability both contribute importantly to the economy's growth and employment prospects in the longer term and moderate the variability of output and employment in the short to medium term."

[1] Assistant Professor, Department of Economics, M K Bhavnagar University, Bhavnagar-364002
Email- viju_19782004@yahoo.com
(This paper was presented at 51st Annual Conference of The Indian Econometric Society (TIES) at Patiala University, Punjab during 12-14 December, 2014.) recd. April 16/ frvd. May 16/ srvd. June 16.

The demand for money[1] is at the heart of how policy should be conducted effectively. In most developed and developing countries, policymakers have frequently questioned whether the money demand is stable over a period of time.The demand for money has been an important part of economics from the origin of the subject. Though, very little attention was paid to demand for money before 1920s. This apparent lack of attention has appeared to have particularly changed since the Great Depression of early 1930's and with the publication of John Maynard Keynes, in 1936, of The General Theory. These events have fascinated special attention in monetary theory consequently and equally have been focused on the demand for money.

It raises issues such as "What are the roles of monetary policy in causing an economic boom or recession?" "What is the role of money in society?" "How is a monetary policy transmitted to the real sector?", "Can money be used as a tool to stimulate development and growth, especially in the developing world?" "Who holds the money? Why is it held?" etc. The debate usually centres on whether easy or tight monetary policies are preferable. In other words, should money and credit be plentiful and inexpensive or scarce and expensive?

These issues call for an appropriate analysis of the use of money and the functioning of monetary policy instruments. This is more so for developing countries where the problems of growth and development appear to be of a particular nature and are compounded by structural bottlenecks and rigidities, movement towards flexible exchange rate regime, globalisation of the capital markets, financial market liberalisation and innovation. The problem in the developing countries is how more or less quantity of money can be used to stimulate economic growth and development.

In this analysis, attempt has been made to formulate demand for money function for various specifications in error correction mechanism, causality and stability issues in India.

2. RECENT THEORETICAL DEVELOPMENTS IN THE LITERATURE

Is the demand for money irrelevant in the process of monetary policy procedures?[2] To provide important insights on this issue, we review some recent developments on the demand for money. In a classic study, Poole (1970) showed that under certaintyequivalent outcomes of optimal monetary policy, either the rate of interest or monetary aggregates is identical (see Duca and VanHoose, 2004, p. 249). He argued that the selection of the monetary policy instruments should depend on the stability of money demand. Instability in the money demand function could be an indication of an interest rate based policy. It seems that, Pooles' analysis is important because it identifies the importance of understanding money demand for the design and formulation of optimal monetary policy. Friedman (1975) opposed the inferences made by Poole (1970) and instead argued that monetary aggregates has no direct role in the process of monetary policy. He asserted that monetary aggregates are observed to serve better as intermediate targets. Friedman also considered other macroeconomic variables as potential instruments for monetary policy, among others is the indicator indexes suggested by Brunner and Meltzer (1967).[3]

Subsequently, Goodfriend (1987) postulated that if central banks treat interest rate volatility as their prime objective, then expectations of induced future price level adjustments can result in price changes simultaneously, thereby generating inherent nonstationarities in the price level (Duca and VanHoose, 2004, p. 250). Goodfriend's interpretations seem to imply that monetary aggregates are useful for monetary policy in achieving inflation stability. Later, VanHoose (1989) criticised this interpretation. Hepointed out that monetary aggregates are endogenous and results emerges as non-stationarity price levels if money supply is targeted. To this end, economies that aim for inflation stability through the monetary aggregates will find it difficult to achieve their targets.

Dotsey and Hornstein (2006) include monetary aggregates to the list of potential indicator variables (see Duca and VanHoose (2004, p. 251). They argued that the stability of money demand is a useful indicator of productivity shocks that are not directly observable to the central banks. Meltzer (2001) analysed the effectiveness of monetary policy transmission mechanism under interest rate based policy.He claims that in interest rate models there is limited recognition of the relevance of real balance effects on aggregate expenditures. He further argues that effects of real monetary base growth on consumption arise independently of expenditure inducing effects of changes in central banks' nominal interest rate instruments (for more details, see Duca and VanHoose, 2004, p. 251).

Moosa (1992) was the first to study the stationarity and the cointegration relationship among the variables of the money demand function in India. He has utilized three types of money supply- cash balances, M1 and M2 to analyze cointegration tests on real money balances, short-term interest rates and industrial production for the first quarter of 1972 to the last quarter of 1990. He found the cointegration relationship among all the money supply aggregates with output and interest rates. Moosa (1992) suggested on the basis of his study that narrow definition of money supply is better for pursuing monetary policy.

Another study of Bhattacharya (1995) used three aggregates of money supply- M1, M2 and M3 for the annual period of 1950 to 1980 along with real GNP, long term and short-term interest rates and performed cointegration tests. He found a cointegrating relationship among the variables only when M1 is utilized and concluded that the long-term interest rates are more sensitive to money demand. He has also estimated an error correction model (ECM) and summarized that the error correction term was significant and negative when M1 was used and held that monetary policy is stable over the long term when money supply is narrowly defined.

Thus, the inferences drawn from these studies appear to be reasonable under current circumstances, however, we argue that the stability of money demand is not a one-shot investigation but rather entails a continued assessment. To this end, a continued research on the empirical front is not irrelevant. Therefore, money demand stability requires a re-investigation using updated data.

3. DATA AND RESEARCH METHODOLOGY

This study comprises of Monthly data over the period of April 1990 to July 2013 a data set of 280 observations for various variables. The data source for the Industrial Production Index (IIP) and the Whole sale Price Index (WPI) as well as for Consumer Price Index (CPI) has been from Economic Survey 2012-13. While the money supply aggregates are collected from various issues of the RBI Bulletin and deflated by WPI. Apart from it, the weighted average of call money rates is taken as the interest rate and considered from various issues of RBI Bulletin. As due to the unavailability of appropriate Income measure for monthly data, various scale variables have been experimented. However, IIP is not the correct measure for Income; it has been considered as the scale variable in the money demand equation. Moreover, a host of other variables like market capitalization measured in BSE-Stock Market deflated with WPI, Average annual price of 10g. Gold price deflated with WPI to capture the wealth effect of the money demand as being the store of value, Gold is generally hedge against the inflation. Therefore, to capture the effects, we also estimate Real Average Gold prices in the money demand equation by deflating Gold price with Average WPI.

To formulate demand for money function, the traditional quantity theory of money (QTM) demand expressed as MV = Py[4] has been considered. As suggested by Friedman (1987) including various determinants of money demand, a simple money demand function can be formulated as $(M/P)^d = f(y, r, E, S)$, where M is money stock, P is general price level, y is real income, r is interest rate, E is exchange rate, S is stock price. The exchange rate and stocks price are included as additional determinants of demand for money. The rationality is that foreign exchange and stocks constitutes a part of portfolio of economic agents. Depreciation in exchange rate may result in further depreciation of the currency, which will force individuals to hold money as foreign currency to avoid possible losses. Net effect of stock price could be either positive or negative. Thus, the demand for real money balances as a function of real income, interest rate, real stock market capitalization, real exchange rate can be specified as[5]

$(M/P)^{d}_{t} = \beta_0 + \beta_1(Y/p)_t + \beta_2 r_t + \beta_3 E_t + \beta_4 S_t + \varepsilon_t$ (1)

where, M is nominal money supply at time t, P is the price level (WPI), Y is nominal income, r_t is the opportunity cost variable , such as interest rates and Et is the real exchange rate, St is the real stock price at time t. In the equation, $(M/P)^{d}_{t}$ represents the real money balance and (Y/p)t is the real income at time t. Theoretically, demand for money is directly related with real income and indirectly with interest rates. However, the sign of the exchange rate andstock price is uncertain. Therefore, by convention, the values of the coefficient of income (β_1) should be positive and interest rate (β_2) is negative but for exchange rate (β_3) and stock market capitalization (β_4) it could be either negative or positive. It calls for empirical estimation of money demand functions and test of stability.The equation 1 can be expressed as a vector of variables, where each variable might be dynamically interrelated. Their long run and short-run equilibrium relationship can be examined using cointegration and error correction model respectively. The results of unit root and cointegration are not produced here for the sake of saving space.

A simple Error Correction Model (ECM) has been proved to be a successful tool in applied money demand research [Sriram, (1999)]. It is a dynamic-error-correction model where the long-run equilibrium is embedded in an equation that captures short-run variation and dynamics [see Kole and Meade, (1995)].Granger (1986) had shown that the concept of stable long-term equilibrium is the statistical equivalence of cointegration and implies the presence of dynamic error-correction term.

Once the series are cointegrated they follow equilibrium pattern in the long run. However, in the short-run they may depart from each other resulting in dis-equilibrium. This can be explained through corresponding error correction model by including stationary residuals from the cointegrating vectors and include its one period lagged values (ECt-1) in an error correction model.

The ECM can be specified as,

$$\Delta M_t = {}_0 + \sum_{j-0}^{n} {}_{1j}\Delta M_{t-j} + \sum_{J=0}^{n} {}_{2j}\Delta y_{t-1} + \sum_{j=0}^{n} {}_{3j} + \sum_{j=0}^{n} {}_{4j}\Delta E_{t-j} + \sum_{j=0}^{n} {}_{5j}\Delta S_{t-j} + {}^{a}EC_{t-1} + U_t \quad (2)$$

where, λ is the coefficient of error correction term. It denotes the speed of convergence towards equilibrium and provides the direction of equilibrium. The expected sign of the coefficient is negative. It means if the model is out of equilibrium, then demand for real money balance come forward from below to restore the equilibrium in the next period. If λ is not statistically significant, implying that the coefficient is equivalent to zero, hence the dependent variable adjusts to the changes in independent variables in the same period during short run.

Granger causality test has been applied to evaluate the temporal causality. Granger causality test says that if the variables are cointegrated then there exists necessarily a causal relationship among them at least in one direction. The causality can be tested using F statistics. Under the null hypothesis of no causality (e.g. from Y to X), if calculated F statistics is greater than critical F statistics with appropriate degrees of freedom and significance level, then reject the null hypothesis against alternative hypothesis.

The stability of demand for money function has been examined by using various recursive residual tests. We have applied CUSUM and CUSUMQ tests, proposed by Brown et. al. (1975) to test the stability of the long run and short-run coefficients. If the plot of CUSUM or CUSUMQ stays within the 5% significance level, then the coefficients estimates are said to be stable.

4.1 ERROR CORRECTION MECHANISM

If the variables are cointegrated, it need not necessarily mean that in the short-run they are always in equilibrium. This departure from the equilibrium relationship in the short-run is explained through error correction term. The error correction term is obtained from the residuals terms of cointegrating equations and plugged into the cointegrating equation with lagged term in first difference. The specified error correction model 2is estimated using OLS methods. The results are reported in table 1.3.

All the coefficients of ECM are found negative and six model specifications are found with significant error terms. All significant error terms are found for both LRM1 and LRM3 indicates there is a short run feedback to money demand function with respect to various specified equations used. And the said equations are highly unstable in the specified model equation. In the first specification it took 554% adjustment to have equilibrium in the money demand function. So, as with specification no.03, it took 556.5 % , specifications no.4 it took 173.76% , for specification no. 5 it took 445.% , specification no.07 it took 453.77% and for specification no. 08 it took 175.35% adjustments in the short run. Thus these tests confirmed the error correction mechanism in monthly data for the whole period.

4.2 GRANGER CAUSALITY TESTS

The bivariate Granger causality test is applied for testing causality. According to Engel-Granger (1987), if the variables are cointegrated, then they are necessarily causally related at least in one direction. Granger causality applied for stationary series only, so we have estimated this for variables with first difference. The bivariate Granger causality tests results are reported in table 1.4.

So far as the whole data series is concerned, the null hypothesis of LRM1 does not Granger cause monetary aggregates has been rejected for CR at various significance level, as reported by F statistics and corresponding P values. It implies that real IIP Granger causes stock market capitalisation but not the reverse . Unidirectional causality found from CR to LRBSMA, LGP to LRBSMA, LRM1 to LGP , LREER to LRBSMA and LRM1 to LRBSMA. Bi-directional relationships exist for LREER to CR and LRBSMA to LIIP which again in confirmation to earlier studies. Moreover, bi-directional causality has been detected for LRM1 to LIIP, LRM3 to LIIP. No causal relationship has been noticed in either direction between exchange rate and real money balances. The result is consistent with regression result, also justified as per the magnitude and sign of the coefficients are concerned.

4.3 STABILITY TESTS

Once variables are cointegrated and causal relationship is established, then stability of the demand for money can be tested applying CUSUM and CUSUMQ tests. From cointegrating equation we can obtain residuals. Considering the coefficients of residual with one period lagged term, estimated an error correction model with the appropriate lagged term of six, for whole data set and then apply both CUSUM and CUSUMQ test on the residual of error correction term. The equation 1 specifies ECM and can be estimated by OLS method. Then apply the CUSUM and CUSUMQ tests on the residual. If graphical plot of the CUSUM and CUSUMQ stays within 5% significance level, then coefficient estimators are said to be stable. The various CUSUM and CUSUMQ plots are given in the Graphs 2.1 to 2.8 for model specifications of 01 to 08 as explained in the table 1.2. From the estimated graphs, it is revealed that most of the model specifications are unstable as evident from the error term significantly observed in the equations. It is further inferred that monthly money demand systems are unstable and needs to have error corrections to restore equilibrium therein.

5. CONCLUDING REMARKS

The main focus of this exercise is to re-evaluate the performance of the money demand specifications in the context of identifying suitable determinants and to test stability of demand for money function. The major concern of this effort is to find out the appropriate channel of monetary transmission mechanism or the routes of the effects of monetary policy operations. The major conclusions that emerge from this analysis and empirical study are as follows.

The income elasticity of the demand for money is positive, statistically significant and in most of the cases greater than unity. This result clearly suggests that there are no economies of scale in holding money and proportionate change in the income requires more than a proportionate change in money to sustain it. Again, the result suggests that money is considered to be a luxury asset in India. This result may be ascribed to the fact that people are willing to forego the premium assets in illiquid form. Moreover, the large value of income indicates that transaction demand for money is important. The statistical insignificance of the interest rate variable implies that the interest rate cannot be a suitable variable for monetary management in India. This implies that interest rate fails to become an appropriate proxy for opportunity cost variable in India. This suggests the importance of physical assets in people's portfolio in the country like India.

The insignificant influence of rate interest on demand for money in India may be attributed to the less developed money and capital markets and due to lack ofrepresentative interest rate. Moreover, the absence of speculative demand for money (which is mainly a function of rate of interest) also seems to be responsible for the insignificant effect of the rate of interest on the demand for money in India.

The demand for real cash balances is positively and significantly related to the movement of real stock prices. The positive relation appears to reflect a wealth and / or transaction effect. Further, in India context, the transaction effect cannot be ruled out and difficult to be separated from the wealth effect. The result also suggests that the demand side has been influenced more directly with the development in the capital market having pronounced bearing on the money market.

The effects of exchange rate on the level of real money balances show different results from earlier studies. It confirms negative effects (absorption reducing effect). This suggests either of the following reasons: (i) the absorption effect is only a temporary short run phenomenon, which peters out in the long run and even on an annual basis, or (ii) the absorption reducing effect is only a recent phenomenon, which may even continue in future.

From the above conclusions, the following implications may be drawn for the effective operation of monetary policy in India. The ineffectiveness of the interest rate shows the lack of response mechanism through the changes in the interest rate. Under such a situation, monetary authorities should go for a complete deregulation of interest rate. The results also show that, the exchange rate is gradually emerging as an alternative route through which monetary impulses could be transmitted to the economy. The asset prices of the capital market provide sufficient clues for becoming the channels of transmission to monetary policy operations. The market capitalisation used as an explanatory variable turned out to be positive significant. This suggests that long term asset's market has emerged as a possible channel of monetary transmission which supports the hypothesis that money is required for investment purposes.

6. ACKNOWLEDGEMENT

This research endeavour is based on my Ph.D. thesis entitled Demand for Money in India- An Econometric Analysis. I would like to thank (anonymous) referees for their insight and comments. It has helped me to improve and reshape this manuscript in a meaningful way.

7. APPENDIX

Monthly Data Analysis from April, 1990-91 to July, 2013-14

TABLE 1.1 - Description of Variables

Variables	Definitions
RM1	Real Narrow Money- Real Narrow money is comprised of currency with the public, demand deposits with banks and other deposits with RBI
RM3	Real Broad Money – which includes M1 and time deposits with banks
IIP	Index number of Industrial Production
CR	Weighted Average of Call/Notice Money Rates
CI	Average Monthly Inflation Rate based on Consumer Price Index
RBSEMCAP	Real market capitalisation (BSE- Stock Market)
REER	Real effective exchange rate (36 country trade weighted)
GP	Average monthly price of 10g Gold at Mumbai Bullion Market
LRM1	Log value of Real Narrow Money
LRM3	Log value of Real Broad Money
LRBSMA	Log value of Real market capitalisation (BSE-Stock Market)
LREER	Log value of Real effective exchange rate (36 country trade weighted)
LGP	Log value of Average monthly price of 10g Gold at Mumbai Bullion Market

Table – 1.2 Money Demand Specifications

Sr. No.	Dependent Variable	Independent Variables
1.	LRM1	LIIP, LREER, LRBSMA, CR
2.	LRM3	LIIP, LREER, LRBSMA, CR
3.	LRM1	LIIP, LREER, LRBSMA, CI
4.	LRM3	LIIP, LREER, LRBSMA, CI
5.	LRM1	LIIP, LREER, LGP, CR
6.	LRM3	LIIP, LREER, LGP, CR
7.	LRM1	LIIP, LREER, LGP, CI
8.	LRM3	LIIP, LREER, LGP, CI

Table – 1.3Estimation of Error Correction Models

ECM Equations	Coefficient of ECM (-1)	T- Statistics	P- values
LRM1, LIIP, LREER, LRBSMA, CR	-0.3294	-5.5405	0.000*
LRM3, LIIP, LREER, LRBSMA, CR	0.0325	-1.5733	0.0116**
LRM1, LIIP, LREER, LRBSMA, CI	-0.3332	-5.565	0.000*
LRM3, LIIP, LREER, LRBSMA, CI	0.0343	-1.7376	0.0434**
LRM1, LIIP, LREER, LGP, CR	-0.2389	-4.453	0.000*
LRM3, LIIP, LREER, LGP, CR	0.0297	-1.7112	0.0482**
LRM1, LIIP, LREER, LGP, CI	-0.2421	-4.5377	0.000*
LRM3, LIIP, LREER, LGP, CI	0.030008	-1.75345	0.0407**

(***, ****Denotes 5% and 1% significant level respectively.)

