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RESEARCH ARTICLES

CONSTRUCTION OF PARTIALLY EFFICIENCY BALANCED DESIGNS

D. K. Ghosh[1] and Sangeeta Ahuja[2]

ABSTRACT

Puri and Nigam (1977) introduced partially Efficiency balanced designs which may be useful for bioassays and factorial experiments. Das (1958) obtained re-inforced incomplete block design by including a certain number of additional treatments in each block of the original incomplete block design. In the present investigation, different methods of constructing partially efficiency balanced design have been discussed by re-enforcing one extra treatment and augmenting blocks.

Keywords: Lattice design, Efficiency Balanced Design, Partially Efficiency Balanced Design, Reinforced Design and Augmenting Blocks.

1. Introduction

Balanced incomplete block designs, Lattice designs and partially balanced incomplete block designs were introduced by Yates (1936) and Bose and Nair (1939). But the drawbacks of these designs are the following (i) BIB designs contains large number of replications and is available only with equi-block sizes and equi-replicated binary designs, (ii) Lattice design is available only for square and cubic treatments, and (iii) PBIB design is not available for unequal block sizes and unequal replications.

To overcome these problems, Calinski (1971), Puri and Nigam (1975a), Willam (1975) introduced efficiency balanced design. Further Puri and Nigam (1977) introduced another class of design known as Partially Efficiency Balanced (PEB) design with m-efficiency classes. The efficiency factor, as referred by Puri and Nigam (1977), associated with every contrasts of i^{th} class is $(1-\mu_i)$ for PEB design with m – associate class. Where μ_i (i= 1, 2, . . . , m) are the eigen values of $\underline{M}_0$ with multiplicities $\rho_i(\sum p_i = v-1)$ and $\underline{M}_0$ is defined as

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$$\underline{M}_0 = \underline{r}^{-\delta}\underline{P} - \underline{1}\ \frac{\underline{r}'}{N} \text{ and } \underline{P} = N\ \underline{K}^{-\delta}\underline{N}'$$

In the present work, we have shown that if the ratio of the variances of the estimates of the treatment contrast corresponding to completely randomized design is E_i (i = 1,2) for all the treatment effects t_i and t_j for $i \neq j$ = 1 to v, then the design is said to be partially efficiency balanced design. A new method for constructing such designs along with their analysis has also been carried out.

1.1 Definition: An arrangement of v treatments in to b blocks of sizes K_1, K_2, . . K_b. ., is said to be PEB design if

i. The i^{th} treatment, (i= 1, 2, . . . , v) is replicated r_i times,

ii. The ratio of the variances of the estimates of any treatment contrast for some of the treatments obtainable through two designs is constant, say, E_1.

iii. Again the ratio of the variances of the estimates of any treatment contrast for some other treatments obtainable through two designs is another constant, say, , provided other design is completely randomized design having same replication sizes. Where and are efficiency factors of the PEB design.

2. Method of Construction

In this investigation we have discussed three different cases for constructing the partially efficiency balanced designs using Balanced incomplete block designs. These are shown below.

Case 2.1 Partially efficiency balanced design with (v+1) treatments.

Consider BIB designs with parameters v, b, r, k. Now reinforcing one extra treatment, say, t_j in each of the b blocks of the BIB design with replication sizes r_i = r (i = 1, 2, ….., v) and r_j = b, we obtain Partially efficiency balanced design with (v+1) treatments with parameters v^1 = v+1, b^1 = b, $r^1 = r_i$ = r for i = 1, 2, . . . , v; r_{v+1} =b, k^1 = k+1 and E_1 = (r + λv) / r(k + 1), E_2 = (b + r) (r + λv) / b(k + 1) (r + λ).

Example 2.1 Consider a BIB design with parameters v=4, b=6, r=3, k=2, and λ=1. The blocks of this design are the following (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4).

Now reinforce the above design by taking one extra treatment, say, 5 in each of the 6 block. The resulting design is a PEB design with parameters v= 5, b=6, = 3 (i=1, 2, 3 ,4), $r_5 = 6$, $k = 3$ and $E_1 = \frac{7}{5}$, $E_2 = \frac{7}{8}$. The blocks of the resulting PEB design are (1, 2, 5), (1, 3, 5), (1, 4, 5), (2, 3, 5), (2, 4, 5), (3, 4, 5).

2.1. Analysis

In two way classification data with n_{ij} observations in the (i, j)th cell (i=1, 2, ..., t, j= 1, 2, ..., b), the reduced normal equation for estimating the treatment effect t_i after eliminating the block effects for two factors, say, treatments and blocks come out as

$$C_{ii}t_i - \sum_{m \neq i} C_{im} t_m = Q_i$$

Where $C_{ii} = n_i - \sum n_{ij}^2 / n_j$; $n_i = \sum_j n_{ij}$; $C_{im} = \sum n_{ij} n_{mj} / n_j$;

$$Q_i = T_i - \sum n_{ij} B_j / n_j$$

Where's Q_i are adjusted treatment totals, T_i is the total of the observations of the i^{th} treatment and B_j is the total of the observations in the j^{th} block.

In this design the reduced normal equations for estimating the treatment effects and t_i and t_j are $\left(r - \frac{r}{k+1} + \frac{\lambda}{k+1}\right)t_i - \frac{\lambda}{k+1}\left(\sum_{m=1}^{v} t_m\right) - \frac{r}{k+1} t_j = Q_i$ (2.1)

and $\left(b - \frac{b}{k+1}\right)t_j - \frac{r}{k+1}\left(\sum_{m=1}^{v} t_m\right) = Q_j$ where i= 1, 2, 3, ... , v; s= v+1 (2.2)

Taking the restriction that $t_i = 0$ and after simplifying the reduced normal equations (2.1) and (2.2), we obtain the estimate of the treatment effects t_i and t_j which are

$$\left(r - \frac{r}{k+1} + \frac{\lambda}{k+1}\right)t_i + \frac{\lambda - r}{k+1} t_j = Q_i$$

and $\left(b - \frac{b}{k+1} + \frac{r}{k+1}\right)t_j = Q_j$

The solution of the estimates of the treatment effects t_i and t_j after simplification, are estimated as $\hat{t}_i = \frac{k+1}{r+\lambda v}\left(Q_i + \frac{r-\lambda}{r(v+1)} Q_j\right)$ (2.3)

and $\hat{t}_j = \frac{k+1}{r+\lambda v} Q_j$ (2.4)

As we know that the sum of squares due to treatments adjusted for block is $\sum_{i=1}^{v+1} \hat{t}_i Q_i$ and hence on putting the solution of the estimates of $\hat{t}_i$ and $\hat{t}_j$, we get the sum of the squares due to adjusted treatments. Variances of the difference of the treatment effects $\hat{t}_i$ and $\hat{t}_j$ are obtained as

$$\text{Var}\left(\hat{t}_i - \hat{t}_m\right) = 2(k+1)\sigma^2 / (r + \lambda v) \qquad (2.5)$$

$$\text{Var}\left(\hat{t}_i - \hat{t}_j\right) = (k+1)(r+\lambda)\sigma^2 / r\,(r + \lambda v). \qquad (2.6)$$

Taking the ratio of the variances of the Partially efficiency block designs corresponding to its randomized block design we obtain the efficiency factors of PEB design E_1 and E_2 where

$E_1 = (r + \lambda v) / r(k+1)$ and $E_2 = (b + r)(r + \lambda v) / b\,(k+1)(r+\lambda)$.

Case 2.2 Partially efficiency balanced design with (v+1) treatments and augmenting blocks.

A BIB design with parameter v, b, r, k, and λ when reinforced by an extra treatment, say t_i in each of b blocks and further augmented by any number ($n \geq 0$) of extra blocks provided each block contains once each of the v treatments in the original BIB design, then the resulting design becomes PEB design with parameters $v' = v$, $b' = b+n$, $r' = r_i = (r+n)$ for i= 1, 2, ..., v, $k' = k_j = k+1$ for j = 1,2,...., b

= v otherwise,

with $E_1 = \dfrac{r + \lambda v + n(k+1)}{(k+1)(r+n)}$ and $E_2 = \dfrac{r(v+1)(b+r+n)[r+\lambda v+n(k+1)]}{b(k+1)(r+n)[(v+1)(r+\lambda)+n(k+1)]}$

If n is taken as zero then Case 2.1 becomes the particular case of Case 2.2.

Example 2.2 Consider a BIB design with parameters v=4, b=6, r=3, k=2, and λ=1. The blocks of the BIB designs are (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4).