Table – 1.4Granger Causality Test

Hypothesis for e.g. H_0 ; Y does not Granger cause X. ; H_1 : Y Granger cause X, Lag = 02:

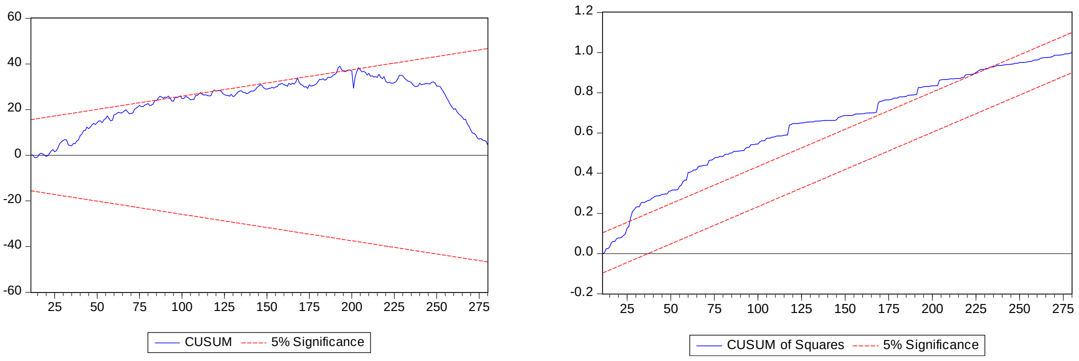
Direction of Causality (Y–>X)	Direction of Causality (Y–>X)	Direction of Causality (Y–>X)	Direction of Causality (Y–>X)	Direction of Causality (Y–>X)	Direction of Causality (Y–>X)	Direction of Causality (Y–>X)	Direction of Causality (Y–>X)
CI–>CR	4.53600**	0.0341	Y	LRBSMA–>LGP	3.568**	0.0295	Y
CR–>CI	0.06140	0.8045	N	LGP–>LRBSMA	0.6470	0.5244	N
LGP–>CR	1.8292	0.1625	N	LREER–>LGP	0.5812	0.5599	N
CR–>LGP	0.7503	0.4732	N	LGP–>LREER	0.6174	0.5401	N
LIIP–>CR	6.0919*	0.0028	Y	LRM1–>LGP	3.0416*	0.0002	Y
CR–>LIIP	2.6440	0.0729	Y	LGP–>LRM1	0.1618	0.8506	N
LRBSMA–>CR	5.2669*	0.0057	Y	LRM3–>LGP	3.2836**	0.0390	Y
CR–>LRBSMA	0.5831	0.5588	N	LGP–>LRM3	1.0371	0.3559	N
LREER–>CR	9.4329*	0.0001	N	LRBSMA–>LIIP	9.06103*	0.0002	Y
CR–>LREER	3.7998*	0.0236	N	LIIP–>LRBSMA	2.5536	0.0797	Y
LRM1–>CR	8.8820*	0.0002	N	LREER–>LIIP	0.5068	0.6030	N
CR–>LRM1	0.4263	0.6533	N	LIIP–>LREER	0.1545	0.8569	N
LRM3–>CR	13.2880*	0.0000	Y	LRM1–>LIIP	14.2894*	0.0000	Y
CR–>LRM3	0.2662	0.7665	N	LIIP–>LRM1	3.6749**	0.0266	Y
LGP–>CI	0.67191	0.4131	N	LRM3–>LIIP	7.2607*	0.0008	Y
CI–>LGP	0.4118	0.5216	N	LIIP–>LRM3	7.7989*	0.0005	Y
LIIP–>CI	0.00251	0.9601	N	LREER–>LRBSMA	4.428**	0.0128	Y
CI–>LIIP	1.7107	0.1920	N	LRBSMA–>LREER	0.6697	0.5126	N
LRBSMA–>CI	0.18029	0.6715	N	LRM1–>LRBSMA	3.2103**	0.0419	Y
CI–>LRBSMA	0.5866	0.5569	N	LRBSMA–>LRM1	1.3720	0.2553	N
LREER–>CI	0.16645	0.6836	N	LRM3–>LRBSMA	3.0246	0.0502	Y
CI–>LREER	5.995**	0.0150	Y	LRBSMA–>LRM3	0.13014	0.8780	N
LRM1–>CI	0.1983	0.8202	N	LRM1–>LREER	0.18233	0.8333	N
CI–>LRM1	0.16485	0.3727	N	LREER–>LRM1	1.114	0.3297	N
LRM3–>CI	1.9E-07	0.9997	N	LRM3–>LREER	0.2202	0.8025	N
CI–>LRM3	0.37573	0.5404	N	LREER–>LRM3	1.3557	0.2595	N
LIIP–>LGP	3.1725**	0.0416	Y	LRM3–>LRM1	5.04315*	0.0071	Y
LGP–>LIIP	0.2243	0.7992	N	LRM1–>LRM3	0.00495	0.9951	N

(***, **** Denotes 5% and 1% significant level respectively.)

Cusum and Cusum Square test

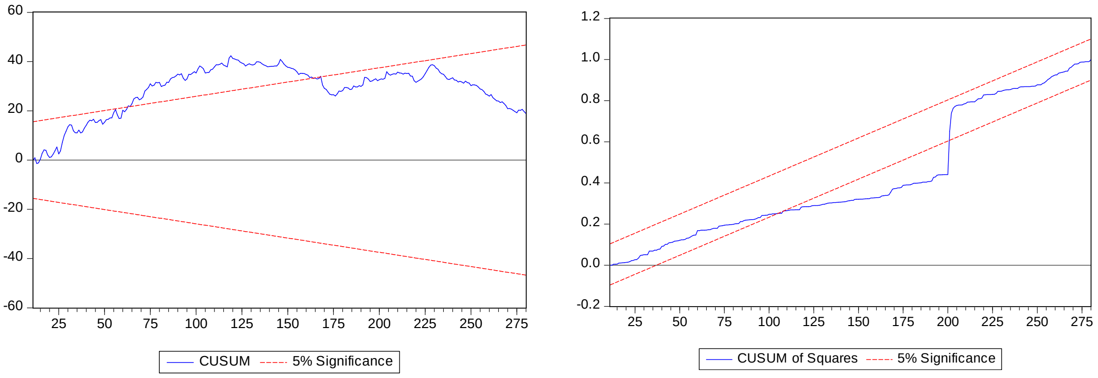
GRAPH 2.1

1. MODEL SPECIFICATION: LRM1, LIIP, LREER, LRBSMA, CR



GRAPH 2.2

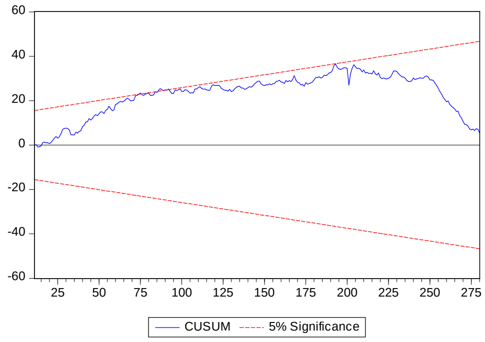
2. MODEL SPECIFICATION: LRM3, LIIP, LREER, LRBSMA, CR

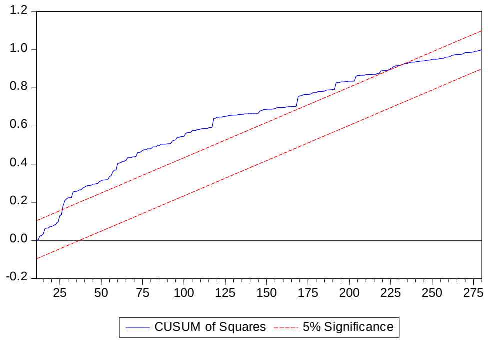


The straight line represents the critical bound at 5% significance level. Figures show CUSUM and CUSUMQ Test: Plot of cumulative sum of square of recursive residual

GRAPH 2.3

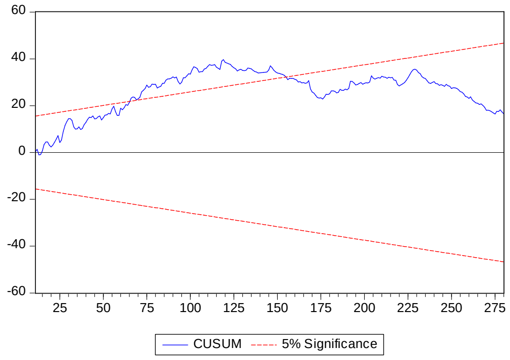
3. MODEL SPECIFICATION: LRM1, LIIP, LREER, LRBSMA, CI

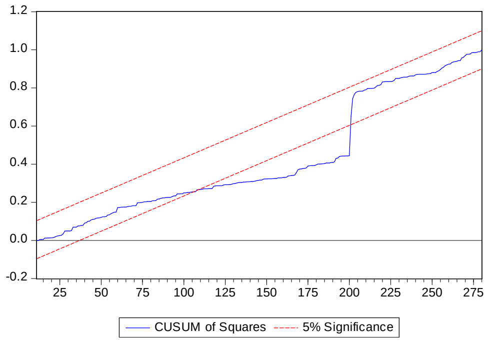




GRAPH 2.4

4. MODEL SPECIFICATION: LRM3, LIIP, LREER, LRBSMA, CI

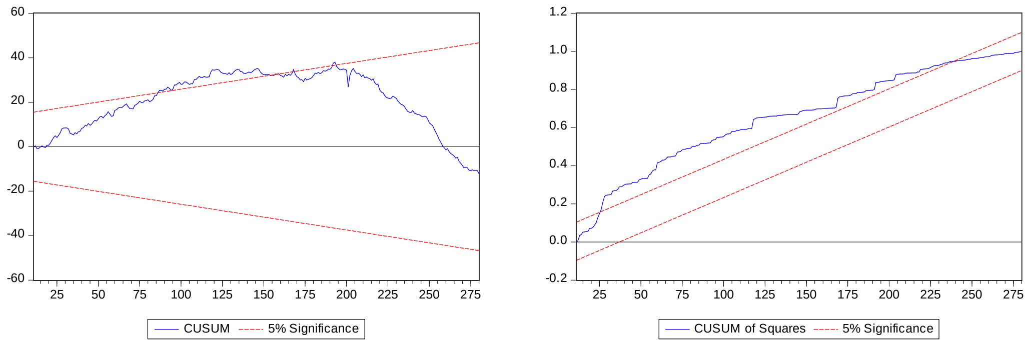




The straight line represents the critical bound at 5% significance level. Figures show CUSUM and CUSUMQ Test: Plot of cumulative sum of square of recursive residual

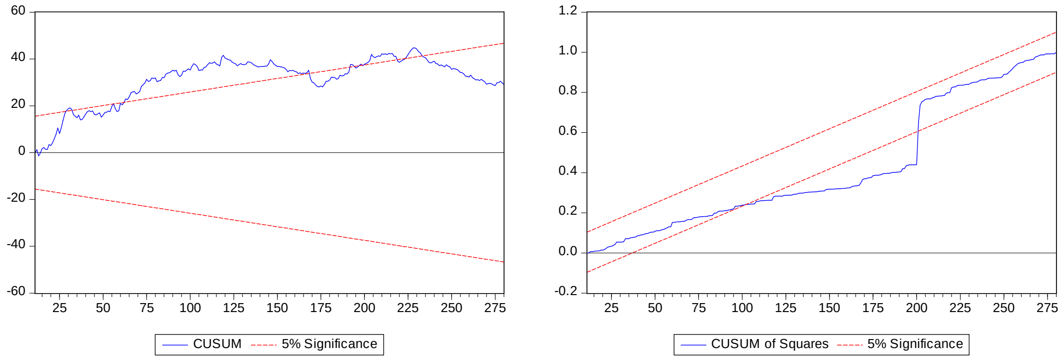
GRAPH 2.5

5. MODEL SPECIFICATION: LRM1, LIIP, LREER, LGP, CR



GRAPH 2.6

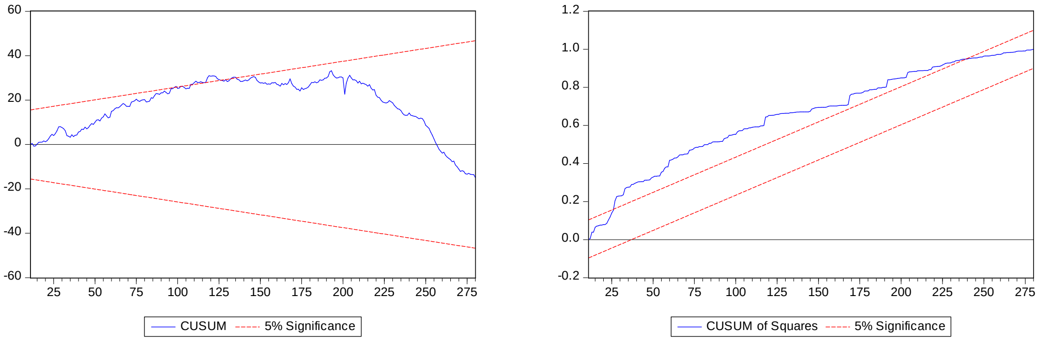
6. MODEL SPECIFICATION: LRM3, LIIP, LREER, LGP, CR



The straight line represents the critical bound at 5% significance level. Figures show CUSUM and CUSUMQ Test: Plot of cumulative sum of square of recursive residual

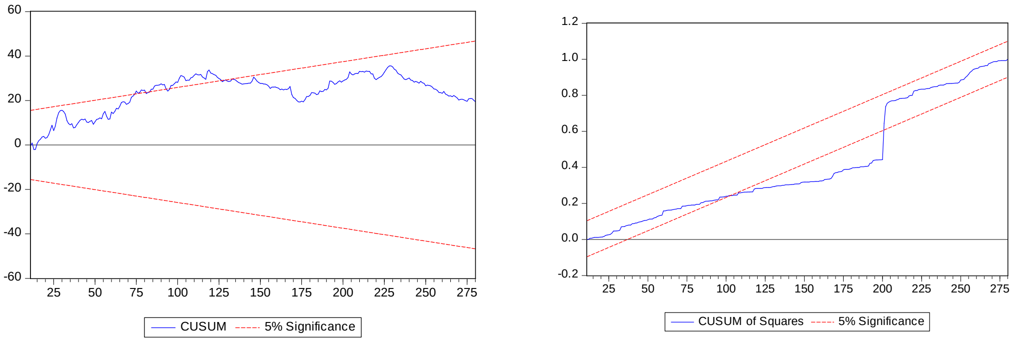
GRAPH 2.7

7.MODEL SPECIFICATION:LRM1, LIIP, LREER, LGP, CI



GRAPH 2.8

8. MODEL SPECIFICATION: LRM1, LIIP, LREER, LGP, CI



The straight line represents the critical bound at 5% significance level. Figures show CUSUM and CUSUMQ Test: Plot of cumulative sum of square of recursive residual

8. SELECTED REFERENCES

[1]. **Arif ,R. R.** (1996), 'Money Demand Stability: Myth or Reality- An Econometric Analysis' Development Research Group, No. 13, Reserve Bank of India, 1999.

[2]. **Bhattacharya, B. B.** (1974), 'Demand and Supply of Money in a Developing Economy: A Structural Analysis for India', Review of Economics and Statistics, Vol.56, No.-4, Nov. 1974, pp. 502.

[3]. **Bhoi, B.K.** (1995), 'Modeling Buffer Stock Money: An Indian Experience', *RBI Occasional Papers*, Vol. 16, No.1, 1995.

[4]. **Biswas, Basudeb and Ram, Rati** (1983), 'Stability of Demand for Money in India: Some Further Evidence' *The Indian Economic Journal*, Vol.31, Jul-Sep. 1983, pp. 77.

[5]. **Brown, R.L., J. Durbin, and J.M. Evans** (1975), 'Techniques for Testing the Constancy of Regression Relations Over Time', *Journal of the RoyalStatistical Society*, Series B, 37, pp. 149-163.

[6]. **Brunner, K. and Allan H. Meltzer** (1967), 'The Meaning of Monetary Indicators', Horwich (ed.), *Monetary Process and Policy*, Richard D. Irwin, Homewood.

[7]. **Deadman, Derekand and Ghatak, Subrata** (1981), 'On the Stability of Demand for Money in India' *The Indian Economic Journal*, Vol.29, Jul-Sept. 1981, pp. 41.

[8]. **Dotsey, M., and Andreas Hornstein** (2006), 'Implementation of Optimal Monetary Policy', *Economic Quarterly*, Federal Reserve Bank of Richmond, Vol. 92, No.2, pp. 113-133.

[9]. **Duca J., and David Vanhoose**(2004), 'Recent Developments in Understanding the Demand for Money', *Journal Economics and Business*, Vol. 56, No. 4, pp.247-272.

[10]. **Engle, R.F., and C.W.J. Granger** (1987), 'Co-Integration and Error Correction: Representation, Estimation, and Testing', *Econometrica*, Vol. 55, pp. 251-256.

[11]. **Friedman, M.** (1975), 'Targets, Instruments and Indicators of Monetary Policy', *Journal of Monetary Economics*, Vol.1, October, pp. 443-473.

[12]. **Friedman, M.**(1987), 'Quantity Theory of Money', *The New Palgrave: A Dictionary of Economics*, Vol.4, p.15.

[13]. **Goodfriend, M.** (1987), 'Interest rate smoothing and price level trend-stationarity', *Journal of Monetary Economics*, Elsevier, Vol. 19, No. 3, May, pp. 335-348.

[14]. **Granger,C.W.J**(1986), 'Developments in the study of Cointegrated Economics Variables' *Oxford Bulletin of Economics and Statistics*, Vol. 48, No. 3, Aug. 1986, pp. 213-28.

[15]. **Gujarati, Damodar**, *Basic Econometrics*, 3rd Edition, McGraw Hill International Editions, New York.

[16]. Inoue, Takeshi, and Hamori Shigeyuki (2008), 'An Empirical analysis of the money demand function in India', *IDE Discussion Papers 166*, Institute of Developing Economies, Japan External Trade Organization (JETRO).

[17]. Hussain, Zakir and Sampath R.K. (1981-82), 'Demand for Money in India', *The Indian Economic Journal*, Vol.29, Jul-Sept. 1981-82, pp. 17.

[18]. Johansen, S. (1988), 'Statistical Analysis of Cointegration Vectors', *Journal of Economic Dynamics and Control*, No. 12, pp. 231-254.

[19]. Johansen, S. and Katarina Juselius (1990), 'Maximum likelihood estimation and inference on Cointegration- with applications to the demand for money', *Qxford Bulletin of Economics and Statistics*, Vol.52, No.2, pp. 169-210.

[20]. Kole, Linda S., and Ellen E. Meade (1995), 'German Monetary Targeting: A Retrospective View', *Federal Reserve Bulletin*, Vol. 81, No. 10, Oct. 1995,pp. 917-31.

[21]. Kulkarni, Kishor G., *Modern Monetary Theory*, McMillan India Limited, New Delhi.

[22]. Laidler, David E.W. (1969), 'The Buffer Stock Notion in Monetary Economics', *The Economic Journal*, Vol. 94, March 1984, pp. 17-34.

[23]. Mahajan, Y. Lal (1979), 'Stability of The Demand for Money in India', *The Indian Economic Journal*, Vol.27, Jul-Sept. 1979, pp. 95.

[24]. Mehra, Y. P. (1992), 'In Search of a Stable, Short-Run M1 Demand Function' *Economic Review*, Vol. 78, May-June 1992, pp. 9-22.

[25]. Meltzer, A.H. (2001), 'Monetary Transmission at Low Inflation: Some Clues from Japan in the 1990s', *Monetary and Economic Studies*, Institute for Monetary and Economic Studies, Bank of Japan, Vol. 19, No. S1, pp. 13-34.

[26]. Moosa, A. Imad (1992), 'The Demand for Money in India: A Cointegration Approach', *The Indian Economic Journal*, Vol.40, July-Sept. 1992, pp. 101.

[27]. Poole W. (1970), 'Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model', *The Quarterly Journal of Economics*, Vol. 84, No. 2, May, pp. 197-216.

[28]. Prasad, Ramchandra U, 'Determinants of Demand for Money in India- An Empirical Analysis' *Research Paper ISID*, New Delhi, 1988.

[29]. Rangarajan, C. and Anoop Singh (1984), 'Reserve Money: Concepts and Policy Implications for India', *Reserve Bank of India Occasional Papers,* June.

[30]. **Rao, B. B. and Rup Singh** (2003), 'Demand for Money in India: 1953-2002', University of South Pacific, Suva, Fiji 2003.

[31]. **Resina, Katafona** (2001), 'Re-examination of The Demand for Money in Fiji ', *Working Paper 2001/03*, Reserve Bank of Fiji, May 2001.

[32]. **Sampath, R. K. and Zakir Hussain**(1981), 'Demand for Money in India', *The Indian Economic Journal*, Vol.29,No. 1, July-Sept. 1981, pp.17.

[33]. **Sriram, S. S.** (1999), 'Survey of Literature on Demand for Money: Theoretical and Empirical Work with Special Reference to Error- Correction Model', *IMF Working Paper*, WP/99/64, 1999.

[34]. **Trivedi, M. S**. (1980), 'Inflationary Expectations and Demand for Money in India (1951-1975)', *The Indian Economic Journal*, Vol.28, July-Sept. 1980, pp. 62.

[35]. **Trivedi, M. S**. (1983), 'Inflationary Expectations and Demand for Money Per Capita Real Balances: Evidence from the Indian Economy', *Artha Vignana*, Vol.25, No. 1, 1983, pp. 21.

[36]. **Trivedi, M. S**. (1984), 'Price Expectations, Price Uncertainty and Demand for Money in India: Some Evidence', From *Liberalism and LDCS*, 1984 pp. 226.

[37]. **VanHoose, D.** (1989), 'Borrowed reserves targeting and nominal income smoothing', *Research Working Paper 89-15*, Federal Reserve Bank of Kansas City.

[38]. **Vasudenan, A**. (1977), 'Demand for Money in India- A Survey of Literature', *RBI Staff Occasional Papers*, Vol. 2, No. 1, 1977.

RESEARCH ARTICLE

INVENTORY SYSTEM WITH DETERIORATION OF ITEMS UNDER PARETO TYPE – I DISTRIBUTION, POWER DEMAND PATTERN AND PARTIAL BACKLOGGING

U. B. Gothi[1] and Ankit Bhojak[2]

ABSTRACT

In the present research paper we have developed an inventory model by assuming random lifespan of the product which follows Pareto type – I Distribution. Looking to the market demand of the product an inventory model is formulated in power demand. Inventory holding cost is assumed to be a linear function of time. Shortages are allowed with partially backlogging which has its variable rate and it is assumed as an exponential function depending on waiting time for the next replenishment. For the developed model, total average cost is minimised so as to determine the optimum order value of quantity and time. The purposed model has been illustrated with its sensitivity analysis.

1. INTRODUCTON

In last few decades, inventory problems have been studied on a large scale. Inventory refers to as a stock of items before reaching to the customers. Food items, Vegetables, Dairy products, pharmaceuticals etc. are few examples of our essential daily required items to fulfil our daily requirements. Deterioration of item is an essential term for inventory system. We can classify the items in our day to day life in two categories so far as the deterioration of items is concerned. Hence to develop inventory model for such items one has to give weightage for appropriate deterioration rate to meet with the real life situation.

Various inventory models have been developed by considering different deterioration rates. Ghare and Schrader [4] developed a model for an exponentially decaying inventory. Goyal and Giri [6] published trends in modeling of deteriorating inventory. Ruxian Li, Hongjie Lan and John R. Mawhinney [14] reviewed on deteriorating inventory study. Kirtan Parmar, Indu Aggarwal and U. B. Gothi [8] formulated order level inventory model for deteriorating item under varying demand conditions. Devyani Chatterji and U. B. Gothi [3] have presented

[1] Head, Department of Statistics, St. Xavier's College (Autonomous), Ahmedabad, Gujarat, India.
[2] Department of Statistics, GLS Institute of Commerce, GLS University, Ahmedabad, Gujarat, India.
(rcd. April'16 / frvd. May'16 / srvd. June'16)

EOQ model for deteriorating items under two and three parameter weibull distribution and constant IHC with partially backlogged shortages. Ankit Bhojak and U. B. Gothi [1] developed inventory models for ameliorating and deteriorating items with time dependent demand and IHC.

Numbers of inventory models have been developed by taking different forms for demand function into consideration. Jinn-Tsair Teng , Liang-Yuh Ouyang and Mei-Chuan Cheng [7] have given an EOQ model for deteriorating items with power form stock-dependent demand. Mishra and Singh [9] presented a computational approach to EOQ model with power form stock dependent demand and cubic deterioration. R. Babukrishnaraj and K. Ramasamy [11] formulated an inventory model with power demand pattern, Weibull distribution deterioration with no shortages. An EOQ inventory model for Weibull distributed deteriorating items with power demand pattern and shortages was analysed by S. P. Singh and V. K. Sehgal [15]. Nandagopal Rajeswari and Thirumalaisamy Vanjikkodi [10] presented an inventory model for items with two parameter Weibull distribution deterioration and backlogging. A production inventory model of power demand and constant production rate where the products have finite shelf-life was formulated by Shirajul Islam Ukil, Mohammad Ekramol Islam and Md. Sharif Uddin [16]. U. B. Gothi, Devyani A. Chatterji and Kirtan C. Parmar [18] have developed an integrated inventory model with exponential amelioration and two parameter Weibull deterioration.

Rao, Begum and Murty [13] obtained optimal ordering policies of inventory model for deteriorating items having generalized Pareto lifetime. Singh, N., Vaish, and Singh, S. R. [17] considered an EOQ model with Pareto distribution for deterioration, trapezoidal type demand and backlogging under trade credit policy. Inventory model pertaining to deteriorating items under two component mixture of Pareto lifetime and selling price dependent demand is developed by Vijayalakshmi, Srinivasa Rao and Nirupama Devi [20].

In this paper, we have made an attempt to develop an EOQ model by considering time to deteriorate as random variable following the two parameter Pareto type – I distribution. The probability density function of two parameter Pareto type – I distribution is given by $f(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{-\beta-1}$; $t \geq \alpha$, where β and α are the parameters with positive real values. The instantaneous rate of deterioration $\theta(t)$ of the non-deteriorated inventory at time t, can be obtained from $\theta(t) = \frac{f(t)}{1-F(t)}$, where $F(t) = 1 - \left(\frac{t}{\alpha}\right)^{-\beta}$ is the cumulative distribution function for the two parameter Pareto type – I distribution. Thus, the instantaneous rate of deterioration of the on-hand inventory is $^{-}(t) = \frac{-}{t}$. Inventory holding cost is considered as a linear function of time. In our attempt power demand pattern is assumed and shortages are allowed to occur where unsatisfied demand is partially backlogged. The developed model is illustrated by means of a numerical illustration with its sensitivity analysis.

1. ASSUMPTIONS

In developing the above stated model, following assumptions are considered to develop this model:

1. The inventory system involves only one item and one stocking point.
2. Replenishment rate is infinite.
3. Lead-time is zero.
4. The deterioration occurs when the item is effectively in stock.
5. The deteriorated items are not replaced during the given cycle.
6. Time horizon period is considered to be infinite.
7. Shortages are allowed to occur and unsatisfied demand is backlogged at a rate $e^{-¢\,(T-t)}$, where the backlogging parameter ¢ is a small positive constant.
8. Inventory holding cost is a linear function of time and it is $C_h = h + r\,t \quad (h, r > 0)$.
9. The demand rate is $R(t) = \frac{d\, t^{\frac{1}{n}-1}}{n\, T^{\frac{1}{n}}}$ following power demand pattern over a time period [0, T], where d is demand size during the planning horizon T and n is a finite positive index.
10. Time to deterioration follows exponential distribution in the time interval [0, μ) and Pareto type – I distribution in the time interval [μ, t_1]

    Thus, the deterioration rate for different time interval is

$$^{-}(t) = \begin{cases} - & ;\ 0 \le t < « \\ \frac{-}{t} & ;\ « \le t \le t_1 \end{cases}$$

11. Deterioration cost, production cost, ordering cost and shortage cost per unit are known and constants.
12. Total inventory cost is a continuous real function which is convex to the origin.

2. NOTATIONS

The following notations are used to develop the proposed model

1. Q (t) : Inventory level of the product at time t.
2. R (t) : Demand rate which is varying over time.
3. θ (t) : Deterioration rate.
4. A : Ordering cost per order during the cycle period.
5. C_h : Inventory holding cost per unit per unit time.
6. C_d : Deterioration cost per unit.
7. C_p : Production cost per unit.
8. C_s : Shortage cost per unit.
9. *l* : Opportunity cost due to lost sale per unit.
10. S : Inventory level at time t = 0.
11. S_1 : Inventory level at time t = μ.
12. S_2 : The maximum inventory level during shortage period.
13. Q : Reorder quantity
14. T : Duration of a cycle.
15 TC : Total cost per unit time.

3. MATHEMATICAL FORMULATION AND SOLUTION

Cycle starts with inventory level of S units. The total time is distributed into three time intervals. In the first time interval [0, μ] the inventory level decreases due to constant deterioration rate and demand rate R(t) and reaches to S_1. In the second time interval $[\mu, t_1]$ the inventory level goes down and reaches to zero due to the effects of Pareto type – 1 deterioration rate and demand rate R(t). Shortages are allowed to occur during the third time interval [, T]. Unsatisfied demand is partially backlogged and the rest is lost. The above mentioned inventory system is presented graphically in Fig. 1.

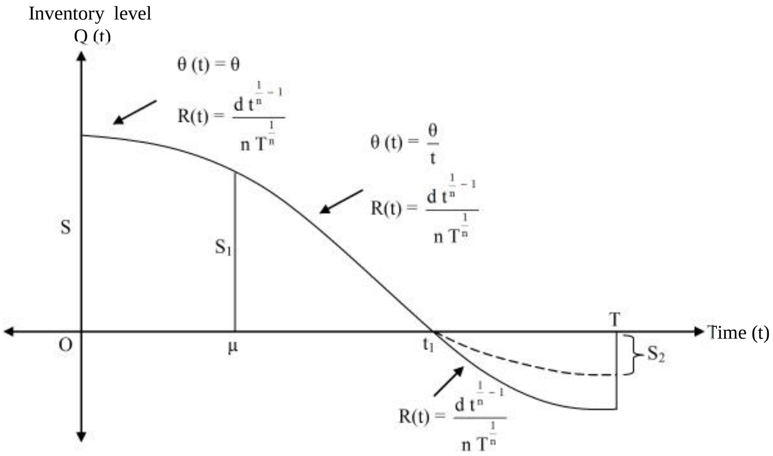


Fig. 1. Graphical presentation of inventory system

Differential equations to obtain Q (t) pertaining to the situations as described above are given by

$$\frac{d\,Q(t)}{d\,t} + \text{¯}\ Q(t) = -\frac{d\,t^{\frac{1}{n}-1}}{n\,T^{\frac{1}{n}}} \qquad 0 \le t \le \text{«} \qquad \text{... (1)}$$

$$\frac{d\,Q(t)}{d\,t} + \frac{\text{¯}}{t}\ Q(t) = -\frac{d\,t^{\frac{1}{n}-1}}{n\,T^{\frac{1}{n}}} \qquad \text{«} \le t \le t_1 \qquad \text{... (2)}$$

$$\frac{d\,Q(t)}{d\,t} = -\frac{d\,t^{\frac{1}{n}-1}}{n\,T^{\frac{1}{n}}}\ e^{-\text{¢}\,(T-t)} \qquad t_1 \le t \le T \qquad \text{... (3)}$$

Under the boundary conditions

$$Q(0) = S \text{ and } Q(t_1) = 0$$

the solutions for the differential equations (1), (2) and (3) are given by

$$Q(t) = S\,(1-\text{¯}\ t) - \frac{d}{T^{\frac{1}{n}}}\left(t^{\frac{1}{n}} - \left(\frac{n\,\text{¯}}{n+1}\right)t^{\frac{1}{n}+1}\right) \qquad \text{... (4)}$$

$$Q(t) = \frac{d}{T^{\frac{1}{n}}\,(1+n\,\text{¯})}\left(t_1^{\frac{1}{n}+\text{¯}}\ t^{-\text{¯}} - t^{\frac{1}{n}}\right) \qquad \text{«} \le t \le t_1 \qquad \text{... (5)}$$

and

$$Q(t) = \frac{d}{T^{\frac{1}{n}}}\left((1-\text{¢}\ T)\left(t_1^{\frac{1}{n}} - t^{\frac{1}{n}}\right) + \frac{\text{¢}}{n+1}\left(t_1^{\frac{1}{n}+1} - t^{\frac{1}{n}+1}\right)\right) \qquad t_1 \le t \le T \qquad \text{... (6)}$$

Substituting $Q(\text{«}) = S_1$ in equations (4) and (5), we get

$$S_1 = S\,(1-\text{¯}\ \text{«}) - \frac{d}{T^{\frac{1}{n}}}\left(\text{«}^{\frac{1}{n}} - \left(\frac{n\,\text{¯}}{n+1}\right)\text{«}^{\frac{1}{n}+1}\right) \qquad \text{... (7)}$$

$$S_1 = \frac{d}{T^{\frac{1}{n}}\left(1 + n^{-}\right)}\left(t_1^{\frac{1}{n}+^{-}}\,«^{-\,-} - «^{\frac{1}{n}}\right) \qquad \text{... (8)}$$

Eliminating from equations (7) and (8), we get

$$S\; = \frac{d}{T^{\frac{1}{n}}\left(1-^{-}\,«\right)}\left(n^{-}\,«^{\frac{1}{n}}\left(\frac{1}{1+n^{-}} - \frac{«}{n+1}\right) + \frac{t_1^{\frac{1}{n}+^{-}}}{«^{-}\left(1+n^{-}\right)}\right) \qquad \text{... (9)}$$

Substituting $Q(T) = -\,S_2$ in equation (6), we get

$$S_2 = \frac{d}{T^{\frac{1}{n}}}\left(\left(1-¢\;T\right)\left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \frac{¢}{n+1}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}\right)\right) \qquad \text{... (10)}$$

The order quantity

$$Q = \left(S + S_2\right)$$

$$\Rightarrow Q = \left(\begin{array}{l} \dfrac{d}{T^{\frac{1}{n}}\left(1-^{-}\,«\right)}\left(n^{-}\,«^{\frac{1}{n}}\left(\dfrac{1}{1+n^{-}} - \dfrac{«}{n+1}\right) + \dfrac{t_1^{\frac{1}{n}+^{-}}}{«^{-}\left(1+n^{-}\right)}\right) + \\ \dfrac{d}{T^{\frac{1}{n}}}\left(\left(1-¢\;T\right)\left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \dfrac{¢}{n+1}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}\right)\right) \end{array}\right) \qquad \text{... (11)}$$

5. COST COMPONENTS

The total cost per replenishment cycle consists of the following cost components

I. Operating Cost (OC)

The operating cost over the period [0, T] is

$$OC \;=\; A \qquad \text{... (12)}$$

II. Inventory Holding Cost (IHC)

The holding cost for carrying inventory over the period [0, T] is

$$IHC \;=\; \int_0^{«}\left(h + r\,t\right) Q(t)\;dt + \int_{«}^{t_1}\left(h + r\,t\right) Q(t)\;dt$$

$$\Rightarrow IHC = \left\{ \begin{array}{l} \int_0^{«} (h + r\,t) \left(S\,(1 - ^{–}\,t) - \frac{d}{T^{\frac{1}{n}}} \left(t^{\frac{1}{n}} - \left(\frac{n^{–}}{n+1} \right) t^{\frac{1}{n}+1} \right) \right) dt \\ + \int_{«}^{t_1} (h + r\,t) \left(\frac{d}{T^{\frac{1}{n}} \left(1 + n^{–}\right)} \left(t_1^{\frac{1}{n}+^{–}}\ t^{-^{–}} - t^{\frac{1}{n}} \right) \right) dt \end{array} \right\}$$

$$\Rightarrow IHC = \left\{ \begin{array}{l} h \left(S\left(« - \frac{^{–}\,«^2}{2} \right) - \frac{d}{T^{\frac{1}{n}}} \left(\frac{«^{\frac{1}{n}+1}}{\frac{1}{n}+1} - \left(\frac{n^{–}}{n+1} \right) \frac{«^{\frac{1}{n}+2}}{\frac{1}{n}+2} \right) \right) \\ +r \left(S\left(\frac{«^2}{2} - \frac{^{–}\,«^3}{3} \right) - \frac{d}{T^{\frac{1}{n}}} \left(\frac{«^{\frac{1}{n}+2}}{\frac{1}{n}+2} - \left(\frac{n^{–}}{n+1} \right) \frac{«^{\frac{1}{n}+3}}{\frac{1}{n}+3} \right) \right) \\ + \frac{d}{T^{\frac{1}{n}} \left(1 + n^{–}\right)} \left(\begin{array}{l} h \left(\frac{t_1^{\frac{1}{n}+^{–}}}{1-^{–}} \left(t_1^{1-^{–}} - «^{1-^{–}} \right) - \frac{1}{\frac{1}{n}+1} \left(t_1^{\frac{1}{n}+1} - «^{\frac{1}{n}+1} \right) \right) \\ + r \left(\frac{t_1^{\frac{1}{n}+^{–}}}{2-^{–}} \left(t_1^{2-^{–}} - «^{2-^{–}} \right) - \frac{1}{\frac{1}{n}+2} \left(t_1^{\frac{1}{n}+2} - «^{\frac{1}{n}+2} \right) \right) \end{array} \right) \end{array} \right\}$$