Now reinforce the above design by taking one extra treatment, say, 5 in each of 6 blocks. Further taking n=1, That is, augment one more block, containing four treatments in the block. In this way using case 2.2, the resulting design becomes a PEB design with parameters v=5, b=7, $r_i = 3$ (i=1, 2, 3, 4), $r_5 = 6$, $k_j = 3$ (j= 1, 2, 3, 4, 5, 6) and $k_7 = 4$ with $E_1 = 5/6$, $E_2 = 50/57$. The blocks of the PEB design are (1, 2, 5), (1, 3, 5), (1, 4, 5), (2, 3, 5), (2, 4, 5), (3, 4, 5), (1, 2, 3, 4).

Case 2.3 PEB design with v treatments with augmenting blocks provided any one treatment is repeated p times in the augmented blocks.

Consider a BIB design with parameters v, b, r, k and λ. Let t augment n>0 extra blocks such that each of the n blocks contain the first treatment, say, t_1, p times (p>0) and each of the other treatment once, then the resulting design becomes PEB design of two associate classes.

Here several series of BIB design can be used for obtaining PEB designs. Some of the series of BIB designs are carried out here. The remaining is left for the readers.

Series I: Let us consider BIB design with parameters $v=b=4\lambda+3$, $r=k=2\lambda+1$ and λ. This becomes PEB design with parameters $v' = 4\lambda+3$, $b' = 4\lambda+3+n$, $r' = r_i = (2\lambda+1+n)$ for $i= 2, 3, \ldots, v$ and $r_1 = (2\lambda+1+np)$, $k = k_j = (2\lambda+1)$ for $j=1, 2, \ldots, b$ and $k_{b+1} = (v - 1 + p)$ using case 2.3.

Example 2.3 Following the case 2.3, we can always construct a PEB design with parameters $v' = 11$, $b'=12$, $r'= r_i = 6$ ($i= 2, 3, \ldots, 11$), $r_1 = 8$, $k' = k_j = 5$ ($j=1, 2, \ldots, 11$), $k_{12} = 13$ from a BIB design with parameters $v=b=11$, $r=k=5$ and $\lambda=2$,. The blocks of PEB design are

(1, 3, 4, 5, 9), (2, 4, 5, 6, 10), (3, 5, 6, 7, 11), (4, 6, 7, 8, 1), (5, 7, 8, 9 2), (6, 8, 9, 10, 3), (7, 9, 10, 11, 4), (8, 10, 11, 1, 5), (9, 11, 1, 2, 6), (10, 1, 2, 3, 7), (11, 2, 3, 4, 8), (1, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11).

Efficiency factors of this design are $E_1 = 287/320$ and $E_2 = 9/10$.

Remarks: It can be seen that efficiency factor are very high.

Series II: Let us construct BIB design with parameters $v=r^2$, $b=t^2+t$, $r=t+1$, $k=t$ and $\lambda=1$. This gives PEB design when it is augmented by n extra blocks such that first treatment t_1 occurs p times in each of the n blocks and other treatments once.

For example, $v=9$, $b=12$, $r=k$, $k=3$, $\lambda=1$ along with $n=1$, $p=3$ gives a PEB design with $E_1 = 96/119$ and $E_2 = 4/5$. The blocks of the PEB design are

(1, 2, 3), (4, 5, 6), (7, 8, 9), (1, 4, 7), (2, 5, 8), (3, 6, 9), (1, 6, 8), (2, 4, 9), (3, 5, 7), (1, 5, 9), (2, 6, 7), (3, 4, 8), (1, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9).

Remarks: In this design if p=4, the resulting design becomes Efficiency balanced (EB) design with E=4/5.

Series III: We can always construct a PEB design from BIB design having parameters v, $b = \frac{v(v-1)}{2}$, $r = v-1$, $k = 2$ and $\lambda = 1$.

Similarly, every series of BIB design give PEB design when augmented by n>0 blocks along with p>0.

We can have the analysis of cases 2.2 and 2.3 similar to that of the analysis of case 2.1.

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THE BEST THINGS ABOUT BEING A STATISTICIAN IS THAT YOU GET TO PLAY IN EVERYBODY'S BACKYARD.

- Tyler

EPQ MODEL FOR IMPERFECT QUALITY ITEMS UNDER CONSTANT DEMAND RATE AND TIME VARYING IHC

U. B. Gothi[1], Devyani A. Chatterji[2]

ABSTRACT

Imperfect quality items are unavoidable in any inventory system due to imperfect production process, natural disasters, damages and many other such reasons. Keeping these facts in view, this paper investigates an Economic Production Quantity (EPQ) model with imperfect quality items and constant defective rate. The life span of the produced item is divided into three stages: preliminary stage, escalation stage and declining stage. Here an inventory model is derived with two different production rates P_1 and P_2 ($>P_1$). Inventory holding cost is constant in the first case and varying over time in the second case. Shortages do not occur. Mathematical model is developed which is presented by an illustration along with its sensitivity analysis.

1. Introduction

The life span of a product begins with the initial product requirement and ends with its extraction from the market of both the product and its support. A new product is first developed and then launched in the market. Once the introduction is successful, an escalation period follows with the wider awareness of the product and increasing sales. The product enters maturity when sales stop growing and demand stabilizes. The life span of a product may be as short as few months or it can be a century or more depending upon the category of the product. During the preliminary stage the company is likely to incur additional costs i.e. advertising cost associated with the initial distribution of the product. These costs coupled with a low sales volume usually make the preliminary stage a period of some loss. Actually, the prime objective of this stage is to create a market and construct the basic demand for the product. The escalation stage is a period of swift returns. Sales increase as more customers become aware of

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the product and its benefits. Hence additional market segments are targeted. The marketing team will enlarge the distribution at this point. This escalation stage is the most money-making stage. Then since the brand awareness is strong, advertising expenditures will be reduced. In due course of time, sales will begin to decline as the market becomes saturated or the product becomes technically obsolete or customers may change taste. The profitability may be maintained for a longer time if the product has developed a brand name. Finally with the declining production volumes, unit costs may boost and gradually no more profit can be earned.

The origin of the Economic Production Quantity (EPQ) model can be traced back to 1918, when E.W. Taft made an extension of the Economic Order Quantity (EOQ) model developed by F.W. Harris. The EPQ model is commonly used in the manufacturing sector for determining the optimal production quantity to minimize total inventory costs. However, some of the assumptions in the model are rarely met and in some cases, are considered to be unrealistic, especially the assumption of perfect quality items. A considerable amount of research work has been carried out to include more realistic assumptions. Salameh and Jaber [16] presented a modified EPQ model that accounts imperfect quality items. Goyal and Cardenas-Barron [5] introduced a practical and simple approach for determining the EPQ of an item, where similar optimal results to the work of Salameh and Jaber [16] were obtained. Pasandideh et al. [14] developed a multi-product EPQ model with imperfect quality items, wherein reworks are allowed with warehouse space limitations. Yoo et al. [17] proposed a profit-maximizing EPQ model that incorporates both imperfect production quality and two-way inspection. Liao et al. [11] studied integrated maintenance and production programs with the EPQ model with imperfect repair and rework upon failure. Haji et al. [6] extended the optimal solution for an inventory problem consisting of a single machine with defectives. They assumed that no shortages are allowed and all defective items are to be reworked. Setup costs for rework and the waiting time of defectives were also considered. Chiu et al.[1] employed mathematical modeling, along with a recursive searching algorithm, to determine the optimal run time for an imperfect finite production rate model with scrap, rework, and stochastic machine breakdown. Mishra[12] analysed an inventory model with a variable rate of deterioration, finite rate of replenishment but no shortage. Only a special case of the model was solved under very restrictive assumptions. Deb and Chaudhuri [2] studied a model with a finite production rate and a time-proportional deterioration rate, allowing for backlogging. Goswami and Chaudhuri [3] assumed that the demand rate, production rate and deterioration rate were all time dependent. Detailed information regarding inventory modelling for deteriorating items was given in the review articles of Nahmias [13] and Rafaat[15]. An order-level inventory model for deteriorating items without shortages has been developed by Jalan and Chaudhuri [7].

Recently, Kirtan Parmar and Gothi U. B.[9] have developed a deterministic inventory model for deteriorating items where time to deterioration has Exponential distribution and with time-dependent quadratic demand. In this model, shortages are not allowed and holding cost is time-dependent. Gothi U. B. and Kirtan Parmar [4] have extended above deterministic inventory model by taking two parameter Weibull distribution to represent the distribution of time to deterioration and shortages are allowed and partially backlogged. Also, Kirtan Parmar and Gothi U. B.[8] have developed an Economic Production Quantity model when deteriorating rate follows three parameter Weibull distribution with constant demand and production rate. In this model, shortages are not allowed and holding cost is time-dependent. Jani, Jaiswal and Shah [8] have developed (S,q_p) system inventory model for deteriorating items.

Based on the extensive literature on EPQ model, it is clearly observed that there is still a growing interest, particularly on the issue of an imperfect production system, which is a real-world manufacturing sector concern. This study presents an imperfect quality product system wherein imperfectly produced items are subjected to rework. Model is developed in section 4 and it is illustrated with sensitivity analysis in section 5 and section 6.