... (13)

III. Deterioration Cost (DC)

The deterioration cost over the period $[0, t_1]$ is

$$DC = C_d \left(\int_0^{«} {}^{–}\ Q(t)\ dt + \int_{«}^{t_1} \frac{^{–}}{t}\ Q(t)\ dt \right)$$

$$\Rightarrow DC = C_d\,^{–} \left(\left(S\,« - \frac{d}{T^{\frac{1}{n}}} \frac{«^{\frac{1}{n}+1}}{\frac{1}{n}+1} \right) + \frac{d}{T^{\frac{1}{n}} \left(1 + n^{–}\right)} \left(\frac{t_1^{\frac{1}{n}+^{–}}}{^{–}} \left(t_1^{-^{–}} - «^{-^{–}} \right) - n \left(t_1^{\frac{1}{n}} - «^{\frac{1}{n}} \right) \right) \right)$$

... (14)

IV. Shortage Cost (SC)

The shortage cost over the period $[t_1 , T]$ is

$$SC = - C_s \int_{t_1}^{T} Q(t)\, dt$$

$$\Rightarrow SC = \frac{C_s\, d}{T^{\frac{1}{n}}}\left(\frac{(1-\text{¢}\ T)}{\frac{1}{n}+1}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}\right) + \frac{\text{¢}}{(n+1)\left(\frac{1}{n}+2\right)}\left(T^{\frac{1}{n}+2} - t_1^{\frac{1}{n}+2}\right) - \left((1-\text{¢}\ T)\ t_1^{\frac{1}{n}} + \frac{\text{¢}}{(n+1)} t_1^{\frac{1}{n}+1}\right)(T - t_1) \right) \quad \text{... (15)}$$

V. Opportunity Cost due to lost sales (LSC)

The opportunity cost due to lost sales during the period $[t_1 , T]$ is

$$LSC = l \int_{t_1}^{T} \left(1 - e^{-\text{¢}\,(T-t)}\right) \frac{d\, t^{\frac{1}{n}-1}}{n\, T^{\frac{1}{n}}}\ dt$$

$$\Rightarrow LSC = \frac{l\ d\ \text{¢}}{(n+1)\, T^{\frac{1}{n}}}\left(n\, T^{\frac{1}{n}+1} + t_1^{\frac{1}{n}}\left(t_1 - (n+1)T\right)\right) \quad \text{... (16)}$$

VI. Purchase Cost (PC)

The purchase cost during the period is

$$PC = C_p\, Q$$

$$\Rightarrow PC = C_p \left(\frac{d}{T^{\frac{1}{n}}(1-\text{¯}\ \text{«})}\left(n^{\text{¯}}\ \text{«}^{\frac{1}{n}}\left(\frac{1}{1+n^{\text{¯}}} - \frac{\text{«}}{n+1}\right) + \frac{t_1^{\frac{1}{n}+\text{¯}}}{\text{«}^{\text{¯}}\ (1+n^{\text{¯}})} \right) + \frac{d}{T^{\frac{1}{n}}}\left((1-\text{¢}\ T)\left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}}\right) + \frac{\text{¢}}{n+1}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1}\right)\right) \right) \quad \text{... (17)}$$

Hence, the total cost per unit time is given by

$$TC = \frac{1}{T}\left(OC + IHC + DC + SC + LSC + PC\right)$$

$$\Rightarrow TC = \frac{1}{T}\left(\begin{array}{l} A + \left\{ \begin{array}{l} h\left(S\left(« - \frac{—«^{2}}{2} \right) - \frac{d}{T^{\frac{1}{n}}}\left(\frac{«^{\frac{1}{n}+1}}{\frac{1}{n}+1} - \left(\frac{n—}{n+1} \right)\frac{«^{\frac{1}{n}+2}}{\frac{1}{n}+2} \right) \right) \\ + r\left(S\left(\frac{«^{2}}{2} - \frac{—«^{3}}{3} \right) - \frac{d}{T^{\frac{1}{n}}}\left(\frac{«^{\frac{1}{n}+2}}{\frac{1}{n}+2} - \left(\frac{n—}{n+1} \right)\frac{«^{\frac{1}{n}+3}}{\frac{1}{n}+3} \right) \right) \\ + \frac{d}{T^{\frac{1}{n}}\left(1 + n—\right)}\left(\begin{array}{l} h\left(\frac{t_1^{\frac{1}{n}+—}}{1-—}\left(t_1^{1-—} - «^{1-—} \right) - \frac{1}{\frac{1}{n}+1}\left(t_1^{\frac{1}{n}+1} - «^{\frac{1}{n}+1} \right) \right) \\ + r\left(\frac{t_1^{\frac{1}{n}+—}}{2-—}\left(t_1^{2-—} - «^{2-—} \right) - \frac{1}{\frac{1}{n}+2}\left(t_1^{\frac{1}{n}+2} - «^{\frac{1}{n}+2} \right) \right) \end{array} \right) \end{array} \right\} \\ + C_d - \left(\left(S« - \frac{d}{T^{\frac{1}{n}}}\frac{«^{\frac{1}{n}+1}}{\frac{1}{n}+1} \right) + \frac{d}{T^{\frac{1}{n}}\left(1 + n—\right)}\left(\frac{t_1^{\frac{1}{n}+—}}{—}\left(t_1^{—} - «^{—} \right) - n\left(t_1^{\frac{1}{n}} - «^{\frac{1}{n}} \right) \right) \right) \\ + \frac{C_s\, d}{T^{\frac{1}{n}}}\left(\begin{array}{l} \frac{\left(1 - ¢\, T\right)}{\frac{1}{n}+1}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1} \right) + \frac{¢}{\left(n+1\right)\left(\frac{1}{n}+2\right)}\left(T^{\frac{1}{n}+2} - t_1^{\frac{1}{n}+2} \right) \\ - \left(\left(1 - ¢\, T\right) t_1^{\frac{1}{n}} + \frac{¢}{\left(n+1\right)} t_1^{\frac{1}{n}+1} \right)\left(T - t_1 \right) \end{array} \right) \\ + \frac{l\, d\, ¢}{\left(n+1\right)T^{\frac{1}{n}}}\left(n\, T^{\frac{1}{n}+1} + t_1^{\frac{1}{n}}\left(t_1 - \left(n+1\right)T \right) \right) \\ + C_p\left(\begin{array}{l} \frac{d}{T^{\frac{1}{n}}\left(1 - —\, «\right)}\left(n—\, «^{\frac{1}{n}}\left(\frac{1}{1+n—} - \frac{«}{n+1} \right) + \frac{t_1^{\frac{1}{n}+—}}{«^{—}\left(1+n—\right)} \right) + \\ \frac{d}{T^{\frac{1}{n}}}\left(\left(1 - ¢\, T\right)\left(T^{\frac{1}{n}} - t_1^{\frac{1}{n}} \right) + \frac{¢}{n+1}\left(T^{\frac{1}{n}+1} - t_1^{\frac{1}{n}+1} \right) \right) \end{array} \right) \end{array} \right)$$

... (18)

Our objective is to determine the optimum values μ^*, t_1^* and T^* of μ, t_1 and T respectively so that TC is minimum. Note that values μ^*, and can be obtained by solving the equations underlying the necessary conditions

$$\frac{\partial(TC)}{\partial «} = 0, \; \frac{\partial(TC)}{\partial t_1} = 0 \;\&\; \frac{\partial(TC)}{\partial T} = 0 \qquad \text{... (19)}$$

such that they can satisfy the following sufficient conditions

$$\left.\begin{array}{l} \left|\begin{matrix} \frac{\partial^2 TC}{\partial «^2} & \frac{\partial^2 TC}{\partial « \partial t_1} & \frac{\partial^2 TC}{\partial « \partial T} \\ \frac{\partial^2 TC}{\partial t_1 \partial «} & \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial T} \\ \frac{\partial^2 TC}{\partial T \partial «} & \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T^2} \end{matrix}\right|_{« = «^*, t_1 = t_1^*, T = T^*} > 0 \\ \left|\begin{matrix} \frac{\partial^2 TC}{\partial «^2} & \frac{\partial^2 TC}{\partial « \partial t_1} \\ \frac{\partial^2 TC}{\partial t_1 \partial «} & \frac{\partial^2 TC}{\partial t_1^2} \end{matrix}\right|_{« = «^*, t_1 = t_1^*, T = T^*} > 0 \\ \text{and} \\ \left.\frac{\partial^2 TC}{\partial «^2}\right|_{« = «^*, t_1 = t_1^*, T = T^*} > 0 \end{array}\right\} \qquad \text{... (20)}$$

The optimal solution of the equations in (19) can be obtained by using Mathematica software. The above developed model is illustrated by means of the following numerical example.

6. NUMERICAL EXAMPLE

Let us consider the following numerical example to illustrate the above developed model. ues of the parameters A = 200, θ = 0.0001, δ = 0.0001, h = 8, r = 3, l = 5, C_s = 3, C_d = 4, C_p = 7, n = 4 and d = 15 (with appropriate units of measurement). We obtain the values μ^* = 0.99 (≈ 1) unit, t_1^* = 15.71 units, T^* = 183.90 units and optimum average total cost = 16.51 units and the optimum reorder quantity Q = 14.93 (≈ 15) units by using Mathematica software.

7. PARTIAL SENSITIVITY ANALYSIS

Partial sensitivity analysis is an essential approach to identify the effect of optimal solution of the model for the changes in its parameter values. In this section, we study the sensitivity of average total cost and reorder quantity Q with respect to the changes in the values of the parameters A, θ, δ, h, r, l, C_s, C_d, C_p, n and d.

This analysis is performed by considering 10% and 20% increase and decrease in each one of the above parameters keeping all other remaining parameters as fixed. The results are presented in the table below. The seventh and the last column of the table show the % change in and % change in Q respectively as compared to the original solution corresponding to the change in parameter values, taken one by one.

Partial Sensitivity Analysis

Para meter	% change	μ (1)	t_1 (2)	T (3)	TC (4)	% change in TC (5)	Q = S + S_2 (6)	% change in Q (7)
A	– 20	0.999878948	14.43148907	159.1640586	16.27792763	– 1.415483794	14.93871633	0.07839246
	– 10	0.999879413	15.07664354	171.3986177	16.3990041	– 0.682204619	14.93296536	0.039865165
	+ 10	0.99988023	16.34000794	196.6881429	16.6168021	0.63685216	14.92085192	– 0.04128597
	+ 20	0.999880592	16.96229878	209.7993338	16.71525385	1.233108525	14.91446264	– 0.08408943
θ	– 20	0.999903867	15.71211928	183.8955054	16.51159618	– 0.000309717	14.92679508	– 0.00147113
	– 10	0.999891853	15.71211469	183.8966138	16.51162175	– 0.00015486	14.92690488	– 0.00073558
	+ 10	0.999867825	15.7121055	183.8988305	16.51167289	0.000154861	14.92712448	0.000735597
	+ 20	0.999855811	15.71210091	183.8999389	16.51169846	0.000309724	14.92723429	0.001471213
δ	– 20	0.999879717	15.52521465	179.6161476	16.54252823	0.187025031	14.94351913	0.110567675
	– 10	0.999879776	15.61621337	181.6964053	16.52720511	0.094223101	14.93540011	0.056176228
	+ 10	0.999879903	15.81347238	186.2346629	16.49584026	– 0.095732776	14.91833615	– 0.05813978
	+ 20	0.999879972	15.92097639	188.7246382	16.47976747	– 0.19307492	14.90933306	– 0.11845382
h	– 20	0.999879689	15.48350238	170.8181931	16.28156844	– 1.393433815	14.93417468	0.047966737
	– 10	0.999879763	15.59621976	177.2843371	16.39873293	– 0.683846891	14.93064637	0.024329641
	+ 10	0.999879915	15.83156918	190.670797	16.62055471	0.659579187	14.92327303	– 0.02506632
	+ 20	0.999879993	15.95502598	197.6174554	16.72567642	1.296231065	14.91941393	– 0.05091944
r	– 20	0.999880823	17.37930366	187.4560914	16.11066646	– 2.428472758	14.92851128	0.010026144
	– 10	0.999880311	16.47650373	185.456108	16.32131129	– 1.152737963	14.92776847	0.005049804
	+ 10	0.999879398	15.05384274	182.6695663	16.68524352	1.051356022	14.92625988	– 0.00505659
	+ 20	0.999878984	14.47908136	181.6948572	16.84479597	2.017658485	14.92550998	– 0.01008037

Parameter	% change	μ (1)	t_1 (2)	T (3)	TC (4)	% change in TC (5)	$Q = S + S_2$ (6)	% change in Q (7)
l	– 20	0.999879838	15.71226746	183.9066747	16.51124443	– 0.002440032	14.92700975	– 0.0000330
	– 10	0.999879838	15.71218877	183.9021983	16.51144588	– 0.001220011	14.92701221	– 0.0000165
	+ 10	0.999879838	15.71203142	183.8932462	16.51184876	0.00122	14.92701715	0.0000165
	+ 20	0.999879838	15.71195275	183.8887706	16.5120502	0.002439989	14.92701961	0.0000330
C_s	– 20	0.999880465	16.73893552	253.2864587	14.00013681	– 15.21053875	14.88720855	– 0.26667172
	– 10	0.999880109	16.14192141	212.6143979	15.28847888	– 7.407912797	14.91081323	– 0.1085378
	+ 10	0.99987962	15.37965326	162.3583986	17.67665671	7.055682396	14.93884186	0.079233368
	+ 20	0.999879437	15.11052675	145.5438127	18.78910189	13.7930185	14.94783833	0.139503077
C_d	– 20	0.999879838	15.71209165	183.8972491	16.51164251	– 0.0000291	14.92701492	0.0000016
	– 10	0.999879838	15.71210087	183.8974856	16.51164492	– 0.0000145	14.9270148	0.0000008
	+ 10	0.999879838	15.71211932	183.8979586	16.51164972	0.0000145	14.92701456	– 0.0000008
	+ 20	0.999879838	15.71212854	183.8981951	16.51165212	0.0000291	14.92701444	– 0.0000016
C_p	– 20	0.999879391	15.04440485	170.7687139	16.39370633	– 0.714289678	14.93326414	0.04186679
	– 10	0.99987962	15.37944331	177.2947589	16.45377475	– 0.35049544	14.93016788	0.021124082
	+ 10	0.999880048	16.04270546	190.5815872	16.56745436	0.337985928	14.92380268	– 0.02151802
	+ 20	0.999880249	16.37151789	197.3505444	16.62131419	0.664178858	14.92052985	– 0.04344361
n	– 20	0.999882372	15.37867173	177.2835905	18.35203822	11.1460163	14.91979864	– 0.04834213
	– 10	0.999880974	15.50627146	179.8001334	17.39415336	5.344748611	14.92401301	– 0.02010898
	+ 10	0.999878905	15.96585009	189.0135958	15.70379308	– 4.892632596	14.92922837	0.014830062
	+ 20	0.99987813	16.2500757	194.829231	14.96572477	– 9.362618493	14.93089941	0.026024853
d	– 20	0.999880764	17.27192735	216.4853518	13.40974109	– 18.78617057	11.92894231	– 20.0848758
	– 10	0.999880271	16.4093935	198.1285322	14.96525324	– 9.365474271	13.42813812	– 10.0413686
	+ 10	0.999879454	15.13478028	172.5235822	18.0505351	9.320013628	16.42567609	10.03992725
	+ 20	0.999879108	14.64775647	163.214415	19.58314756	18.60202187	17.92418501	20.07883288

8. GRAPHICAL PRESENTATION

Graphical presentation for the above partial sensitivity analysis is shown in Fig. 2.

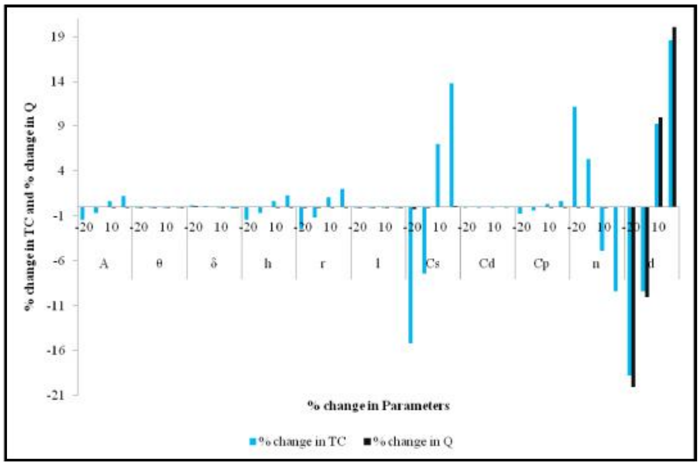


Fig. 2. Graphical presentation of partial sensitivity analysis

9. CONCLUDING REMARKS

From the above sensitivity analysis we may conclude that the total cost TC per time unit is highly sensitive to the changes in the values of the parameters d, C_s and n; moderately sensitive to the changes in the values of the parameters r, h and A and less sensitive to the changes in the values of the parameters $C_{p,}$ θ, δ, C_d, and *l*.

We may also conclude that the reorder quantity Q is highly sensitive to the changes in the values of the parameter d only and not much affected by changes in the rest of the parameters $A_,$ θ, δ, h, r, *l*, C_s, C_d, C_p and n.

Moreover, it can also be observed from Fig. 2. and table for partial sensitivity analysis that there is an opposite change in average total cost TC and reorder quantity Q for parameters A, h, r, C_d, C_p and n where as simultaneous change can be found in average total cost TC and reorder quantity Q for the remaining parameters d, θ, δ, *l* and C_s.

10. ACKNOWLEDGEMENT

We thank Dr. B. B. Jani for his constant support and encouragement to prepare this research paper. We are highly indebted to the referee (anonymous) for the comments and suggestions that helped us in revising thoroughly the earlier draft of this paper.

9. REFERENCES

[1] **Ankit Bhojak and U. B. Gothi. (2016).** Inventory Models for Ameliorating and Deteriorating Items with Time Dependent Demand and IHC. *International Journal of Management and Social Science Research Review, Vol.1, Issue.2* , 85 - 98.

[2] **Ankit Bhojak and U.B. Gothi. (2015).** Inventory Model for Ameliorating and Deteriorating Items with Time Dependent Demand. *Sankhya Vignan, (NSV 11), No.2* , 31- 43.

[3] **Devyani Chatterji and U. B. Gothi. (2015).** EOQ Model For Deteriorating Items Under Two And Three Parameter Weibull Distribution And Constant IHC With Partially Backlogged Shortages. *International Journal of Science, Engineering and Technology Research (IJSETR), Volume 4, Issue 10, October 2015* , 3582-3594.

[4] **Ghare, P. M. and Schrader G. F. (1963).** A model for an exponentially decaying inventory. *Journal of Industrial Engineering* , 238 - 243.

[5] **Giri B.C., Goswami A. and Chaudhuri K.S. (1996).** An EOQ model for deteriorating items with time varying demand and costs. *Journal of the Operational Research Society* , 1398-1405.

[6] **Goyal S.K. and Giri B.C. (2001).** Recent trends in modeling of deteriorating inventory. *European Journal of Operational Research* , 1-16.

[7] **Jinn-Tsair Teng , Liang-Yuh Ouyang and Mei-Chuan Cheng. (2005).** An EOQ Model for Deteriorating Items with Power Form Stock-Dependent Demand. *Information and Management Sciences Vol. 16, No.1* , 1-16.

[8] **Kirtan Parmar, Indu Aggarwal and U. B. Gothi. (2015).** Order level inventory model for deteriorating item under varying demand condition. *Sankhya Vignan, (NSV 11)* , 20-30.

[9] **Mishra S. S.and Singh P.K. (2011).** A Computational Approach to EOQ Model with Power form Stock Dependent Demand and Cubic Deterioration. *American Journal Of Operation Research , 1(1)* , 5-13.

[10] **Nandagopal Rajeswari and Thirumalaisamy Vanjikkodi. (2012).** An Inventory Model for Items with Two Parameter Weibull Distribution Deterioration and Backlogging. *American Journal of Operations Research,2* , 247-252 .

[11] **R. Babu Krishnaraj and K. Ramasamy. (2012).** An Inventory Model With Power Demand Pattern, Weibull Distribution Deterioration And Without Shortages. *The Bulletin of Society for Mathematical Services and Standards, Vol.2* , 33-37.

[12] **R. Begum, R. R. Sahoo, S. K. Sahu and M. Mishra. (2010).** An EOQ Model for Varying Items with Weibull Distribution Deterioration and Price-dependent Demand. *Journal of scientific research, Res. 2 (1)* , 24-36.

[13] **Rao, K. S., Begum, K. J., and Murty, M. V. (2007).** Optimal ordering policies of inventory model for deteriorating items having generalized Pareto lifetime. *Current Science-Bangalore- 93(10)* , 1407-1411.

[14] **Ruxian Li, Hongjie Lan and John R. Mawhinney. (2010).** A Review on Deteriorating Inventory Study. *J.Service Science & Management, 3* , 117-129.

[15] **S. P. Singh and V. K. Sehgal. (2011).** An EOQ Inventory Model For Weibull Distributed Deteriorating Items With Power Demand Pattern And Shortages. *JP Journal of Mathematical Sciences, Volume 1, Issue 2* , 99-110.

[16] **Shirajul Islam Ukil, Mohammad Ekramol Islam and Md. Sharif Uddin. (2015).** A Production Inventory Model of Power Demand and Constant Production Rate Where the Products Have Finite Shelf-Life. *Journal of Service Science and Management* ,

874-885.

[17] **Singh, N., Vaish, B., and Singh, S. R. (2010).** An EOQ Model with Pareto Distribution for Deterioration, Trapezoidal Type Demand and Backlogging under Trade Credit Policy. *The IUP Journal of Computational Mathematics, 3(4)* , 30-53.

[18] **U. B. Gothi, Devyani A. Chatterji and Kirtan C. Parmar. (2016).** An Integrated Inventory Model With Exponential Amelioration And Two Parameter Weibull Deterioration. *Journal of Statistics and Mathematical Engineering, MAT Journals,Vol.2, No. 2* .

[19] **U.B. Gothi and Devyani A. Chatterji. (2015).** EPQ model for Imperfect Quality Items Under Constant Demand Rate and Varying IHC. *Sankhya Vignan NSV11* , 7 - 19.

[20] **Vijayalakshmi. G, Srinivasa Rao. K and Nirupama Devi. K. (2014).** Inventory Model for Deteriorating Items Having Two Component Mixture of Pareto Lifetime and Selling Price Dependent Demand. *International Journal of Scientific & Engineering Research, Volume 5, Issue 7, July* , 254-262.

[21] **Wu J. W. and Lee W. C. (2003).** An EOQ inventory model for items with Weibull distributed deterioration, shortages and time-varying demand. *Information and Optimization Science, 24* , 103-122.

RESEARCH ARTICLE

KAPLAN MEIER METHOD FOR COMPARISON OF SURVIVAL TIME

D. K. Ghosh[1]

ABSTRACT

In this investigation Kaplan Meier method is used to analyze the Breast cancer data collected from Rajkot cancer Hospital for the period 2008 to 2009. Further an attempt is made to compare the survival time of two groups namely either treatment received or treatment not received by the patients. Moreover it is concluded that the Mean survival time of patients who do not receive treatment (any one out of the four) is quite less than the Mean survival time of patients who received any one of the treatment . However it is also noticed that the standard errors of the estimate of the mean in case of not receiving the treatment are more than the those received for all the cases.

1 INTRODUCTION

In this investigation a study is made for the survival analysis of breast cancer data collected from **Rajkot Cancer hospital for the period 2008 to 2009**. R x C contingency tables was prepared and then Chi square tests are used to find the dependence of stage at the diagnosis and other various factors affecting the cause of breast cancer. Kaplan-Meier method is used to observe the survival analysis of breast cancer.

Kaplan-Meier procedure is a very strong technique which is used to analyze clinical data. Let T be a random variable for a cancer patient's survival time. Since T denotes survival time hence its all values are non-negative. i.e., T can be any number ($T \geq 0$). Let t be any specified value of interest for random variable **T**. Let δ denote (0, 1) random variable indicating either censorship or failure. (i.e., $\delta=1$ for failure and $\delta=0$ for censored.) It is to be noted that if the patient has not died then the censorship is the only remaining possibility for that patient's survival time (i.e. $\delta=0$). Censorship has occurred if and only if one of the following will happen.

(i) A patient survives until the study ends.
(ii) A patient is lost to follow up.
(iii) A patient withdraws given study period.

[1.] Department of Statistics Saurashtra University Rajkot, Gujarat, India
(rcd. April '16 / frvd. May '16 / srvd. June '16)

Here a study is made only for censored data.

Survival function

The Survival function S(t) is defined as the probability that a patient Survives longer than some specified time t. i.e. S(t) = P(T>t).

Kaplan- Meier in 1958 studied non-parametric estimation from incomplete observations. They also studied the survival time in censored data. Cox regression is used to find relationship between survival time and its corresponding possible explanatory variables. In general survival analysis is carried out to analyze statistical data corresponding to survival time. Here survival time is considered as a time taken when an end event occurs in the data set. Therefore, one can say that it is the time to event. This is also known as end point or failure time data. In medical event and non-medical events, time to events contains survival time. Survival time in a medical event can be expressed as survival time until death or until the patient is being discharged from hospital.

Komarek et. al. (2005), studied in detail about the survival time in a medical event of tooth experiences in case of longitudinal oral cancer study. Perrigot et. al.(2004) discussed that the survival time is an important variable for the development of franchising. In all these cases the data contain censored observation in which the end point has not happened in every observation. i.e., Information is known for limited duration. However the censoring time is the most important information in finding cumulative survival probability for the survival analysis. The main advantage of using survival analysis is to analyze censored data.

Ghosh, et.al.(2015) studied the survival analysis of breast cancer patients to find survival function which indicates the survival of the patients in terms of the months. Recently Ghosh and Suryawanshi (2015) have studied the survival analysis of Breast cancer using proportional Hazard regression.

In this investigation a comparison is made for the survival times in terms of Mean and median estimates of the breast cancer patients whom either the treatments are given or not given.

The survival function is a number of individuals with survival time, which is at least t time periods divided by the number of individuals in the study. Let n_j denote no. of individuals alive just before time $t_{(j)}$ and d_j = number of deaths at for t , and k= 1, 2, … , r.

Now Kaplan-Meier estimate of the survival function can be defined as the product limit estimate expressed as,

$$S(\hat{t}) = \prod_{j=1}^{k} \left(\frac{n_j - d_j}{n_j} \right) \qquad (1.1)$$

In this study let us consider the following

(1) To find the percentage of survival at any time of interest.

(2) To compare the survival time of two groups namely surgical and non surgical etc.

In this investigation Kaplan-Meier method is used to obtain the Survival function, Survival time for the patient suffering with Breast cancer in Saurashtra-Kutch region. Here two study groups are considered as breast cancer women, say, who have either gone for surgery or not gone for surgery. Moreover, the survival time of the two studied groups are compared by using Log-Rank test. This is computed with the help of SPSS using equation(1.1).

Here it is concluded that, if the patients are not operated, (that is, not gone for surgery, Radiotherapy, Hormonal therapy and Chemotherapy) then their survival time is reducing drastically. However if they were operated, (that is, gone for surgery, Radiotherapy, Hormonal therapy and Chemotherapy) then their survival time is increasing slowly.

2. METHODOLOGY

In medical Statistics, especially for cancer problem, the main interest for finding survival analysis is to find death of patient which is due to disease. Hence to carry out the survival analysis, the status of the data is divided into two groups-namely censored data and uncensored data. The occurrence of censored observation may be due to the following few reasons.

(1) The patients are still alive at the end of study for which the critical event did not occur.

(2) The patients follow up information are lost. This is due to patient's reluctance to turn for the treatment on the study day.

(3) The event occurs but the cause is unrelated to the disease.

However, the status of the patients is recorded to indentify whether observations (Patients) is either a censored data or an uncensored data. For Simplification of analysis purpose we code '0' for censored data and code '1' for uncensored data. i.e. code 1 for an event of dying from the disease. Normally a factor is used to indicate whether the observation is in treatment group or control group. e.g. If there are two groups i.e. one treatment group and other control group then the factor '0' is referred for the control group and '1' for the treatment group. However if there are two treatment groups and one control group then '0' is referred for control group and '1' and '2' is referred for treatment one and treatment two group respectively. In this investigation the main interest of study is death due to breast cancer. In our study there are variables like treatment surgery, treatment chemotherapy, treatment radiotherapy, treatment hormonal therapy which are given to the cancer patients.

The variable treatment surgery is coded as (i) treatment surgery taking place and (ii) not taking place, treatment chemotherapy taking place and not taking place etc.

SPSS techniques are used to carry out the statistical analysis. In this investigation our main focus is on Kaplan Meier procedure. The Kaplan Meier Method is used to analyze censored and uncensored data for survival time. Moreover it is also used to compare, two treatment groups on their survival time. Thus Kaplan Meier techniques is the Uni variate version of the survival analysis.

3. STATISTICAL ANALYSIS

In this study, breast cancer data are collected for the year 2008-09 from Rajkot Cancer hospital. Here 315 patients suffering from Breast cancer are considered for the survival analysis study. It is found that out of 315 patients, 75 breast cancer patients died and 240 patients are censored during the treatment for the period 2008 – 09. This is shown in Table 3.1 below.

Table 3.1

		Frequency	Percent	Cumulative Percent
Valid	Censored	240	76.2	76.2
	Died	75	23.8	100.0
	Total	315	100.0	

4. SURVIVAL ANALYSIS FOR THE PERIOD 2008-09

Kaplan Meier (KM) method is useful to compare the survival function of two or more groups. By using log-rank test, it is concluded that whether or not two or more survival functions are the same based on computer result. The null hypothesis of log-rank test is "All survival functions are the same".

In this study, an attempt is made to consider the four treatments namely Surgery, Radiotherapy, Hormonal therapy and chemotherapy respectively, either applied or not applied on the breast cancer patients. This will be dealt with one by one in sections 4.1, 4.2, 4.3 and 4.4 respectively.

4.1 COMPARISONS OF SURVIVAL TIME OF TREATMENT SURGERY

Here those patients are considered who have either taken surgery or not gone for surgery. That is, we have two cases for study. Next mean, median, standard error and their 95% confidence intervals are computed using SPSS which are shown in various tables given below.

Table 4.1.1 shows the number of cases for the two categories in Treatment surgery, the patients who have not received treatment surgery which coded as 'No' and patient who have received treatment surgery which are coded by 'Yes'. Thus, there are 89 breast cancer patients who have not received treatment surgery and 226 breast cancer patients who have received treatment surgery.

Table 4..1.1 Case Processing Summary

Treatment Surgery	Total N	N of Events	Censored	
			N	Percent
No	89	31	58	65.2%
Yes	226	44	182	80.5%
Overall	315	75	240	76.2%

The Mean Survival time for patients who did not receive treatment surgery is 45.036 months and Mean Survival time for patients who have received treatment surgery is 64.921 months. The Median survival time of patients who do not receive treatment surgery is 41.10 months and Median survival time of patients who received treatment surgery is 75.90 months.

Table 4.1.2 Means and Medians for Survival Time

Treatment Radiotherapy	Mean				Median			
	Estimate	Std. Error	95% Confidence Interval		Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound			Lower Bound	Upper Bound
No	56.16	2.539	51.190	61.142	.	.	.	.
Yes	65.97	2.146	61.772	70.185	75.90	10.43	55.454	96.346
Overall	60.54	1.772	57.073	64.017	75.90	12.09	52.199	99.601

Table 4.1.3 shows the result of log-rank test with p-value 0.000, which is less than 0.05. Hence the null hypothesis of log rank test is rejected, which indicates that there is significant difference between the two groups of treatment surgery. Hence survival times of the patients who did not receive treatment surgery and patients who received treatment surgery are different.

Table 4.1.3 Overall Comparisons

	Chi-Square	df	Sig. (p value)
Log Rank (Mantel-Cox)	8.778	1	.003

4.2 Comparisons of Survival times of treatment Radiotherapy

Here those patients are considered who have either taken treatment Radiotherapy or not gone for Radiotherapy. That is, we have two cases for study. Next mean, median, standard error and their 95% confidence intervals are computed using SPSS which are shown in various tables.

Table 4.2.1 shows the number of cases for the two categories in Treatment Radiotherapy, the patients who did not receive treatment Radiotherapy, coded as 'No' and patient who received treatment Radiotherapy, coded by 'Yes'. Thus, there are195 breast cancer patients who have not received treatment Radiotherapy and 120 breast cancer patients who have received treatment Radiotherapy.

Table 4.2.1 Case Processing Summary

Treatment Hormonal Therapy	Total N	N of Events	Censored	
			N	Percent
No	235	65	170	72.3%
Yes	80	10	70	87.5%
Overall	315	75	240	76.2%

The Mean Survival time for patients who did not receive treatment Radiotherapy is 56.166 months and Mean Survival time for patients who received treatment Radiotherapy is 65.978 months. The Median survival time of patients who received treatment Radiotherapy is 75.90 months.

Table 4.2.2 : Means and Medians for Survival Time

Treatment Radiotherapy	Mean				Median			
	Estimate	Std. Error	95% Confidence Interval		Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound			Lower Bound	Upper Bound
No	56.16	2.539	51.190	61.142	.	.	.	.
Yes	65.97	2.146	61.772	70.185	75.90	10.43	55.454	96.346
Overall	60.54	1.772	57.073	64.017	75.90	12.09	52.199	99.601

Table 4.2.3 shows the result of log-rank test with p-value 0.003, which is less than 0.05. Hence the null hypothesis of log rank test is rejected, which indicates that there is significant difference between the two groups of treatment Radiotherapy. Hence survival times of patient who did not receive treatment Radiotherapy and patients who received treatment Radiotherapy are different.

Table 4.2.3 Overall Comparisons

	Chi-Square	df	Sig. (p value)
Log Rank (Mantel-Cox)	8.778	1	.003

4.3 COMPARISONS OF SURVIVAL TIMES OF TREATMENT HORMONAL THERAPY

Here those patients are considered who have either taken Hormonal therapy or not gone for Hormonal therapy. That is, we have two cases for study. Next mean, median, standard error and their 95% confidence intervals are computed using SPSS which are shown in various tables.

Table 4.3.1 shows the number of cases for the two categories in Treatment Hormonal Therapy, the patients who did not receive treatment Hormonal Therapy, coded as 'No' and patient who received treatment Hormonal Therapy, are coded by 'Yes'. Thus, there are 235 breast cancer patients who have not received treatment Hormonal Therapy and 80 breast cancer patients who have received treatment Hormonal Therapy.

Table 4.3.1 Case Processing Summary

Treatment Hormonal Therapy	Total N	N of Events	Censored	
			N	Percent
No	235	65	170	72.3%
Yes	80	10	70	87.5%
Overall	315	75	240	76.2%

The Mean Survival time for patients who did not receive treatment Hormonal Therapy is 55.859 months and Mean Survival time for patients who received treatment Hormonal Therapy is 68.178 months. The Median survival time of patients who did not receive treatment Hormonal Therapy is 75.90 months.

Table 4.3.2 Means and Medians for Survival Time

Treatment Hormonal Therapy	Mean				Median			
	Estimate	Std. Error	95% Confidence Interval		Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound			Lower Bound	Upper Bound
No	55.859	2.233	51.482	60.235	75.900	12.202	51.984	99.816
Yes	68.178	1.981	64.295	72.061	.	.	.	.
Overall	60.545	1.772	57.073	64.017	75.900	12.093	52.199	99.601

It is to be noticed that estimation is limited to the largest survival time if it is censored.

Table 4.3.3 shows the result of log-rank test with p-value 0.000, which is less than 0.05. Hence the null hypothesis of log rank test is rejected, which indicates that there is significant difference between the two groups of treatment Hormonal Therapy. Hence survival times of patient who did not receive treatment Hormonal Therapy and patients who received treatment Hormonal Therapy are different.

Table 4.3.3 Overall Comparisons

	Chi-Square	df	Sig. (p. value)
Log Rank (Mantel-Cox)	16.328	1	.000

4.4 COMPARISONS OF SURVIVAL TIMES OF TREATMENT CHEMOTHERAPY

Here we consider those patients who have either taken treatment Chemotherapy or not gone for Chemotherapy. That is, we have two cases for study. Next mean, median, standard error and their 95% confidence intervals are computed using SPSS which are shown in various tables.

Table 4.4.1 shows the number of cases for the two categories in Treatment Chemotherapy, the patients who did not receive treatment Chemotherapy, are coded as ‘No’ and patient who received treatment Chemotherapy, are coded by ‘Yes’. Thus, there are100 breast cancer patients who did not receive treatment Chemotherapy and 215 breast cancer patients who received treatment Chemotherapy.