2. Notations

Following notations are used in our developed model.

1. R is the constant demand rate.
2. W is the constant defective rate.
3. C_h is the inventory holding cost per unit per unit time.
4. C_d is the cost of defective item per unit per unit time.
5. A is the operating cost which is fixed.
6. P_c is the unit production cost.
7. $Q(t)$ is the inventory level at time t ($\geq$ 0).
8. T is the fixed duration of a production cycle.
9. x is the proportion of defective items from regular production $(0<x<0.1)$.
10. TC is the Total Cost per unit time.

3. Basic Assumptions

The model is developed under the following assumptions.

1. The demand rate is known to be constant and continuous.
2. As soon as the items are produced they are added to the inventory.
3. Shortages are not allowed to occur.
4. We consider only a single product items.
5. The production rate is always greater than or equal to the sum of the demand rate and the rate of defectives i.e. $P \geq R+W$.

6. In the first case IHC is constant and in the second case IHC is a linear function of time i.e. $C_h = h + rt$ $(h > 0, r > 0, t > 0)$.
7. Lead-time is zero.
8. Time horizon is finite.
9. No repair or replacement of the imperfect quality items takes place during a given cycle.
10. Total cost function is a real and continuous function.

4. Mathematical Formulation and Analysis

There is no stock at the beginning of the cycle and production starts. The initial production rate is P_1 during the time period $[0, t_1]$. The stock attains a level Q_1 at time $t = t_1$. During $[t_1, t_2]$, the production rate increases to $P_2(>P_1)$ and the inventory level reaches to the level S at time $t=t_2$. Production is switched off after time t_2. During $[t_2, T]$, the inventory level gradually decreases mainly to meet demand only. The inventory level will reach zero at time t=T.

The graphical presentation is shown in Figure 1.

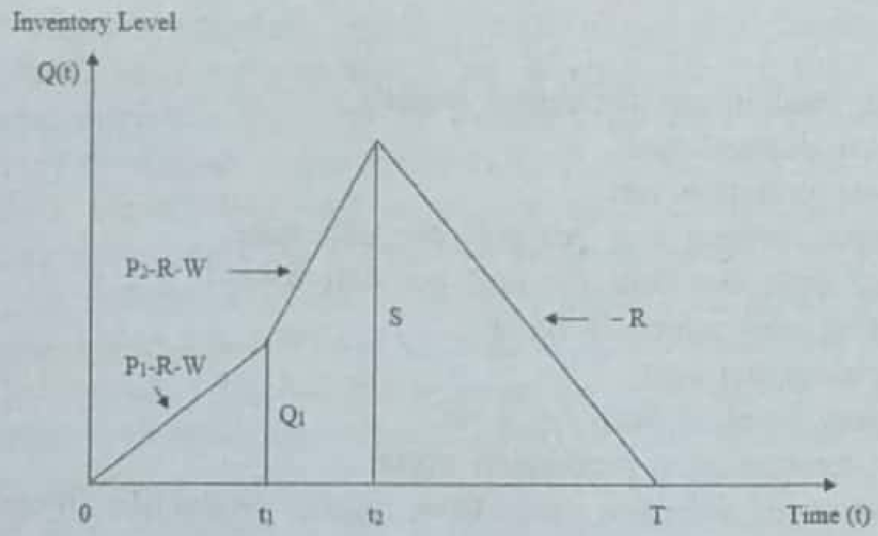

Figure – 1: Graphical Presentation of the Inventory System

The rate at which the inventory level changes during cycle period [0,T] can be explained by the following differential equations.

$$\frac{dQ(t)}{dt} = P_1 - R - W \qquad (0 \le t \le t_1) \tag{1}$$

$$\frac{dQ(t)}{dt} = P_2 - R - W \qquad (t_1 \le t \le t_2) \tag{2}$$

$$\frac{dQ(t)}{dt} = -R \qquad (t_2 \le t \le T) \tag{3}$$

Using the boundary conditions Q(0)=0, $Q(t_1)=Q_1$, $Q(t_2)=S$ and Q(T)=0, the solutions of the differential equations (1), (2) and (3) are

$$Q(t) = (P_1 - R - W)t \qquad (0 \le t \le t_1) \tag{4}$$

$$Q(t) = Q_1 + (P_2 - R - W)(t - t_1) \qquad (t_1 \le t \le t_2) \tag{5}$$

$$Q(t) = S + R(t_2 - t) \qquad (t_2 \le t \le T) \tag{6}$$

At t= t_1, Q(t)=Q_1 and hence from equation (4) we get,

$$t_1 = \frac{Q_1}{P_1 - R - W} \tag{7}$$

At t= t_2, Q(t)=S and hence from equation (5) we get,

$$S = Q_1 + (P_2 - R - W)(t_2 - t_1) \Rightarrow t_2 = \frac{S - Q_1}{P_2 - R - W} + \frac{Q_1}{P_1 - R - W} \tag{8}$$

At t= T, Q(t)=0 and hence from equation (6) we get,

$$T = \frac{S - Q_1}{P_2 - R - W} + \frac{Q_1}{P_1 - R - W} + \frac{S}{R} \tag{9}$$

The total cost consists of the following costs.

1) Operating Cost

$$OC = A \quad \text{(Fixed)} \tag{10}$$

2) Production Cost

$$PC = p_c\left[P_1 t_1 + P_2(t_2 - t_1)\right] \Rightarrow PC = p_c\left[\frac{P_1 Q_1}{P_1 - R - W} + \frac{P_2(S - Q_1)}{P_2 - R - W}\right] \tag{11}$$

3) Cost of Defective Items

$$DC = C_d x\left[P_1 t_1 + P_2(t_2 - t_1)\right] \Rightarrow DC = C_d x\left[\frac{P_1 Q_1}{P_1 - R - W} + \frac{P_2(S - Q_1)}{P_2 - R - W}\right] \qquad (12)$$

We consider the following two cases.

Case I: When the inventory holding cost C_h is constant

4) Inventory Holding Cost

$$IHC = C_h\left[\int_0^{t_1} Q(t)dt + \int_{t_1}^{t_2} Q(t)dt + \int_{t_2}^{T} Q(t)dt\right]$$

$$= C_h\left[\int_0^{t_1}(P_1 - R - W)t\,dt + \int_{t_1}^{t_2}\{Q_1 + (P_2 - R - W)(t - t_1)\}dt + \int_{t_2}^{T}\{S + R(t_2 - T)\}dt\right]$$

$$\Rightarrow IHC = C_h\left[\begin{array}{l}\frac{1}{2}\{(P_1 + P_2) - 2R - 2W\}t_1^2 - \{Q_1 + (P_2 - R - W)t_2\}t_1 + \frac{1}{2}(P_2 - 3R - W)t_2^2\\ +(Q_1 - S + 2RT)t_2 + ST - RT^2\end{array}\right] \qquad (13)$$

Substituing value of t_1, t_3 and T from equations (7), (8) and (9) into equation (13) we get,

$$IHC = c_h\left[\begin{array}{l}\frac{1}{2}\{(P_1 + P_2) - 2R - 2W\}\left(\frac{Q_1}{P_1 - R - W}\right)^2\\ -\left\{Q_1 + (P_2 - R - W)\left(\frac{S - Q_1}{P_2 - R - W} + \frac{Q_1}{P_1 - R - W}\right)\right\}\left(\frac{Q_1}{P_1 - R - W}\right)\\ +\frac{1}{2}(P_2 - 3R - W)\left(\frac{S - Q_1}{P_2 - R - W} + \frac{Q_1}{P_1 - R - W}\right)^2\\ +\left\{Q_1 - S + 2R\left(\frac{S - Q_1}{P_2 - R - W} + \frac{Q_1}{P_1 - R - W} + \frac{S}{R}\right)\right\}\left(\frac{S - Q_1}{P_2 - R - W} + \frac{Q_1}{P_1 - R - W}\right)\\ +S\left(\frac{S - Q_1}{P_2 - R - W} + \frac{Q_1}{P_1 - R - W} + \frac{S}{R}\right) - R\left(\frac{S - Q_1}{P_2 - R - W} + \frac{Q_1}{P_1 - R - W} + \frac{S}{R}\right)^2\end{array}\right] \qquad (14)$$