Table 4.4.1 Case Processing Summary

Treatment Chemotherapy	Total N	N of Events	Censored	
			N	Percent
No	100	29	71	71.0%
Yes	215	46	169	78.6%
Overall	315	75	240	76.2%

The Mean Survival time for patients who did not receive treatment Chemotherapy is 46.65 months and Mean Survival time for patients who received treatment Chemotherapy is 64.176 months. The Median survival time of patients who received treatment Chemotherapy is 75.90 months. The Median survival time who did not receive Chemotherapy is 61.20.

Table 4.4.2 **Means and Medians for Survival Time**

Treatment Chemotherapy	Mean				Median			
	Estimate	Std. Error	95% Confidence Interval		Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound			Lower Bound	Upper Bound
No	46.650	3.53	39.732	53.568	61.200	.	.	.
Yes	64.176	1.85	60.532	67.821	75.900	.	.	.
Overall	60.545	1.77	57.073	64.017	75.900	12.093	52.199	99.601

Table 4.4.3 shows the result of log-rank test with p-value 0.000, which is less than 0.05. Hence the null hypothesis of log rank test is rejected, which indicates that there is significant difference between the two groups of treatment Chemotherapy. Hence survival times of patient who did not receive treatment Chemotherapy and patients who received treatment Chemotherapy are different.

Table 4.4.3 **Overall Comparisons**

	Chi-Square	df	Sig. (p. value)
Log Rank (Mantel-Cox)	14.428	1	.000

5. CONCLUDING REMARKS

From this study it can be concluded that the Mean survival time of patients who did not receive treatment (for all the four cases) is quite less than the Mean survival time of patients who received the treatment. Moreover it can also be concluded that the standard errors of the estimate of the mean in case of not receiving the treatment are more than the treatment received for all the cases. The survival time for the Breast cancer patients after receiving the treatment Hormonal therapy is the highest (68.78 months) followed by Radiotherapy (65.97 months), Surgery (64.921 months), and finally Chemo therapy (64.176 months).

6. ACKNOWLEDGEMENT

I am highly indebted to the referee (anonymous) for reviewing my paper which has helped me to revise thoroughly the earlier submitted drafts of my paper.

7. REFERENCES

[1] **Ghosh, D.K., Gupta, V.K. and Ahuja, Sangeeta (2015).** A study of Survival analysis of Breast cancer and its statistical analysis. *Indian Journal of Statistics and application.* 4, 1, 31 – 51.

[2] **Ghosh, D. K. and Suryawanshi, A. M. (2015).** Survival analysis using Cox Proportional Hazard regression. *International journal of Agricultural and statistical sciences.* 11, 2, 341 – 348.

[3] **Kaplan, E.L. and Paul, Meier (1958).** Non parametric estimation from incomplete observations. *Journal of American statistical Association*, 53, 457-481.

[4] **Komarek, A., Lesaffre, E, Harkanen,T., Declerck, D. and Virtanen, J. (2005).** A Bayesian analysis of Multivariate doubly interval censored dental data. *Biostatistics*, 6 (1), 145 – 155.

[5] **Perrigot, R., Cliquet, G. and Mesbah, M. (2004).** Possible Applications of Survival Analysis in Franchising Research. *The International Review of Retail, Distribution and Consumer Research.* 14, 1, 129 – 143.

RESEARCH ARTICLE

STATISTICAL ANALYSIS FOR ONION CROP PRODUCTION, AREA UNDER CULTIVATION AND EXPORT FOR INIDA

H. M. DIXIT* and P.M.PARMAR**

ABSTRACT

Onion is widely used vegetable food across India in all economic classes of people. Therefore many times its retail price becomes a political issue also. Volatility in price of onion is due to many reasons. Price is directly related with its amount of crop production and demand. Amount of crop production is a function of its area under cultivation. Frequent change in area under cultivation is called crop pattern. In this article an effort is made to determine trend of crop production of onion, its area under cultivation and its export from India. Multiple regression analysis is carried out among area under cultivation, crop production and export of onion from India.

1. INTRODUCTION

For any country or region agriculture sector is of fundamental importance, because it is belived as a spinal chord for the entire economy. If this is understood then & then only other sectors can also develope and it has definitely impact upon GDP growth of that country. This also helps to make the country self-sufficient and have sustainability for the country in terms of its agricultural production.

In agricultural sector, its development depends upon the changes occurring for different crop production, area under cultivation and agricultural productivity. If agricultural productivity is increasing then it helps in changes improving the area under cultivation of the respective crop. Of course this also depends upon the geographical, economic and social factors related with that country or region.

In our country onion production is probably one of the most important item due to its usage in everyday life for almost everybody. In India this crop is taken in two seasons, monsoon and winter. The states like Maharastram MP, Karnataka, Gujarat, Bihar etc, are the major onion crop producing states. Since onion is also very useful for human health, it is regarded as "kasturi" for poor and middle class people.

Onion crop production many times observes uncertainty due to a number of reasons. Since demand for Onion is important for day today life, when shortage occurs it is

* (Arts, Science & Commerce College Pilvai (N.G.)
** (H. L. College Of Commerce, Ahmedabad (Guj.Uni.)
(rcd. April '16 / frvd. May '16 / srvd. June '16)

also to be imported from other countries. However during bumper crop production it is to be exported to other countries also.

In this study, an attempt is made to analyze the pattern of onion crop production, its area under cultivation and its export by means of statistical regression analysis. Conclusions are drawn on the basis of this analysis.

2. DATABASE

The following analysis is carried out on the basis of statistical information pertaining to area under cultivation and crop production collected from department of agricultural and co-operation (Horticulture Division) and amount of export (in metric tonnes) from ICAR. during the period 2001-2014.

3. METHODOLOGY

The following models are considered for area under cultivation, crop production and export of onion from India.

Model A :- Regression models pertaining to area under cultivation, crop production and export of onion

Model A – 1 : -

Let us define $(A)_{x_i}$ = Area under cultivation for the year x_i (where i = 1,2,3…..13)

Then we consider $(A)_{x_i} = \alpha_1 \exp[\beta_1 X_i + u_{X_i}]$as the regression model pertaining to crop production of onion, where $_1$ and $\hat{a}_1$ are the parameters of the model and u_X denotes, the disturbance term for the year x_i

We want to fit this model for the relevant data regarding area under cultivation of Onion. Let us consider log-linear transformation for the above model, then equation takes the form

$$\text{Ln } (A)x_i = \text{Ln } \alpha_1 + \beta_1 X_i + u_{x_i}$$

We define $\text{Ln}(A)_{x_i} = Y_{x_i}$

$\text{Ln } \alpha_1 = A_1$

$\beta_1 = B_1$ and

$u_{x_i} = Z_{x_i}$ then

$Y_{x_i} = A_1 + B_1 X_i + u_{x_i}$ is the log-linear form for the above regression model. Under the usual assumptions, we can obtain OLSE of A_1 and B_1 given by A_1 and B_1 by using the standard formulae. The estimated values of y_x for given X_i are computed from

$$\hat{Y}_{x_i} = \hat{A}_1 + \hat{B}_1 X_i$$

We can also examine the significance of the estimated regression coefficients and carry out ANOVA for the fitted model.

MODEL A – 2 :-

Let us define $(P)_{X_i}$ = Crop production of onion in India. for the year X_i (where i = 1,2,3…13). Then we consider $(P)_{x_i} = \alpha_2 \exp[\beta_2 X_i + u_{x_i}]$ as the regression model pertaining to crop production of onion in India, where α_2 and β_2 are the parameters of the model and u_{x_i} denotes the disturbance term for the year X_i

We want to fit this model for the relevant data regarding crop production of onion in India., Let us consider log- linear transformation for the above model, then equation takes the form.

$$\text{Ln } (P)_{x_i} = \text{Ln } \alpha_2 + \beta_2 X_i + u_{x_i}$$

We define $\text{Ln } (P)_{x_i} = Y_{x_i}$

$\text{Ln } \alpha_2 = A_2$

$\beta_2 = B_2$ and

$u_{x_i} = Z_{X_i}$ then

$Y_{x_i} = A_2 + B_2 X_i + Z_{x_i}$ is the log – linear form for the above regression model. Under the usual assumptions we can obtain OLSE of A_2 and B_2 by $\hat{A}_2$ and $\hat{B}_2$ by using the standard formula. The estimated values of y_x for given x_i are computed from.

$$\hat{Y}_{x_i} = \hat{A}_2 + \hat{B}_2 X_i$$

We can also examine the significance of the estimated regression coefficients and do ANOVA for the fitted model.

MODEL A – 3 :-

Let us define $(EX)_{X_i}$ = Export of onion from India for the year x_i (where i=1,2,3,….13). Then we consider $(EX)_{X_i} = \alpha_3 \exp [\beta_3 X_i + u_{x_i}]$ as the regression model pertaining to export of onion in India, where α_3 and β_3 are the disturbance term for the year x_i.

We want to fit this model for the relevant data regarding export of onion in India. Let us consider log-linear transformation for the above model, then equation takes the form

$$\text{Ln } (EX)_{x_i} = \text{Ln } \alpha_3 + \beta_3 x_i + u_{x_i}$$

We define $\text{Ln}(EX)x_i = Y_{X_i}$

$\text{Ln } \alpha_3 = A_3$

$\beta_3 = B_3$ and

$u_{x_i} = Z_{X_i}$ then

$Y_{X_i} = A_3 + B_3 X_i + Z_{X_i}$ is the log – linear form for the above regression model. Under the usual assumptions we can obtain OLSE of A_3 and B_3 given by $\hat{A}_3$ and $\hat{B}_3$ by using the standard formulae. The estimated values of Y_x for given X_i are computed from $\hat{Y}_{x_i} = \hat{A}_3 + \hat{B}_3 X_i$

We can also examine the significance of the estimated regression coefficients as well as ANOVA for the filtted model.

MODEL B :- Multiple regression models pertaining to onion crop production, area under cultivation and export of onion from India.

MODEL B – 1 :- Relationship between Production (P) and Area under cultivation (A) of onion in India as per passage of time.

$$(\mathrm{P})_{x_i} = \sim_0 \left(\mathrm{A}_{\mathrm{x_i}}\right)^{\sim 1} \cdot \exp\left[\sim_x x_i + u_{x_i}\right]$$

Taking natural logarithm on both sides,

$\mathrm{Ln(P)}_{x_i} = \mathrm{Ln}\alpha_0 + \alpha_1 \mathrm{Ln(A)}_{x_i} + \alpha_2 X_i + u_{x_i}$

Here α_0 , α_1 and α_2 are the structural parameters of the model and OLS estimates of these parameters can be obtained and they can be tested for their statistical significance.

MODEL B – 2 :- Relationship between Export (EX) and Area under cultivation (A) of onion in India as per passage of time.

$$(\mathrm{EX})_{x_i} = {}_0\mathrm{A}^{\;1} \exp\left[{}_2 x_i + u_{x_i}\right]$$

Taking natural logarithm on both sides, then

$\mathrm{Ln(EX)}_{X_i} = \mathrm{Ln}\ \beta_0 + \beta_1\ \mathrm{Ln(A)}_{x_i} + \beta_2 X_i + u_{x_i}$

Here β_0, β_1 and β_2 are the structural parameters of the model and OLS estimates of these parameters can be obtained which can be tested for their statistical significance.

4. STATISTICAL APPLICATIONS AND CONCLUSIONS

Theoretical models discussed above can be applied as under.

Model A – 1

Area under cultivation of onion against time

$Y_x = -1.0225 + 0.08765X_i$

$t = (-20.173)^{**}\ (13.726)^{**}$

$R^2 = 0.9448,\ F = 188.39^{**}\ ,\ n = 13$

For the above regression model it is found that model is statistically significant. About 94.48% variation is explained by the model. Elasticity is 0.08765, which indicates that due to unit change in time, area under cultivation of onion in India, increases by about 8.76% per year.

Model A – 2

Crop production of onion against time

$Y_x = 1.2832 + 0.1268X_i$

$t = (19.406)^{**}\ (15.226)^{**}$

$R^2 = 0.9547\ ,\ F = 231.84^{**}\ ,\ n = 13$

Regression model for crop production of onion suggests that 95.47% variation is explained by the model. It seems that unit change in time accounts for 12.68% increment in crop production of onion in India.

Model A – 3

Export of onion against time

$$Y_x = 1.6411 + 0.1142X_i$$
$$t = (13.85)^{**} \quad (7.10)^{**}$$
$$R^2 = 0.8346\ ,\ F = 50.44^{**}\ ,\ n = 12$$

The regression model for export of onion is found to be statistically significant. On an average the logarithmic value of regressand export of Onion increases by 11.42% with unit change in regressor (i.e. time) and 83.46% variation is explained by the model.

After studying the above models, it appears that growth rate of crop production is higher than area under cultivation and export of onion from India.

Model B - 1

Crop production against area under cultivation and time

$$Ln(P)_{xi} = 2.3269 + 1.020\ Ln(A)_{xi} + 0.0373X_i$$
$$t = (8.741)^{**} \quad (3.973)^{**} \quad (1.613)$$
$$R^2 = 0.9824\ ,\ F = 279.64^{**}\ ,\ n = 13$$

Above model shows statistical significance of relationship amongst crop production of onion, area under cultivation of onion and time. All partial regression coefficients except time are found to be significant. The value of R^2 shows that about 98.24%.variation is explained by the model. Theoretically there should be positive relationship between area under cultivation and crop production of onion, which is justified by the above model as regression coefficient attached to area under cultivation is found to be positive.

Model B – 2

Export against area under cultivation and time

$$Ln(EX)_x = 0.9415 - 0.7284\ (A)x + 0.1803X_i$$
$$t = (1.3840) \quad (-1.044) \quad (2.763)^{*}$$
$$R^2 = 0.8524\ ,\ F = 25.99\ ,\ n = 12$$

Even though R^2 is high the above model can not be considered to be significant.

5. ACKNOWLEDGEMENT

We thank the referee for his review of this paper which has helped us in its revision.

6. REFERENCES :-

[1] **Allen RGD (1990),** Mathematical Analysis for Economists (Chapman & Hill)
[2] **Arthsankalan (2016),** Issue 557.
[3] **Damodar Gujratai (1994),** Econometrics (Prentice Hall Co.)
[4] **Draper & Smith (1992),** Econometric Analysis (John Wiley & Co).
[5] **James Murphy (1973),** Introductory Econometrics (Richard D. Irwin Inc.)
[6] **Johnston (1995),** Econometric Methods (John Wiley & Co.)
[7] **Potluri Rao And R.l.miller (1972),** Applied Econometrics (Prentice Hall Co.)

TECHNICAL NOTE

MAHALABONIS D² STATISTIC*

Devyani Chatterji**

ABSTRACT

The Mahalanobis Distance is a measure of the distance between a point P and a distribution D^*. This was introduced by P. C. Mahalanobis in 1936. It is a multidimensional generalisation of the idea of measuring how many standard deviations away P is from the mean of D^*. This distance is zero if P is the mean of D^* and grows as P moves away from the mean: along each principal component axis, it measures the number of standard deviations from P to the mean of D^*. If each of these axis is rescaled to have unit variance, then Mahalanobis distance corresponds to standard Euclidean distance in the transformed space. Mahalanobis distance is thus unitless and scale-invariant and takes into account the correlations of the data set. The purpose of this article is to give some idea and indicate for applications of this measure in general.

1. INTRODUCTION

The early works of Mahalanobis in Statistics and the development of the D^2 Statistic, in particular, may be accounted to four sources of motivation-

(1) The papers in Anthropometry published in early volumes of Biometrika.

(2) The influence of Prof. Brajendra Nath Seal.

(3) The data on the Anglo-Indians of Calcutta obtained through Mr. Annandale and

(4) His own curiosity on the race origins and race mixture, especially with regards to the inhabitants of Bengal.

These early papers provided statistical methods in order to analyse sets of anthropological data and specifically to assess similarity and dissimilarity between two populations.

After his Tripos examination in 1915, Mahalanobis met his tutor W. H. Macaulay in the college library, who showed him some new bound volumes of Biometrika. Mahalanobis was interested and purchased a set of Biometrika and brought it to India.

* Assistant Professor, Department of Statistics, S.M.Patel Institute of Commerce, GLS University, Ahmedabad, Gujrat, INDIA.

** We are highly indebted to wikipedia (the free encyclopedia) and relevant selected references to present this review article. (rcd. May'16 / frvd. May'16 / srvd. June'16)

The papers in these journals dealing with biological and anthropological data had an immediate influence on him. During 1920-21, he wrote papers on anthropometric constants for Bengal caste data, a noteon the criterion that two samples come from the same population and another note on statistical constants for an Anglo-Indian sample.

Besides, being actively interested in statistical problems relating to agriculture, meteorology and education, Mahalanobis was also deeply involved in the questions relating to racial mixture, racial origins and the assessment of group differences. During 1922-1936, he wrote 15 papers on these subjects leading to his key paper on the generalised distance in 1936. After 1936, he was deeply involved in concrete statistical problems relating to the

Socio-economic conditions in India and engaged in many sample surveys. He returned to the discriminatory problem in 1949 and wrote a joint paper with D. N. Majumdar and C. R. Rao. This revealed some of the important ideas of Mahalanobis. Somesh Dasgupta (1995) has given an extensive study on the evolution of D^2Statistic. Many authors have done notable research work relating to these concepts.

2. WHAT IS MAHALANOBIS DISTANCE?

In Statistics, we sometimes measure 'nearness' or 'farness' in terms of the scale of the data. Often 'scale' means 'standard deviation'.

For univariate data, we may say that an observation that is one standard deviation from the mean is closer to the mean than the observation that is three standard deviations away.

For many distributions, such as the normal distribution, this choice of scale also makes a statement about probability. Specifically, it is more likely to observe an observation that is aboutone standard deviationfrom the mean than it is to observe one that is several standard deviations away. This is because the probability density function is higher near the mean and nearly zero as you move many standard deviations away.

For normally distributed data, you can specify the distance from the mean by computing the so called z score. For a value x, z score of x is the quantity $z = \frac{x - «}{\pm}$ where « is the population mean and ± is the population standard deviation. This is a dimensionless quantity which can be interpreted as the number of standard deviations that x is away from the mean.

Thus, if χ is $N\left(«, \pm^2\right)$, z is $SN\left(0,1\right)$ and $z^2 = \left(\frac{x - «}{\pm}\right)^2$ has chi-square distribution with 1 d.f. ,we may write it as $z^2 = \left(x - «\right)'\left(\pm^2\right)^{-1}\left(x - «\right)$. Based upon this concept we can define now **Mahalanobis distance** as under:

The Mahalanobis distance of an observation $\underline{x}' = (x_1, x_2, ..., x_N)$ from a set of observations with mean $\underline{«}' = («_1, «_2, ..., «_N)$ and covariance matrix S is defined as

$$D_M(\underline{x}) = \sqrt{(\underline{x} - \underline{«})' S^{-1} (\underline{x} - \underline{«})}$$

Mahalanobis distance can also be defined as a dissimilarity measure between two random vectors $\underline{x}$ and $\underline{y}$ of the same distribution with the covariance matrix S as

$$d(\underline{x}, \underline{y}) = \sqrt{(\underline{x} - \underline{y})' S^{-1} (\underline{x} - \underline{y})}$$

If the covariance matrix is Identity matrix, Mahalanobis distance reduces to the Euclidean distance.

If the covariance matrix is diagonal then the resulting distance measure is called a **normalised Euclidean distance** given by $d(\underline{x}, \underline{y}) = \sqrt{\sum_{i=1}^{N} \frac{(x_i - y_i)^2}{S_i^2}}$ where S_i is the standard deviation of x_i and y_i over the sample set.

Mahalanobis distance is preserved under full rank linear transformations of the space spanned by the data. This means that if the data set has a non trivial null space, Mahalanobis distance can be computed after projecting the data (non-degenerately) down onto any space of appropriate dimension for the data.

Mahalanobis distance is closely related to leverage statistic h by means of the relation

$$D^2 = (N-1)\left(h - \frac{1}{N}\right)$$

The Mahalanobis distance accounts for the variance for each variable and the co-variance between variables. Geometrically it does this by transforming the data into standardised uncorrelated data and computing the ordinary Euclidean distance for the transformed data. In this way, it is like a univariate z score. It provides a way to measure distances that takes into account the scale of the data.

In order to use the Mahalanobis distance to classify a test point belonging to one of the N classes, one first estimates the covariance matrix of each class, usually based upon the samples known to belong to each class.

Then given a test sample, one computes the Mahalanobis distance to each class and classifies the test point as belonging to the class for which the Mahalanobis distance is minimal.

3. SOME CONCEPTS RELATED WITH MEASURE D

(1) **Measure D of caste-distance**

$$D = \frac{1}{p}\sum_{i=1}^{p}\frac{\left(m_i - m_i'\right)^2}{S_i^2}$$

where m_i and m_i' are the means of the ith measurement in the two groups respectively, S_i^2 is the pooled variance of the ith measurement and p is the number of measurements.

(2) **Karl Pearson's Coefficient of racial likeness**

$$C = \frac{1}{p}\sum_{i=1}^{p}\left(\frac{nn'}{n+n'}\right)\frac{\left(m_i - m_i'\right)^2}{S_i^2} - 1$$

where n and n' are the sample sizes in two groups.

This measure is influenced by sample sizes and fails to measure the degree of divergence between the two groups.

(3) To account for the above, later **Mahalanobis modified this measure D** and suggested the following measure given by

$$D' = D - \left(\frac{1}{n} + \frac{1}{n'}\right)$$

with its asymptotic variance

$$\pm^2_{D'} = \frac{4}{p}\left(\frac{n+n'}{nn'}\right)\overline{D} + \frac{2}{p}\left(\frac{n+n'}{nn'}\right)^2$$

where $\overline{D}$ is the mean of D'.

(4) **Positional Index**

Consider a group G in relation to a given list of n groups. Compute the distance D of the group G from each of the groups in the list and rank these measures in accordance of their values. Let C be a collection of m groups selected from the given list excluding G and r be the average of these rank values of the groups in C. Then the **Positional Index** of G for C is defined to be

$$P = \frac{n+1-2r}{n-m} \times 100$$

It is clear that P varies from -100 to +100 and a high value of P indicates that the group G is relatively closer to the given collection C of groups.

(5) **Generalised Distance**

Let there be two multivariate normal populations $N_p\left(\underline{\mu}_1, \Sigma\right)$ and $N_p\left(\underline{\mu}_2, \Sigma\right)$, then **Generalised distance** between the two populations was defined by Mahalanobis by

$$\Delta^2 = \frac{1}{p}\left(\underline{\mu}_1 - \underline{\mu}_2\right)' \Sigma^{-1} \left(\underline{\mu}_1 - \underline{\mu}_2\right)$$

The sample analog of Δ^2 when $\sum$ is known is given by

$$D_1^2 = \frac{1}{p}\left(\underline{\overline{X}}_1 - \underline{\overline{X}}_2\right)' \Sigma^{-1} \left(\underline{\overline{X}}_1 - \underline{\overline{X}}_2\right)$$

where $\underline{\overline{X}}_1$ and $\underline{\overline{X}}_2$ are the mean vectors obtained from random samples of sizes n_1 and n_2 taken from these two populations. Later, he modified the above D_1^2 to estimate Δ^2 unbiasedly by

$$D^2 = \frac{1}{p}\left(\underline{\overline{X}}_1 - \underline{\overline{X}}_2\right)' \Sigma^{-1} \left(\underline{\overline{X}}_1 - \underline{\overline{X}}_2\right) - \frac{2}{\bar{n}} \text{ where } \frac{2}{\bar{n}} = \frac{1}{n_1} + \frac{1}{n_2}$$

Mahalanobis derived the first four central moments of D^2 by the relations

$$E(D^2) = \Delta^2, \quad \mu_2(D^2) = \frac{8}{p\bar{n}}\left(\Delta^2 + \frac{1}{\bar{n}}\right), \quad \mu_3(D^2) = \frac{32}{p\bar{n}^2}\left[3\Delta^2 + \frac{1}{\bar{n}}\right],$$

$$\mu_4(D^2) = \frac{192}{p\bar{n}^2}\left[\left(\Delta^2 + \frac{2}{\bar{n}}\right)^2 + \frac{4}{p\bar{n}}\left(2\Delta^2 + \frac{1}{\bar{n}}\right)\right]$$

When both the mean vectors and dispersion matrix are unknown, the sample version of Δ^2 was suggested by

$$D_2^2 = \frac{1}{p}\left(\underline{\overline{X}}_1 - \underline{\overline{X}}_2\right)' S^{-1} \left(\underline{\overline{X}}_1 - \underline{\overline{X}}_2\right)$$

where S is the estimate of $\sum$. When $\sum$ is diagonal, this pooled estimate of $\sum$ is also diagonal matrix and correspondingly D_2^2 is modified.

.4. SOME APPLICATIONS

Mahalanobis distance is widely used in **cluster analysis** and **classification techniques**. It is closely related to **Hotelling's** T^2 **distribution** used for multivariate statistical testing and also to **Fisher's Discriminatory Analysis** which is used for supervised classification.

Mahalanobis distance and leverage are often used to **detect outliers**, especially in the development of linear regression models. A point that has a greater Mahalanobis distance from the rest of the sample population of points is said to have higher leverage since it has a greater influence on the slope or coefficients of the regression equation.

Mahalanobis distance is also used to determine **multivariate outliers**.

Some advanced applications can be listed as under:

(1) Mahalanobis distance for functional data with applications to classification (2013).

(2) Detecting an Anomalous traffic Attached Area based upon Entropy distribution and Mahalanobis distance (2014).

(3) Financial applications of the Mahalanobis Distance (2014) etc.

5. ACKNOWLEDGEMENTS

I thank Dr. B. B. Jani for his encouragement to prepare this review paper. I am highly indebted to the referee (anonymous) for reviewing and revising the earlier draft of this paper and for giving their comments and suggestions.

6. SELECTED REFERENCES:

[1] **Bayarjargal, D. and Cho, G. (2014).** Detecting an anomalous traffic attack area based on entropy distribution and Mahalanobis Distance. *International Journal ofSecurity and its Applications*, Vol. 8, No. 2, pp. 87-94.

[2] **Bose, R. C. (1936).**On the exact distribution and moment coefficients of the D^2 Statistic.*Sankhya*, 2, pp. 143-154.

[3] **Bose, R. C. and Roy, S. N. (1938).**The distribution of studentised D^2 Statistic. *Sankhya*, 4, pp. 19-38.

[4] **Das Gupta, S. (1995).** The Evolution of D^2 Statistic ofMahalanobis.

[5] **Flitzmoser, P., Riemann, C. and Garrett, R. G. (2003).**Multivariate outlier detection in detection in exploration geochemistry.*Technical Report TS-03-5*, Statistical Department, Vienna University of Technology, Austria.

[6] **Hotelling, H. (1931).**The generalisation of student's ratio.*The Annals Mathematical Statistics*, 2,pp. 360-378.

[7] **Joseph, E., Galeno, P. and Rosa, E. L. (2013).** The Mahalanobis distance for functional data with applications to classification. *Journal of Mathematical Statistics.* ar14: 1304.4786, Vol. 17.

[8] **Mahalanobis, P. C. (1927).** Analysis of race mixture in Bengal, *Journal and*

Proceedingsof the Asiatic Society of Bengal, 23, pp. 301-333.

[9] **Mahalanobis, P. C. (1928).**On the need of standardisation on the living.*Biometrika 20A*,pp. 1-31.

[10] **Mahalanobis, P. C. (1936).** On the Generalised Distance in Statistics. *Proceedings of National Institute of Science,* India, 2, pp. 49-55.

[11] **Mahalanobis, P. C., Bose, R. C. and Roy, S. N. (1937).**Normalisation of statistical variates and the use of rectangular co-ordinates in the theory of sampling. *Sankhya,* 3, pp. 1-34.

[12] **Mahalanobis, P. C. (1938).**On the distribution of Fisher's taxonomic coefficient. *Proceedins of Indian Science Congress,* Calcutta 52.

[13] **Mahalanobis, P. C., Majmudar, D. N. and Rao, C. R. (1949).** Anthropometric survey of the united provinces, 1941, a statistical study, *Sankhya,*9, pp. 90-324.

[14] **Mahalanobis, P. C. (1956).** Statistics must have a purpose. *Presidential Address, Third Pakistan Statistical Conference,* Lahore.

[15] **Ramsay, J. O. and Silverman, B. W. (2005).** Functional Data Analysis, 2nd Edition, *Springer,* New York.

[16] **Rao, C. R. (1947).**The problem of classification and distance between two populations.*Nature,* 159, 30.

[17] **Rao, C.R. (1948).** Tests of significance in multivariate analysis. *Biometrika,* 35, pp. 58-79.

[18] **Stockl, S. and Hanke, M. (2014).** Financial Applications of the Mahalanobis distance, *Applied Economics and Finance,* Vol. 1, No. 1, pp. 78-84.

JACOB (JAMES) BERNOULLI*

H. D. BUDHDHATTI**

(6 Jan. 1654 - 27 Dec. 1705)

Jacob Bernoulli was born in Basel, Switzerland. Following his father's wish, he studied theology and entered the ministry. But contrary to the desires of his parents, he also studied mathematics and astronomy. He traveled throughout Europe from 1676 to 1682, learning about the latest discoveries in mathematics and the sciences under leading figures of the time. This included the works of Hudde, Robert Boyle, and Robert Hooke. During this time he also produced a theory of comets.

Bernoulli returned to Switzerland and began teaching mechanics at the University in Basel from 1683. In 1684 he married Judith Stupanus; and they had two children. During this decade, he also began a fertile research career. His travels allowed him to establish correspondence with many leading mathematicians and scientists of his era, which he maintained throughout his life. During this time, he studied the new discoveries in mathematics, including Christiaan Huygens's De ratiociniis in aleae ludo, Descartes' Geometrie and Frans van Schooten's supplements of it. He also studied Isaac Barrow and John Wallis, leading to his interest in infinitesimal geometry. Apart from these, it was between 1684 and 1689 that many of the results that were to make up Ars Conjectandi were discovered.

He was appointed professor of mathematics at the University of Basel in 1687, remaining in this position for the rest of his life. By that time, he had begun tutoring his brother Johann Bernoulli on mathematical topics. The two brothers began to study the calculus as presented by Leibniz in his 1684 paper on the differential calculus in "Nova Methodus pro Maximis et Minimis" itemgue targentibus published in Acta Eruditorum. They also studied the publications of von Tschirnhaus. It must be understood that Leibniz's publications on the calculus were very obscure to mathematicians of that

* This article is adopted by net collection from wikipedia (the free encyclopedia). We are highly indebted for this assistance.

** Ex. CSO, Head, Statistics Dept., GSRTC, Ahmedabad. (rcd. April 2016/ rvd. June 2016)

time and the Bernoullis were the first to try to understand and apply Leibniz's theories. He remained as Professor of Mathematics at the University of Basel for more than 18 years.

Jecob's Doctoral advisers were **Nicholas Malebranche** and **Peter Werenfels**. He also had doctoral students whose names are : **Johanan Bernoulli, Jacob Hermann** and **Nicholaus I. Bernoulli**. He had greatly influenced Gottfried Leibnitz.

After making magnanimous performance by his pioneering works throughout his life at a very early age of 50 years only. Jacob Bernoulli died in 1705 at Basel, Switzerland. Bernoulli chose a figure of a logarithmic spiral and the motto Eadem mutata resurgo ("Changed and yet the same, I rise again") for his gravestone; the spiral executed by the stonemasons was, however, an Archimedean spiral, "Jacob Bernoulli wrote that the logarithmic spiral 'may be used as a symbol, either of fortitude and constancy in adversity, or of the human body, which after all its changes, even after death, will be restored to its exact and perfect self'."

PIONEERING WORKS

Jacob Bernoulli's first important contributions were a pamphlet on the parallels of logic and algebra published in 1685, work on probability in 1685 and geometry in 1687. His geometry result gave a construction to divide any triangle into four equal parts with two perpendicular lines.

By 1689 he had published important work on infinite series and published his law of large numbers in probability theory. Jacob Bernoulli published five treatises on infinite series between 1682 and 1704 The first two of these contained many results, such as the fundamental result that $\sum \frac{1}{n}$ diverges, which Bernoulli believed were new but they had actually been proved by Mengoli 40 years earlier. Bernoulli could not find a closed form for $\sum \frac{1}{n^2}$, but he did show that it converged to a finite limit less than 2. Euler was the first to find the sum of this series in 1737. Bernoulli also studied the exponential series which came out of examining compound interest.

In May 1690 in a paper published in Acta Eruditorum, Jacob Bernoulli showed that the problem of determining the isochrone is equivalent to solving a first-order nonlinear differential equation. The isochrone, or curve of constant descent, is the curve along which a particle will descend under gravity from any point to the bottom in exactly the same time, no matter what the starting point. It had been studied by Huygens in 1687 and Leibniz in 1689. After finding the differential equation, Bernoulli then solved it by what we now call separation of variables. Jacob Bernoulli's paper of 1690 is important for the history of calculus, since the term integral appears for the first time with its integration meaning. In 1696 Bernoulli solved the equation, now called the **Bernoulli differential equation**,

$$y' = p(x)y + q(x)y^n.$$

Jacob Bernoulli also discovered a general method to determine evolutes of a curve as the envelope of its circles of curvature. He also investigated caustic curves and in particular he studied these associated curves of the parabola, the logarithmic spiral and epicycloids around 1692. The lemniscate of Bernoulli was first conceived by Jacob Bernoulli in 1694. In 1695 he investigated the drawbridge problem which seeks the curve required so that a weight sliding along the cable always keeps the drawbridge balanced.

Jacob Bernoulli's most original work was Ars Conjectandi published in Basel in 1713, eight years after his death. The work was incomplete at the time of his death but it is still a work of the greatest significance in the theory of probability. In the book Bernoulli reviewed work of others on probability, in particular work by van Schooten, Leibniz, and Prestet. The Bernoulli numbers appear in the book in a discussion of the exponential series. Many examples are given on how much one would expect to win playing various games of chance. The term Bernoulli trial resulted from this work. There are interesting thoughts on what probability really is:

... probability as a measurable degree of certainty; necessity and chance; moral versus mathematical expectation; a priori an a posteriori probability; expectation of winning when players are divided according to dexterity; regard of all available arguments, their valuation, and their calculable evaluation; law of large numbers ...

Bernoulli was one of the most significant promoters of the formal methods of higher analysis. Astuteness and elegance are seldom found in his method of presentation and expression, but there is a maximum of integrity.

Discovery of the mathematical constant

Bernoulli discovered the constant e by studying a question about compound interest which required him to find the value of the following expression (which is in fact e)

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

One example is an account that starts with \$1.00 and pays 100 percent interest per year. If the interest is credited once, at the end of the year, the value is \$2.00; but if the interest is computed and added twice in the year, the \$1 is multiplied by 1.5 twice, yielding $\$1.00 \times 1.5^2 = \$2.25$. Compounding quarterly yields $\$1.00 \times 1.25^4 = \$2.4414...$, and compounding monthly yields $\$1.00 \times (1.0833...)^{12} = \$2.613035...$.

Bernoulli noticed that this sequence approaches a limit (the force of interest) for more and smaller compounding intervals. Compounding weekly yields \$2.692597..., while compounding daily yields \$2.714567..., just two cents more. Using n as the number of compounding intervals, with interest of 100%/n in each interval, the limit for large n is the number that Euler later named e; with continuous compounding, the account value will reach $2.7182818.... More generally, an account that starts at $1, and yields (1+R) dollars at simple interest, will yield e^R dollars with continuous compounding.

Jacob Bernoulli is very well known since ages for his contributions which may be mentioned briefly as under :

1. Bernoulli differential equation
2. Bernoulli numbers
3. Bernoulli's formula
4. Bernoulli polynomials
5. Bernoulli map
6. Bernoulli trial
7. Bernoulli Process
8. Bernoulli scheme
9. Bernoulli operator
10. Hidden Bernoulli model
11. Bernoulli sampling
12. Bernoulli (Binomial) distribution
13. Bernoulli random variable
14. Bernoulli's Golden theorem
15. Bernoulli's Ineauality
16. Lemniscate of Bernoulli etc.

He is regarded as the incomparable Mathematician. He was honoured for his dedicated work and writing by means of offering the most reputed membership of **Royal Academies of Paris and Berlin.**

We end up this brief biography by offering our salute (giving pranams) to this great dignitary who always remains immotal by his fantastic research works which spread enormous source of inspiration to all of us.

SELECTED REFERENCES

[1] Bernoulli Jacob, Tractatus de seriebus Infinitis (http://www.kubkou.se/pdf/mh/jacobb.pdf.) (pdf)

[2] **Hoffman, J.E. (1970-80)** *Bernoulli, Jakob (Jacaues) Dictionary of scientific Biography 2, New Yorks,* Charles Scribner's Sons P. 46-51

[3] **I. Schneider (2005)** Jacob Bernoullie, Ars Conjectandi (1713), Landmark writings in western Mathematics 1640-1940, Elsevier, P. 88-104

[4] **Livio, Mario (2002)** The golden ratio : the story of phil. the world's most astonishing number, Brodway Books (ISBN 970-0-7679-0815-3)

[5] **O'Connor, John J., Robertson, Edmund F. Mac** Tutor History of Mathematics archive Uni. of Andrews.