Hence, the total cost per unit time for the time period [0,T] is given by

$$TC = \frac{1}{T}[OC + PC + DC + IHC]$$

$$\Rightarrow TC=\frac{1}{\left[\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}+\frac{S}{R}\right]}\left[\begin{array}{l} A+P_c\left[\frac{P_1Q_1}{P_1-R-W}+\frac{P_2(S-Q_1)}{P_2-R-W}\right]+C_d\theta\left[\frac{P_1Q_1}{P_1-R-W}+\frac{P_2(S-Q_1)}{P_2-R-W}\right] \\ +C_h\left\{\begin{array}{l} +\frac{1}{2}\{(P_1+P_2)-2R-2W\}\left(\frac{Q_1}{P_1-R-W}\right)^2 \\ -\left\{Q_1+(P_2-R-W)\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}\right)\right\}\left(\frac{Q_1}{P_1-R-W}\right) \\ +\frac{1}{2}(P_2-3R-W)\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}\right)^2 \\ +\left\{Q_1-S+2R\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}+\frac{S}{R}\right)\right\}\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}\right) \\ +S\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}+\frac{S}{R}\right)-R\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}+\frac{S}{R}\right)^2 \end{array}\right\} \end{array}\right] \quad (15)$$

Our objective is to determine optimum values of Q_1 and S in order that the total cost TC will be minimum. The optimum values Q_1^* and S^*, for which the total cost TC is minimum, can be obtained by solving the equations $\frac{\partial TC}{\partial Q_1}=0$ and $\frac{\partial TC}{\partial S}=0$ which satisfy the sufficient condition $\left\{\left(\frac{\partial^2 TC}{\partial Q_1^2}\right)\left(\frac{\partial^2 TC}{\partial S^2}\right)-\left(\frac{\partial^2 TC}{\partial Q_1 \partial S}\right)^2\right\}_{\left(\substack{Q_1=Q_1^* \\ S=S^*}\right)}>0$

Case II : When the inventory holding cost (C_h) is a linear function of time

$$IHC=\int_0^{t_1}(h+rt)Q(t)dt+\int_{t_1}^{t_2}(h+rt)Q(t)dt+\int_{t_2}^{T}(h+rt)Q(t)dt$$

$$\Rightarrow IHC=\left\{\begin{array}{l} h\left[\begin{array}{l}\frac{1}{2}\{(P_1+P_2)-2R-2W\}t_1^2-\{Q_1+(P_2-R-W)t_2\}t_1+\frac{1}{2}(P_2-3R-W)t_2^2 \\ +(Q_1-S+2RT)t_2+ST-RT^2\end{array}\right] \\ +r\left[\begin{array}{l}\frac{1}{2}\{(P_1+P_2)-2R-2W\}t_1^3+\left\{\frac{Q_1}{2}-\frac{7}{6}(P_2-R-W)t_1-\frac{S}{2}+\frac{RT}{2}\right\}t_2^2 \\ +\left\{\frac{(P_2-R-W)}{3}-\frac{R}{2}\right\}t_2^3+\left\{\frac{1}{3}(P_2-R-W)t_2-\frac{Q_1}{2}\right\}t_1^2+\frac{1}{2}\left(S+Rt_2)T^2\right)-\frac{RT^3}{3}\end{array}\right]\end{array}\right\}$$

Hence the total cost per unit time for the period [0,T] is

$TC=\frac{1}{T}[OC+PC+DC+IHC]$ which is given by the following equation (16).

$$
TC=\frac{1}{\left[\begin{array}{l}\frac{S-Q_1}{P_2-R-W}\\+\frac{Q_1}{P_1-R-W}+\frac{S}{R}\end{array}\right]}\left[\begin{array}{l}
A+p_c\left[\frac{P_1Q_1}{P_1-R-W}+\frac{P_2(S-Q_1)}{P_2-R-W}\right]\\
+C_dx\left[\frac{P_1Q_1}{P_1-R-W}+\frac{P_2(S-Q_1)}{P_2-R-W}\right]\\
+h\left[\begin{array}{l}
\frac{1}{2}\{(P_1+P_2)-2R-2W\}\left(\frac{Q_1}{P_1-R-W}\right)^2\\
-\left\{Q_1+(P_2-R-W)\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}\right)\right\}\left(\frac{Q_1}{P_1-R-W}\right)\\
+\frac{1}{2}(P_2-3R-W)\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}\right)^2\\
+\left\{Q_1-S+2R\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}+\frac{S}{R}\right)\right\}\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}\right)\\
+S\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}+\frac{S}{R}\right)-R\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}+\frac{S}{R}\right)^2
\end{array}\right]\\
+r\left[\begin{array}{l}
\frac{1}{2}\{(P_1+P_2)-2R-2W\}\left(\frac{Q_1}{P_1-R-W}\right)^3\\
+\left\{\frac{Q_1}{2}-\frac{7}{6}(P_2-R-W)\left(\frac{Q_1}{P_1-R-W}\right)-\frac{S}{2}+\frac{RT}{2}\right\}t_2^2\\
+\left\{\frac{(P_2-R-W)}{3}-\frac{R}{2}\right\}\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}\right)^3\\
+\left\{\frac{1}{3}(P_2-R-W)\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}\right)-\frac{Q_1}{2}\right\}\left(\frac{Q_1}{P_1-R-W}\right)^2\\
+\frac{1}{2}\left(S+R\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}\right)\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}+\frac{S}{R}\right)^2\right)\\
-\frac{R}{3}\left(\frac{S-Q_1}{P_2-R-W}+\frac{Q_1}{P_1-R-W}+\frac{S}{R}\right)^3
\end{array}\right]
\end{array}\right] \quad (16)
$$

It may be noted that we can derive the equation for TC function for case I in particular by putting $h=C_h$ and $r=0$ in equation (16). Our objective is to determine optimum values of Q_1 and S in order that the total cost TC will be minimum. The optimum values Q_1^* and S^*, for which the total cost TC is minimum, can be obtained by solving the equations $\frac{\partial TC}{\partial Q_1}=0$ and $\frac{\partial TC}{\partial S}=0$ using appropriate mathematical software, which satisfy the sufficient condition $\left\{\left(\frac{\partial^2 TC}{\partial Q_1^2}\right)\left(\frac{\partial^2 TC}{\partial S^2}\right)-\left(\frac{\partial^2 TC}{\partial Q_1 \partial S}\right)^2\right\}_{\left(\substack{Q_1=Q_1^* \\ S=S^*}\right)}>0$.

5. Illustration

Let us consider the following example to illustrate the above developed model. We consider the following values of the parameter for P_1 = 6000, P_2 = 9000, R = 4500, W = 30, A = 200, h = 2, r = 3, p_e = 10, C_d = 10, = 0.01 (with appropriate units).

We find the optimal values of Q_1 = 344. 8886470 units, S = 400.3506125 units and total optimal cost TC = 46752.82552 units.

6. Sensitivity Analysis

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. In this section, we have studied the sensitivity of the total cost per time unit TC with respect to the changes in the values of the parameters P_1, P_2, R, W, A, h, r, p_e, C_d and x.

The sensitivity analysis is performed by considering variation in each one of the above parameters keeping all the remaining other parameters as fixed. The results are presented in **Table–1.** Here the last column contains the percentage change in TC as compared to the original solution.

Table–1: Partial Sensitivity Analysis

Parameter	Change	Q_1	S	TC	% change in TC
P_1	– 10 %	272.6562670	323.7136222	46599.81032	-0.33
	– 5 %	312.5152105	366.0723637	46685.32084	-0.14
	+ 5 %	371.9653521	428.9531365	46808.08607	0.12
	+10 %	395.0803230	453.3310527	46854.46289	0.22
P_2	– 10 %	345.1257585	394.3149661	46753.44745	0.0013
	– 5 %	345.0010203	397.4907858	46753.12024	0.0006
	+ 5 %	344.7868902	402.9393734	46752.55874	-0.0006
	+10 %	344.6943176	405.2938617	46752.31613	-0.0011
R	– 10 %	373.9277256	437.7571009	42261.09725	-9.61
	– 5 %	361.3396140	420.9927426	44511.36781	-4.79
	+ 5 %	324.0764894	375.3063905	48984.10298	4.77
	+10 %	298.1564208	345.0714757	51203.11857	9.52
W	– 10 %	348.1470733	398.0189056	46731.13581	-0.05
	– 5 %	346.5202075	399.1857484	46741.98504	-0.02
	+ 5 %	343.2523543	401.5135167	46763.65702	0.02
	+10 %	341.6112912	402.6744777	46774.47969	0.05
A	– 10 %	328.2196762	384.3785453	46691.97076	-0.13
	– 5 %	336.6919785	392.4944741	46722.73910	-0.06
	+ 5 %	352.8314875	407.9673833	46782.27969	0.06
	+10 %	360.5396461	415.3627274	46811.14569	0.12
h	– 10 %	352.6393691	411.8629762	46716.79922	-0.08
	– 5 %	348.9209720	406.3036473	46733.98088	-0.04
	+ 5 %	340.9591857	394.6200873	46771.41957	0.04
	+10 %	337.1306070	389.0994032	46789.77151	0.08
r	– 10 %	350.2797402	407.0332316	46741.60267	-0.02
	– 5 %	347.5346968	403.6311096	46747.25930	-0.01
	+ 5 %	342.3351765	397.1839659	46758.30517	0.01
	+10 %	339.8684650	394.1241065	46763.70196	0.02
p_e	– 10 %	347.8191992	397.7145102	42230.74108	-9.67
	– 5 %	346.3574271	399.0338447	44491.79030	-4.84
	+ 5 %	343.4128093	401.6648309	49013.84659	4.84
	+10 %	341.9298765	402.9765061	51274.85365	9.67
C_d	– 10 %	344.9180873	400.3243060	46707.60496	-0.1
	– 5 %	344.9033718	400.3374555	46730.21527	-0.05
	+ 5 %	344.8739259	400.3637652	46775.43574	0.05
	+10 %	344.8591996	400.3769217	46798.04604	0.1
x	– 10 %	344.9180873	400.3243060	46707.60496	-0.1
	– 5 %	344.9033718	400.3374555	46730.21527	-0.05
	+ 5 %	344.8739259	400.3637652	46775.43574	0.05
	+10 %	344.8591996	400.3769217	46798.04604	0.1