BOOK REVIEW

Title : Principals of Econometrics

Authors : Neeraj R. Hatekar

Publisher : Sage Publication India Pvt. Ltd., New Delhi (2016)

Price : Rs. 525/-, Page 439

It is easier to make difficult, something easy, but...
it is difficult to make easy, something difficult.

The above fact is reflected in this textbook. Author is professor of Econometrics at department of Economics, Mumbai University. This textbook is designed for beginners, especially those who believe that study of economics is incomplete without understanding econometrics.

According to Prof. G. S. Madal a"The application of statistical and mathematical methods is for the analysis of economic data, with the purpose of giving empirical content to economic theories and thus verifying them or refusing them."

As stated before this textbook is designed for the beginners with the purpose of acquainting them with the subject eventhough they may not have little or no background in mathematics and stastistics. As the reader goes through this book, he or she feels that the author is successful to achieve his goal.

Moreover, this book is augmented with one or more features. Every numerical example is equipped with R language. Today one can not think of econometrics without software application. R is a widely popular programming language that is free to use.

The entire text consists of six chapters. First three chapters discuss elementary statistical methods in a very lucid manner which are prereuisite for any student or researcher before doing his work with econometric analysis. Here the speciality is about the approach (e.g. standard deviation is compared to distance etc.) we know that estimation and testing of hypothesis is very widely used in many desciplines. However very few persons can understand and interprete precisely correctly its spirit. This text enables its reader to have some clear idea of hypothesis testing.

In econometric analysis manytimes researchers encounter with multicolinearity, heteroscadasticity, authuocorrelation and specification issues. Here you will find sufficient basic discussion about them.

In my view, this text is a welcome move by its learned author and definitely it serves the purpose for which it is meant for.

Date : 10/06/2016

Dr. H. M. Dixit
Head, Statistics Dept.,
Arts and Commerce College, PILWAI (N.G.)

SV NEWS LETTER

K. Muralidharan*

Congratulations to Prof. *Dr. D. K. Ghosh*

Professor D. K. Ghosh completed his Graduation and Post Graduation in Statistics with first class from Bhagalpur University, Bhagalpur, Bihar. He worked for his **Ph.D. thesis** under **Professor M. N. Das** specialized in Design of experiments form 1981 to 1984 at Indian Statistical Institute, New Delhi. **Dr. Ghosh is now nominated for Fellow of Royal Statistical Society (FRSS) in 2016 and Fellow of Gujarat Science Congress in 2010.**

Dr. Ghosh holds the following positions**: UGC BSR Faculty Fellow; Professor and Head, Dep. of Statistics, Saurashtra University, Rajkot, Gujarat, India;** Coordinator: Post Graduate Diploma in Hospital Management; In charge Professor and Head: Dep. of Human Right and International Humanitarian Law; Director: Internal Quality assurance Cell(IQAC); Adjunct Faculty: Banasthali University, Rajasthan.

He possesses the following academic experiences: Research Investigator, Bhagalpur University (I.C.S.S.R. project), Bhagalpur, from May, 1976 to October 1976; Senior Scientific Assistant Tea Research Institute, (C.S.I.R), Jorhat, Assam, from October, 1978 to June, 1985; Research fellow, C.M.F.R.I., (I.C.A.R. project) Cochin, from November 1976 to September 1978; Lecturer, Department of Statistics, Saurashtra University, Rajkot, from July 1985 to October 1986; Reader, Department of Statistics, Saurashtra University, Rajkot, from November 1986 to October 1994; Professor and Head, Department of Statistics, Saurashtra University, Rajkot, from November 1994 to November 2013; Coordinator: Post Graduate Diploma in Hospital management from 2006 to Nov 2013.

He has the following administrative experience: Head of the Department of Statistics, Saurashtra University, Rajkot; Coordinator Post Graduate Diploma in Hospital Management;

Senate, Syndicate and Ex Academic Council Members of Saurashtra University;

• Professor and Head, Dept. of Statistics M. S. University, Vadodara-380 002 India)

Four years as zonal advisory board member of Life Insurance Corporation of India; Advisory Board Committee of Employment bureau, Saurashtra University Campus, Rajkot; Chairman of Ethic society of Rajkot cancer Hospital and Research; Member of Ethic society of Gokul Hospital and Research, Rajkot for more than five years; Member of Ethic society of Cancer Hospital and Research, Rajkot for more than ten years; Center coordinator for assessment works of B.A./B.Sc./M.A./M.Sc./B.E. etc. for more than 10 years; Organization of the campus interview of T.C.S., Kendal Pharmacy, SPSS and other Pharmacy industries for more than five years; Member of Board of studies in many Universities; Referred and referring research papers and books for National and International journals; Selection committee members in several universities; paper setters and examiners in universities, institutes and other places; Evaluating M. Phil dissertation and Ph.D. thesis of many Universities; Advisory board member of Income tax Office of Saurashtra Region since 2012.

His Research interest areas are **Design and analysis of experiments, Construction of various designs, Optimal Designs, Statistical Computing, Operations research, Estimation theory, Biostatistics and Sampling theory etc.**

During his academic career, he has taught the following topics: Analysis of variance, Design of experiments, Construction of design, robust design, Multivariate statistics, Regression analysis, Linear model, Sample survey, mathematical statistics, Statistical Inference, Computer organization, Computer programming language LIKE: BASIC, FORTRAN, COBOL, PASCAL. C and C^{++} , Operations research, Econometrics, Quantitative techniques, computer based optimization techniques etc.

Dr. Ghosh has published more than 105 research papers in Design of experiments, Estimation theory, Sampling theory, Operations research and Biostatistics. Most of them are published in International and reputed national journals.

Dr. Ghosh has supervised **one M. Phil. and thirty two Ph.D.** students in Statistics as a guiding teacher. He published three technical reports. He is author of three books **Distribution Theory, Factorial Experiments, and Bio Statistics** published by Saurashtra University Rajkot. One chapter each in different topics is published in following three different books: (a) Statistical Methods and Practices: Recent advantages, Editor: N. Balakrishna etc. Narosha Publication; (b) Recent advantages in Mating Designs, Editor: L. S. Kaushik and R. C. Hasija, Dhanpatrai Publication; (c) Government and Technological Innovation for Policy Design and Precision, IGI publication, USA.

He has received the following awards:(i) CSIR's, JRF and SRF, (ii) Hariom

Ashram Award of S. P. University, V. V. Nagar in 1995, (iii) John C. Pace visiting scholar at Dept. of Math- Statistics, The West Florida University Pensacola, USA, (iv) Dollarrai Mankad award for Excellence in Research for 2009-2010 and 2010 - 2011,(v) Best awarded paper by Gujarat Science Congress Conference in 1993, 1995 and 2004.

He has visited following International Universities for collaborative research works: The University of west Florida in 1996, 2000, 2003, 2004, 2012; University of Manitoba, Winnipeg in 2003, 2004, 2005, 2009; University of Ohio, Columbia in 2005; University of Chicago, USA in 2009; St. Louis University, St. Louis, USA in 2014; University of Marquette, Milwaukee, USA in 2014 and Central Florida University, Orlando. USA in 2014. He did collaborative research works with professor Kageyama of Hiroshima University, Japan, Professor Bagui of West Florida University, USA, Professor Mondal of Manitoba University, Canada, Professor Ashish Das of IIT Mumbai, and has published good number of research papers with them.

He participated in the following International conferences abroad: Conference at the University of North Carolina, Chapel Hill, Sept. 1996, One day seminar at The University of South Alabama, Mobile, AL, 1996; One day seminar at Arizona State University, Tempe, Arizona, 1996, Four days seminar at The University of west Florida, Pensacola, FL, Nov. 2000, Six great Lake symposium, Western Michigan University, Kalamazoo, Nov. 2000, Two days seminar at The University of west Florida, Pensacola, FL, Nov. 2004,Two day seminar at University of Manitoba, Winnipeg, Canada, May, 2003, Four day workshop at University of Manitoba, Winnipeg, Canada, 4th to 7th August 2004, One day seminar at University of Manitoba, Winnipeg, Canada, September 2005, Five day seminar at Institute of Polytechnique, Portugal, Lisbon , 27th July to 31st July, 2010 etc.

He has reviewed the articles for the following journals: Gujarat Statistical Review; Sankhya Vignan, Assam Statistical Review; Pure and applied Science; Opsearch; Institute of Mathematical Statistics; Utilitas Mathematica; International Jour. of Management and Systems; Statistical methods; Journal of Indian Statistical association; International Mathematical Society ;Australian Journal of statistics; Society of Statistics, Computer and Applications; Jour. Of Applied Statistics; J. of Statistical planning and Inference; Communication in Statistics, Theo – Meth; International J. of Agricultural and Statistical Sciences; Journal of Modern Applied Statistical Methods; Australian and Newzeland journal of Statistics etc.

He is Editorial board member in the following journals: Pure and applied Science (Statistics section only); Assam Statistical Review: Associate Editor; Journal of Statistics, Computer and Applications: Ex Associate Editor; Journal of Tissue culture: Ex Associate Editor; Reviewer of American Mathematical Society; Journal of ISPS(Design of Experiments section); Journal Of Rajasthan Statistical Association: Associate Editor; Indian Jour. of Statistics and Application: Associate Editor; International J. Agricultural and Statistical Sciences: Associate Editor and Journal of Applied sciences Sankhya Vignan etc.

He works as Executive committee member in the following societies: Vice President: Gujarat Statistical Association.; Indian Science Congress from 1996 - 1997, and 2002 – 2003; Indian Society of Agricultural Statistics; Vice-President: Society of Statistics, Computer and applications; General Secretary: Indian Society of Probability and Statistics (2010 to 2013).

He is professional member in the following association: Life member of Indian Society of Agricultural Statistics; Life member of Indian Society of Probability and Statistics; Life member of Gujarat Statistical Society; Life member of Indian Statistical Association Pune; Life member of Indian science congress; Life member of Sankhya; Annual member of I.M.S. Bulletin, U.S.A; Life member of Calcutta Statistical Bulletin, Kolkata; Life member of Societies of Statistics Computer and applications; Indian Society of Medical Statistics and Royal Statistical Society.

He has completed the following two major projects: (1) **UGC major project** "Preparation of Table of PBIB design with two associate classes" in 1996 and (2) **DST Major project** "A study of Survival Analysis of Breast Cancer and its Statistical analysis" in 2016.

Dr. Ghosh has presented many research papers, delivered 50 invited talks and chaired 31 sessions at the National and International conferences. He also delivered more than 50 invited lectures under UGC visiting professor and other schemes at various universities in India and abroad.

We are extremely happy by his above achievements and express our hearty congratulations and best wishes to Prof. Dr. D. K. Ghosh.

READERS FORUM

A. M. PATEL*

FEEDBACK

- **Dr. D. K. Ghosh**

  It is splendid team work with unbiased review system for articles. Quick response is wlecome. Invite more specialised articles for publication.
- **Dr. P. S. Pandya**

  Very good compilation and inspiring articles. There is a need to establish linkage between theoreticians and practitioners. Huge data are always available with DES and researchers can be benefited by them.
- **S. D. Parikh**

  SV Journal is really good and I get all issues regularly. One complaint is about the publishing authority - GSA. There is no communication nor any programmes since long. I am very sorry to report this, but earlier for many years I have worked with GSA, that is why I feel myself depressed by this event.
- **T. M. Durairajan**

  Thanks for updating membership proforma. This was needed since long. SV Journal has consistently kept rapport with the readers. Keep it up.
- **Dr. P. H. Thakar**

  I am extremely happy to observe that SV Journal is radipdly getting momentum. I am quite confident that this growth will continue further to establish rapport between theoreticians and users of statistics.
- **Prof. S. B. Vora (Ex. V.C., G.U.)**

  Having experience for about 49 years as Editor of the Journal *"Arth Sankalan"* I know how difficult it is to publish an academic journal regularly and punctually.I must congratulate the editor and his team for their efforts. Keep it up. Best wishes.
- **Dr. B. H. Prajapati**

  I have observed that mostly theoretical articles are published. There is definitely

* Rtd. Principal, H. K. Commerce College, Ahmedabad and Ex. Secretary, Gujarat Vidyasabha and Brahmchariwadi Trust, Ahmedabad.

a need for review articles as well as informative articles based upon applied research work. Authors can be awarded and encouraged for better work. Seminars, workshops and conferences should be organised by GSA, the publishing authority of SV Journal. Since long, there is no such activity. Some special lectures can be organised by GSA by inviting eminent statisticians and academicians from IASRI / ISI / etc.

- **Dr. Hemal Pandya**
  Since Last issue of SV (Dec. 2015) there is a welcome effort of publishing articles under the caption statistics and management. Article by Dr. A. C. Brahmbhatt was extremely good indeed. I suggest to bring in some more such articles in the journal and also the research papers connected with applications. It is definitely good team work.
- **Dr. K. S. Shah**
  Please invite more informative articles as all do not read research papers. Articles / papers in Gujarati language can be taken for publishing as this is journal of GSA.
- **Dr. Pradeep Jhala**
  I have been a life member of GSA since its inception. I am very happy to note that the annual conferences of GSA organised earlier are very educative, provide latest development in the subject. We get a very prestigious platform to present and learn manythings in the subject. This journal SV published by GSA has a reputation and there is unbiased review system with indeed good team work.
- **Dr. Vina Vani**
  I am proud to feel that SV is a jounal published by GSA from Stat. Dept. G.U. It is my department where I have worked earlier, and then my growth started. Excellent team work. My best wishes to Editor and his colleagues.
- **Dr. Bhushan Bhatt**
  Since long my link with Dr. B. B. Jani (Editor) has been maintained through this journal SV. It covers many areas such as Management, Economics, Computer Applications etc. along with Statisties. My best wishes to the team.

INSTRUCTIONS TO AUTHORS

Editorial board invites research articles, brief summary of research project reports, review articles, informative articles, research and project reports, research notes, class room notes, statistical querries and other problems of interest as well as any relevant informations that can significantly highlight the applications part pertaining to statistics subject.

Accordingly the editorial board welcomes articles in the fields of agricultural and industrial statistics, operations research and operations research management, economics and econometrics, theoretical statistics, SQC, Information and coding theory, statistical planning, Biometrics, computer programming applications, environmental statistics, demography etc.

TWO hard copies of the manuscript should be sent to **Dr. B. B. Jani, Editor, Sankhya Vignan,** at B/14, Bansidhar Apartments, Mirambica School Road, Naranpura, Ahmedabad-380013. (India)

Please also send your article by sending email of Sankhya Vignan **svgsa2015@gmail.com** for quick action and response.

The manuscript should be typed on bond paper in double space with sufficient margins on all the sides of the paper. The title of the article should include name(s) and address of the authors. All references should be listed at the end of the article and should be numbered in alphabetical order of authors. It should be produced in an appropriate manner indicating the years of publication, details of publications etc and should be as concise and compact as possible.

Please note that all submitted articles are reviewed before its publication.

- Inquiries regarding subscription and other particulars regarding **Sankhya Vignan** should be addressed to the **Editorial Secretary**. You may also mail to **drjayesh.purohit@gmail.com**
- **GSA membership** fees and other querried should be address to **Secretary, GSA, C/o. Statistics Dept., Gujarat University, Ahmedabad 380009 (INDIA)**
- Each author will receive a copy of the published journal. The first named author will receive 10 copies of offprints.
- All payment can be made either by cash or M.O. or Bank draft in favour of Gujarat Statistical Association, payable at Ahmedabad.

(P.T.O.)

Note : In order to meet with the administrative, printing and postal expences, it is decided by the editorial board to accept advertisements from interested resources. Normal rates of advertisements are as under :

Last full cover page	**Rs. 7,000/-**
Last inner coverpage	**Rs. 5,000/-**
Second Colour full page	**Rs. 5,000/-**
Inner Colour half page	**Rs. 3,000/-**
Inner full page	**Rs. 2,000/-**
Inner half page	**Rs. 1,000/-**
Inner quarter page	**Rs. 500/-**

Statistics Day : 29th June 2016

P.C. MAHALANOBIS

NOTES

Gujarat Statistical Association

Established : 1969

[Registered under Public Trust Act of 1950 (Bombay)]

R. No. E2502 A'bad-1974

The objective of the association is primarily to promote statistical ideas in pure and applied fields in the form of study, teaching and research in statistics.

The membership of GSA consists of Life / Institutional / ordinary members.

Membership	Fees	
	In India	Abroad
Insitutional Member (for 3 years)	₹ 2,000/-	US $ 500
Life Member	₹ 1,500/-	US $ 300
Ordinary Member (p.a.)	₹ 500/-	US $ 50

Annual Subscription for Gujarat Stastistical Review / Sankhya Vignan for individuals, Institutions and library is ₹ 500 (in India), US $ 50 (Abroad)

JACOB BERNOULLI

Jacob Bernoulli (also known as **James or Jacques** 6 January 1654 - 27 December 1705) was one of the most prominent mathematicians in the Bernoulli family. He was an early proponent of Leibnizian Calculus and had sided Leibniz during the Leibniz-Newton Calculus Controversy. He is known for his numerous contributions to calculus and along with his brother Johnan, was one of the founders of calculus of variations.

However his most famous important contribution was in the field of probability, where the derived the first version of the **Law of Large numbers** in his work **Ars Confectandi**. He is also famous for discovering the number $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$

Several researches are named after him such as (1) Bernoulli's diff. equation (2) Bernoulli numbers (3) Bernoulli's formula (4) Bernoulli Polynomials (5) Bernoulli's inequality (6) Bernoulli map (7) Bernoulli trial (8) Bernoulli Process (9) Bernoulli Scheme (10) Bernoulli Operator (11) Hidden Bernoulli model (12) Bernoulli sampling (13) Bernoulli distribution (14) Bernoulli random variable (15) Bernoulli's Golden Theorem (16) Lamniscate of Bernoulli etc.

He was a member of Royal Academies of Paris and Berlin.

***(Brief Biographical sketch is given inside the journal)**

This page is specially donated by Prof. Shailesh Teredesai (Ex. Head), Statistics Dept., S. M. Patel Insitute of Commerce, GLS, Ahmedabad-380 009.

Printed Matter **BOOK-POST**

(Journal of GSA, Ahmedabad)

To,

From :

Gujarat Statistical Association,

C/o. Statistics Dept. Gujarat University, Ahmedabad-380 009. (INDIA)