7. Graphical Presentation

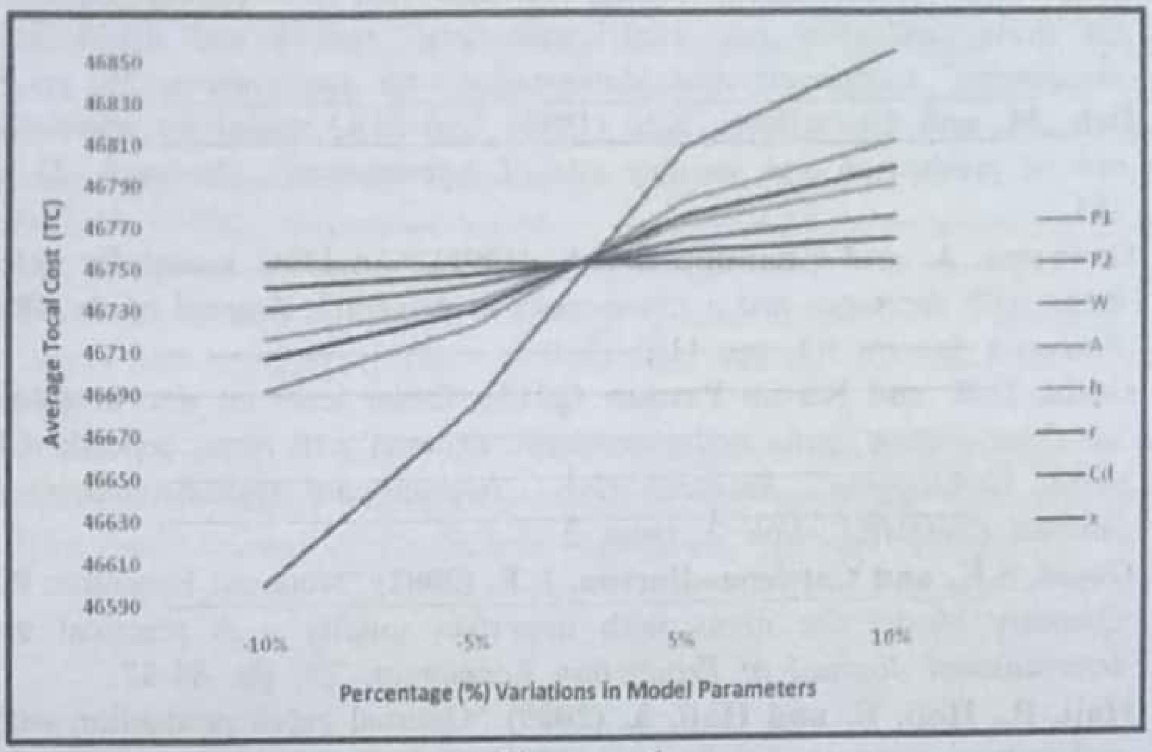

Figure - 1

8. Conclusions

- From Table-1 it is observed that as the production rate P_1 increases by 5% there is a significant increase in both the inventory accumulations Q_1 and S; total cost TC also increases significantly. When production rate P_2 increases by 5%, Q_1 decreases with a very small rate and there is a significant increase in S; while total cost TC decreases insignificantly. When the demand rate R increases by 5%, both the inventory accumulations Q_1 and S decrease significantly while total cost TC increases significantly at every stage. Finally, when the defectives W increases by 5%, Q_1 decreases but S increases insignificantly; whereas there is a significant increase in the total cost TC.
- Thus it may be concluded from Figure–1 that the cycle length T is highly sensitive to the changes in the values of P_1, moderately sensitive to changes in the values of P_2, R and W and less sensitive to the changes in the values of h, x and A.

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"ORDER LEVEL INVENTORY MODEL FOR DETERIORATING ITEM UNDER VARYING DEMAND CONDITION"

Kirtan Parmar[1], Indu Aggarwal[2], U. B. Gothi[3]

ABSTRACT

In this paper we have developed an inventory model for deteriorating items under linear time dependent demand function and also time dependent IHC. Two parameter Weibull distribution is assumed for deterioration of items. The model developed is illustrated with its sensitivity analysis.

1. Introduction

Inventory is defined as an idle resource which helps us to run business successfully and effectively. The inventory product can be classified into three categories based upon their shelf life. They are (a) obsolescence (b) deterioration (c) without deterioration. Deterioration is the damage caused due to spoilage, dryness, etc. Deterioration of an item is a realistic situation associated with any inventory system. It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches to zero. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. For items such as steel, hardware, glassware and toys, the rate of deterioration is so low that there is little need for considering deterioration in the determination of the economic lot size. But some items such as blood, fish, strawberry, alcohol, gasoline, radioactive chemical, medicine and food grains (i.e., paddy, wheat, potato, onion etc.) exercise remarkable deterioration over time. It has been observed that the failure of many items may be expressed in terms of Weibull distribution.

Ghare and Scharder [7] first formulated a mathematical model with a constant deterioration rate. Wee [18] developed EOQ models to allow deterioration and an exponential demand pattern. Emmone [6] established a replenishment model for

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radioactive nuclide generators. Ajantha Roy [1] developed an inventory model where demand rate is a function of selling price. Vipin Kumar, S.R. Singh, Sanjay Sharma [16] have developed a production inventory model with time dependent demand and partial backlogging. S.K.Ghosh and K.S.Chaudhuri [13] have considered Quadratic time demand in their developed inventory model. The assumption of the constant deterioration rate was relaxed by Covert and Philip [5], who used a two-parameter Weibull distribution to represent the distribution of time to deterioration.

This model was further generalised by Philip [11] by taking three-parameter Weibull distribution for deterioration. A two-parameter weibull distribution to represent the distribution of time to deterioration was considered by Zhao Pei-xin [19], Azizul Baten and Anton Abdulbasah kamil [2], etc.

Varying demand pattern plays an important role in the inventory management. Demand may be constant, time-varying, stock-dependent, price-dependent etc. The constant demand is valid, only when the phase of the product life cycle is matured and also for finite time period. Wagner and Whitin [17] discussed the discrete case of the dynamic version of EOQ model. Inventory models for deteriorating items under different demand patterns were considered by Bahari-Kashani [3], Goswami and Chaudhuri [8]. R.P.Tripathi [12] has considered a model under time-varying demand rate and IHC.

It is very common to assume IHC to be constant. But in realistic situation holding cost need not always be constant. Many researchers like C.K.Tripathy and U.Mishra [4], V.K.Mishra and L.Shingh [15], Ajantha Roy [1], etc. have discussed the model under time dependent inventory holding cost.

Kirtan Parmar and U. B. Gothi [9] have developed an order level inventory model for deteriorating item under quadratic demand with time dependent IHC. U. B. Gothi and Kirtan Parmar [14] have developed a deterministic inventory model by taking two parameter Weibull distribution to represent the distribution of time to deterioration and shortages are allowed and partially backlogged. Also, Kirtan Parmar and U. B. Gothi [10] have developed an EPQ model of deteriorating items using three parameter Weibull distribution with constant production rate and time varying holding cost.

In this paper we have developed an inventory model for deteriorating items with varying demand conditions under the assumptions given in section-3 below.

2. Notations

We use the following notations for the developed mathematical model

1. Q(t) : Inventory level of the product at time t ($\geq$ 0).
2. R(t) : Demand rate varying over time.
3. $\theta(t)$: $\alpha\beta t^{\beta-1}$ Two parameter Weibull distribution for the rate of deterioration. (unit/unit time) (where $0 < \alpha < 1$, $\beta > 0$).
4. A : Ordering cost per order during the cycle period.
6. C_h : Inventory holding cost per unit per unit time.
7. C_d : Deterioration cost per unit per unit time.
8. T : Duration of a cycle.
9. TC(T) : Total cost per unit time.

3. Assumptions

The model is developed under the following assumptions

1. Inventory system deals with a single item.
2. The annual demand rate is a function of time which is $R(t) = \lambda t^{-p}$ ($\lambda > 0$ and $0 < p < 1$)
3. Holding cost is a linear function of time expressed by C_h = h+rt ($h > 0$, $r > 0$, $t > 0$)
4. Shortages are not allowed to occur.
5. Lead-time is zero.
6. We consider finite time horizon period.
7. Replenishment rate is infinite.
8. No repair or replacement of the deteriorated items takes place during a given cycle.
9. Total inventory cost is a real, continuous function which is convex to the origin.

4. Mathematical Model And Analysis

At the beginning of the cycle period the on hand inventory level is S units and it gradually reduces to zero at the end of cycle period T. This is due to demand and deterioration of the units during this cycle. Let Q(t) be the on-hand inventory at time t ($0 \leq t \leq T$). In the interval (0, μ) the stock will decrease due to the demand of units and in the interval (μ, T) it will decrease due to the effect of deterioration as well as demand. At time T inventory level reaches zero and fresh procurement takes place thus reaching to order level S.

The graphical presentation is shown in **Figure 1**.

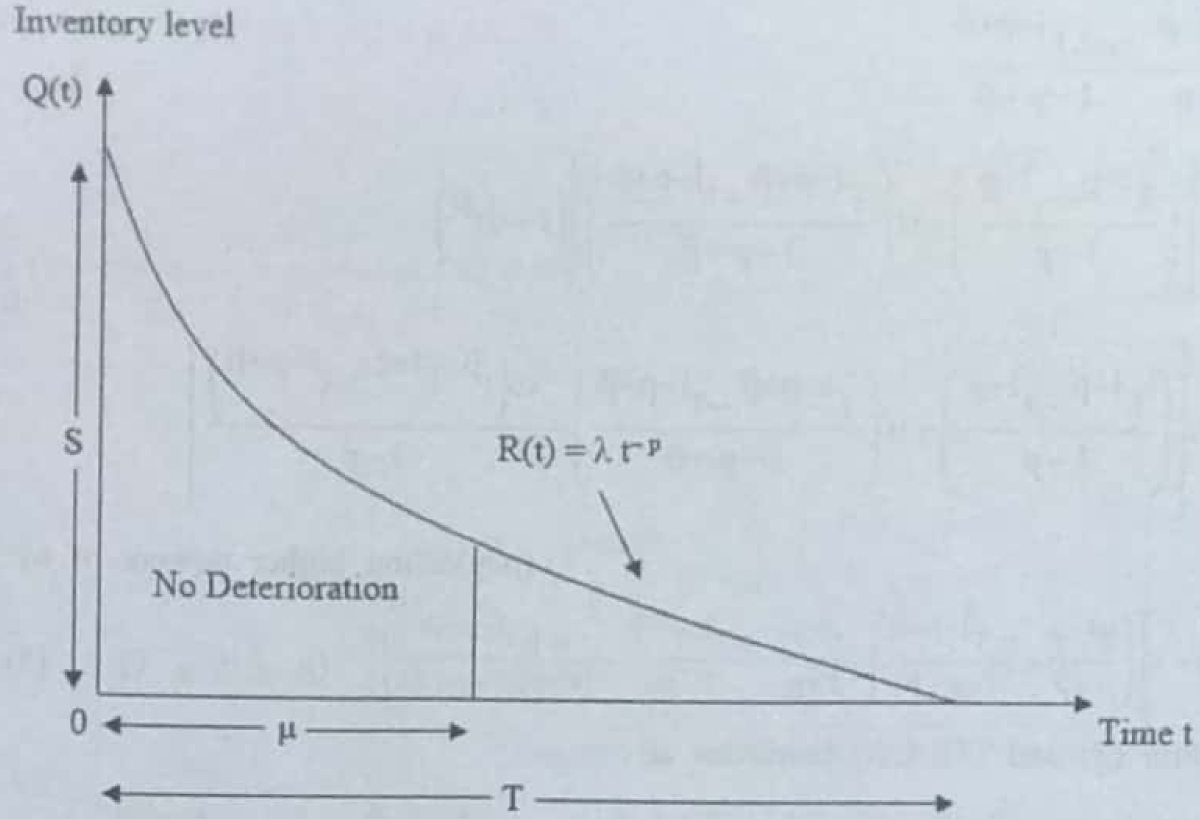

Figure – 1: Graphical presentation of the Inventory system

The rate of change of inventory during the interval (0, μ) and (μ, T) is governed by the following differential equations

$$\frac{dQ(t)}{dt} = -\lambda t^{-p} \qquad (0 \le t \le \mu) \tag{1}$$

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = -\lambda t^{-p} \qquad (\mu \le t \le T) \tag{2}$$

Now solving equation (1) with boundary condition Q(0) = S we get,

$$\Rightarrow Q(t) = S - \frac{\lambda t^{1-p}}{1-p}, \quad (0 \le t \le \mu) \tag{3}$$

Similarly, solution of linear differential equation (2) is given by

$$e^{\alpha t^{\beta}} Q(t) = -\frac{\lambda t^{1-p}}{1-p} - \frac{\alpha \lambda t^{1-p+\beta}}{1-p+\beta} + c_1 \qquad \text{(neglecting higher powers of } \alpha) \qquad (4)$$

Now solving equation (4) with boundary condition Q(T) = 0 we get,

$$c_1 = \frac{\lambda T^{1-p}}{1-p} + \frac{\alpha \lambda T^{1-p+\beta}}{1-p+\beta}$$

$$Q(t) = \lambda \left[\left(\frac{T^{1-p} - t^{1-p}}{1-p} \right) + \alpha \left(\frac{T^{1-p+\beta} - t^{1-p+\beta}}{1-p+\beta} \right) \right] \left(1 - \alpha t^{\beta}\right)$$

$$= \lambda \left[\left(\frac{T^{1-p} - t^{1-p}}{1-p} \right) + \alpha \left(\frac{T^{1-p+\beta} - t^{1-p+\beta}}{1-p+\beta} \right) - \frac{\alpha \left(t^{\beta} T^{1-p} - t^{1-p+\beta} \right)}{1-p} \right]$$

(neglecting higher powers of α)

$$\Rightarrow Q(t) = \lambda \left[\left(\frac{T^{1-p}}{1-p} + \frac{\alpha T^{1-p+\beta}}{1-p+\beta} \right) - \frac{t^{1-p}}{1-p} - \frac{\alpha t^{\beta} T^{1-p}}{1-p} + \frac{\alpha \beta t^{1-p+\beta}}{(1-p)(1-p+\beta)} \right] \quad (\mu \le t \le T) \qquad (5)$$

In equation (3) and (5), Q(t) coincides at $t = \mu$

$$S = \frac{\lambda \mu^{1-p}}{1-p} + \lambda \left[\left(\frac{T^{1-p}}{1-p} + \frac{\alpha T^{1-p+\beta}}{1-p} \right) - \frac{\mu^{1-p}}{1-p} - \frac{\alpha T^{1-p} \mu^{\beta}}{1-p} + \frac{\alpha \beta \mu^{1-p+\beta}}{(1-p)(1-p+\beta)} \right]$$

$$\Rightarrow S = \lambda \left[\frac{\left(1 - \alpha \mu^{\beta}\right) T^{1-p}}{1-p} + \frac{\alpha T^{1-p+\beta}}{1-p+\beta} + \frac{\alpha \beta \mu^{1-p+\beta}}{(1-p)(1-p+\beta)} \right] \qquad (6)$$

The total cost per unit time comprises of the following costs

1) **Ordering Cost**

   OC = A (7)

2) **The deterioration cost during the period [0, T]**

$$DC = C_d \left[S - \lambda \int_{\mu}^{T} R(t)dt \right]$$

$$\Rightarrow DC = C_d\left[S - \frac{\lambda}{1-p}\left(T^{1-p} - \mu^{1-p}\right)\right] \tag{8}$$

3) Inventory Holding Cost

$$IHC = \int_0^{\mu}(h+rt)Q(t)dt + \int_{\mu}^{T}(h+rt)Q(t)dt$$

$$= \int_0^{\mu}(h+rt)\left[S - \frac{\lambda t^{-p}}{1-p}\right]dt + \lambda\int_{\mu}^{T}(h+rt)\left[\left(\frac{T^{1-p}}{1-p} + \frac{\alpha\, T^{1-p+\beta}}{1-p}\right) - \frac{t^{1-p}}{1-p} - \frac{\alpha\, t^{\beta}\, T^{1-p}}{1-p} + \frac{\alpha\,\beta\, t^{1-p+\beta}}{(1-p)(1-p+\beta)}\right]dt$$

$$\Rightarrow IHC = \left[\begin{array}{l} hS\mu + \frac{rS\mu^2}{2} \\ -\frac{\lambda h\mu^{2-p}}{(1-p)(2-p)} \\ -\frac{\lambda\, r\, \mu^{3-p}}{(1-p)(3-p)} \end{array}\right] + \lambda\left\{\begin{array}{l} h\left[\begin{array}{l}\left(\frac{T^{1-p}}{1-p} + \frac{\alpha T^{1-p+\beta}}{1-p+\beta}\right)(T-\mu) - \frac{\left(T^{2-p} - \mu^{2-p}\right)}{(1-p)(2-p)} \\ -\frac{\alpha T^{1-p}\left(T^{\beta+1} - \mu^{\beta+1}\right)}{(1-p)(\beta+1)} + \frac{\alpha\beta\left(T^{2-p+\beta} - \mu^{2-p+\beta}\right)}{(1-p)(1-p+\beta)(2-p+\beta)}\end{array}\right] \\ +r\left[\begin{array}{l}\left(\frac{T^{1-p}}{1-p} + \frac{\alpha T^{1-p+\beta}}{1-p+\beta}\right)\frac{\left(T^2 - \mu^2\right)}{2} - \frac{\left(T^{3-p} - \mu^{3-p}\right)}{(1-p)(3-p)} \\ -\frac{\alpha T^{1-p}\left(T^{\beta+2} - \mu^{\beta+2}\right)}{(1-p)(\beta+2)} + \frac{\alpha\beta\left(T^{3-p+\beta} - \mu^{3-p+\beta}\right)}{(1-p)(1-p+\beta)(3-p+\beta)}\end{array}\right]\end{array}\right\} \tag{9}$$

Hence, the total cost per unit time is

$$TC(T) = \frac{1}{T}(OC + DC + IHC)$$

$$TC(T)=\frac{1}{T}\left\{\begin{array}{l} A+C_d\left[S-\frac{\lambda}{1-p}\left(T^{1-p}-\mu^{1-p}\right)\right]+\left[\begin{array}{l} hS\mu+\frac{rS\mu^2}{2}-\frac{\lambda h\,\mu^{2-p}}{(1-p)(2-p)} \\ -\frac{\lambda r\,\mu^{3-p}}{(1-p)(3-p)} \end{array}\right] \\ +\lambda\left\{\begin{array}{l} h\left[\begin{array}{l} \left(\frac{T^{1-p}}{1-p}+\frac{\alpha T^{1-p+\beta}}{1-p+\beta}\right)(T-\mu)-\frac{\left(T^{2-p}-\mu^{2-p}\right)}{(1-p)(2-p)} \\ -\frac{\alpha T^{1-p}\left(T^{\beta+1}-\mu^{\beta+1}\right)}{(1-p)(\beta+1)}+\frac{\alpha\beta\left(T^{2-p+\beta}-\mu^{2-p+\beta}\right)}{(1-p)(1-p+\beta)(2-p+\beta)} \end{array}\right] \\ +r\left[\begin{array}{l} \left(\frac{T^{1-p}}{1-p}+\frac{\alpha T^{1-p+\beta}}{1-p+\beta}\right)\frac{\left(T^2-\mu^2\right)}{2}-\frac{\left(T^{3-p}-\mu^{3-p}\right)}{(1-p)(3-p)} \\ -\frac{\alpha T^{1-p}\left(T^{\beta+2}-\mu^{\beta+2}\right)}{(1-p)(\beta+2)}+\frac{\alpha\beta\left(T^{3-p+\beta}-\mu^{3-p+\beta}\right)}{(1-p)(1-p+\beta)(3-p+\beta)} \end{array}\right] \end{array}\right\} \end{array}\right\} \quad (10)$$

Our objective is to determine optimum value of T so that TC(T) is minimum. The optimum value of T for which total cost TC(T) is minimum, is the solution of equation $\frac{\partial TC(T)}{\partial T}=0$ giving T* satisfying the sufficient condition $\left[\frac{\partial^2 TC(T)}{\partial T^2}\right]_{T=T^*}>0.$ (11)

The optimal solution of the equations (10) can be obtained by using appropriate software. This has been illustrated by the following numerical example.

5. Numerical Example

Let us consider the following example to illustrate the above developed model. We consider the following values of the parameters for A = 350, h = 1, r = 0.5, μ = 5, C_d = 15, λ = 10, α = 0.05, β = 2 & p = 0.2 (with appropriate units).

Considering the above results given in (10) and (11), solution is obtained by using appropriate software. We obtain the optimal value of T = 6.087113121 units and optimal total cost TC = 225.8802312 units.

6. Sensitivity Analysis

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. In this section, we study the sensitivity for the cycle length (T) and total cost per time unit (TC) with respect to the changes in the values of the parameters A, h, r, μ, C_d, λ, α, β, and p.

We consider 10% and 20% increase or decrease in each one of the above parameters keeping all other remaining parameter as fixed. The results are presented in **Table – 1**. The last column of table shows the % change in TC as compared to the original solution corresponding to the changes in parameters values.

Table – 1: Partial Sensitivity Analysis

Parameter	% change	T	TC	% changes in TC
A	– 20	6.009022336	214.3065011	– 5.12
	– 10	6.048361913	220.1120445	– 2.55
	+ 10	6.125295246	231.6120811	+ 2.54
	+ 20	6.162926612	237.3085687	+ 5.06
h	– 20	6.142643148	220.7360943	– 2.28
	– 10	6.114445238	223.3174463	– 1.13
	+ 10	6.060605413	228.4252243	+ 1.13
	+ 20	6.034883422	230.9531559	+ 2.25
r	– 20	6.188637351	220.6197693	– 2.33
	– 10	6.136325361	223.2804664	– 1.15
	+ 10	6.040689492	228.4239075	+ 1.13
	+ 20	5.996787453	230.9157731	+ 2.23
μ	– 20	5.515036094	224.3750818	– 0.67
	– 10	5.789040806	225.2353680	– 0.29
	+ 10	6.408590985	226.5025623	+ 0.28
	+ 20	6.752116681	227.2454679	+ 0.60
C_d	– 20	6.006913741	202.3817574	– 10.40
	– 10	6.049056987	214.1498335	– 05.19
	+ 10	6.121672661	237.5783289	+ 05.18
	+ 20	6.153214076	249.2485038	+10.35
λ	– 20	6.181541239	192.1150754	– 14.95
	– 10	6.129503380	209.0220779	– 07.46
	+ 10	6.051908764	242.7017487	+ 07.45
	+ 20	6.022202136	259.4950782	+ 14.88
α	– 20	6.256301937	223.5525647	– 1.03
	– 10	6.165331392	224.7967499	– 0.48
	+ 10	6.019065189	226.8330857	+ 0.42
	+ 20	5.959274123	227.6781559	+ 0.80
β	– 20	6.873815071	215.8620810	– 4.44
	– 10	6.446521260	221.1192951	– 2.11
	+ 10	5.799663980	229.9218306	+ 1.79
	+ 20	5.578297947	233.1770053	+ 3.23
p	– 20	6.013060654	230.8875502	+ 2.22
	– 10	6.049394710	228.3389543	+ 1.09
	+ 10	6.126294121	223.5113310	– 1.05
	+ 20	6.167022449	221.2324314	– 2.06

7. Graphical Presentation

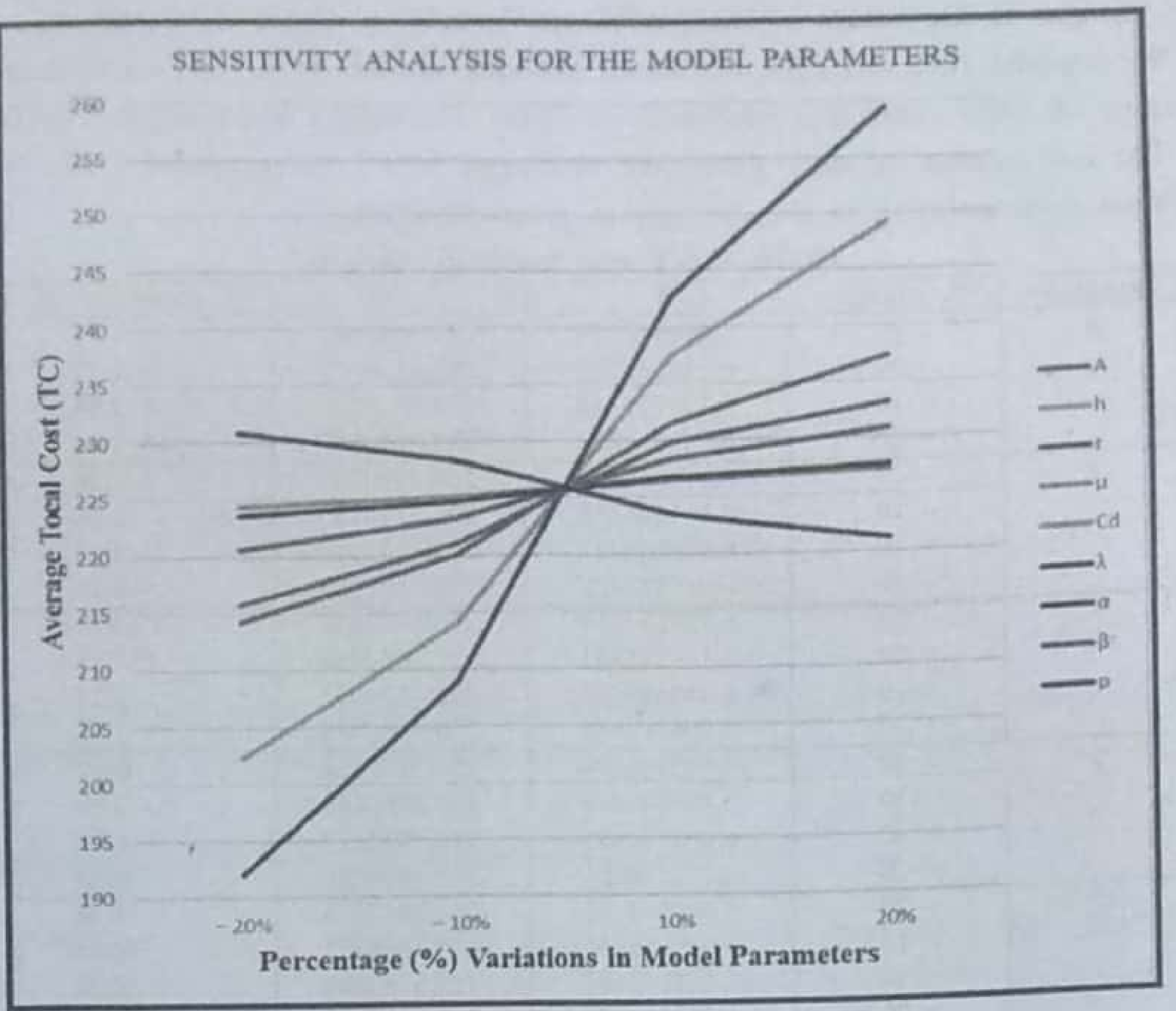

Figure – 2

8. Conclusions

From the above sensitivity analysis we may conclude that the total cost per time unit (TC) is highly sensitive to changes in the values of λ and C_d, moderately sensitive to changes in the values of A, h, r, β, p and less sensitive to changes in the values of α and μ.

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REGRESSION ANALYSIS FOR THE SECTORAL POWER CONSUMPTION IN GUJARAT STATE

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ABSTRACT

Electricity is a form of energy and it is an inevitable requirement for sustainable growth and development for any nation. Therefore it becomes necessary to have a close look at its installed capacity , generation of electricity ,consumption, etc. Electricity is consumed by many sectors like domestic, commercial, agricultural, railway, etc. Each sector has its own significance and they all together play integrated role in nation's development. Some political decisions compel the government to provide subsidy to state electricity board. Moreover theft of electricity and transmission loss makes it costlier.

In this paper an attempt is made to establish trend models for various sectors. Moreover multiple regression models are developed for some prime sectors. This effort may be useful for policy of energy conservation and energy audit also.

KEY WORDS Sectoral Consumption, PPCI, Policy Planning

1. Introduction

Electricity has invaded our lives and has become the most vital in almost all the aspects of society today. Its usage is inevitable in our everyday life. Broadly speaking it is extremely useful for transport sector, domestic, communication, services, Industrial manufacturing, Agricultural sector, entertainment fields, etc. Like almighty God it is Omnipresent.

Gujarat state is considered to be a very fast growing and developing state in India and it is having sufficiently surplus power generation which is 23887.54 (Megawatts) out of which the installed total capacities are 18841.32 MW for Thermal, 559.32 MW

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for Nuclear, 772.0 MW for Hydro and 3714.90 MW for renewable energy respectively as per latest available figures in 2013.

In this paper we want to study about the electricity consumption in different Sectors during the year 1989 to 2011 for Gujarat state. The study also includes the per capita consumption of electricity in Gujarat state during these years.

Statistical modeling is applied to highlight the existing situation and forecasting is done for successive years. This can be helpful for executive as well as government policy planning decisions.

Statistical Methodology and Analysis are given in sections 2 and 3,and section 4 gives the projections and conclusions based upon our analysis. Relevant database is given in Appendix A and B.

2. Statistical Methodology

The following regression model is considered to study the per capita consumption of electricity in Gujarat state during the years 1989-2011.

MODEL (A) Regression model for per capita consumption of electricity in Gujarat state.

Let us define PC_{xj}= Per capita consumption of electricity for period X_j where (j=1,2,3,...13)

Then we consider $PC_{xj} = \alpha + \beta X_j + \in_{xj}$ as the linear regression model pertaining to per capita consumption of electricity, where α and β are the parameters of the model and $\in_{xj}$ denotes the disturbance term for period X_j

We want to fit this model for the relevant data pertaining to the per capita consumption of electricity in Gujarat state.

Under the usual assumptions we can obtain OLSE of α and β given by $\hat{\alpha}$ and $\hat{\beta}$ by using the standard formulae. Then the estimated values of $\hat{p}C_{xj} = \hat{\alpha} + \hat{\beta}X_j$ can be obtained.

We can also examine the significance of the estimated regression coefficients and carry out ANOVA for the fitted model.

Per capita power consumption indicator (PPC Indicator) is worked out with 1989-90 as the base year.(see Appendix A)

MODEL (B) Regression Models pertaining to sectoral consumption of electricity in Gujarat state

The following regression model is considered to study certain sectoral consumption of electricity related to Gujarat state.

Let us define S_{ixj}=i[th] Sectoral Consumption for the period X_j ,where i=1,2,3,4,5,6,7,8,9 and j=1,2,3,…11

Then we consider $S_{ixj} = \alpha \exp [\beta X_j + U_{xj}]$ as the regression model pertaining to the particular Sectoral Consumption. Where á and â are the parameters of the model U_{xj} denotes the disturbance term for the year x_j (j=1,2,3…11)

We want to fit this model for the relevant data regarding the particular sectoral consumption for Gujarat state. Let us consider log -linear transformation for the above model so that the above equation takes the form as under:-

$\text{Ln } S_{ixj} = \text{Ln } \alpha + \beta X_j + u_{xj}$ Where i=1,2,3,…9 and j=1,2,3,…11

We define $\text{Ln } S_{ixj} = Y_{ixj}$

$\text{Ln } \alpha = A$, $\beta = B$ and $u_{xj} = Z_{xj}$

Then $Y_{ixj} = A + B X_j + Zx_j$ is the log-linear form for the above regression model. Under the usual assumptions we can obtain OLSE of A and B given by $\hat{A}$ and $\hat{\beta}$ by using the standard formulae. Then estimated values of Y_{ixi} where i=1,2,3…9 for given x_j are computed from

$$\hat{Y}_{ixi} = \hat{A} + \hat{\beta} X_j$$

We can also examine the significance of the estimated regression coefficients and carry out ANOVA for the fitted model

MODEL(C) Multiple Regression Models for Sectoral consumption of electricity in Gujarat state.

Let us consider the following notations and symbols for the models considered. General structural setup is given for the models as under.

Let Y_{ixj}= i[th] sectoral Consumption component of electricity for year X_j, where (i= 1,2,3) and (j=1,2,3,…11)

Therefore

y_{1xj}= sectoral Consumption component of electricity for domestic purpose.

y_{2xj}= sectoral Consumption component of electricity for Commercial purpose.

y_{3xj}= sectoral Consumption component of electricity for industrial purpose.

y_{9xj}= Aggregate Consumption component of electricity for all the sectors .

We define Y_{9xj}= Ln y_{9xj}

And Z_{ixj} = Ln y_{ixj} (i=1,2,3)

(e.g. when i=1, Z_{1xj}=Ln y_{1xj} which gives the natural logarithm value of domestic consumption component of electricity for the period X_j and so on.)

X_j= Respective year of consumption component of electricity corresponding to Y_i.

Particular models under this study are mentioned below:-

Model C-1

Next we pose a Log-Linear regression model as under.

$$Y_{9xj}=\beta_0+ \beta_1 Z_{1xj}+ \beta_2 Z_{2xj}++ \beta_3 Z_{3xj}+ \beta_4 X_j+U_{xj}$$

Where U_{xj} is the disturbance term.

Model C-2

$$Y_{9xj} = \beta_0+ \beta_1 Z_{1xj}+ \beta_2 Z_{2xj}+ \beta_3 X_j+U_{xj}$$

Where U_{xj} is the disturbance term.

The OLS estimates of β_0, β_1, β_2, β_3 and β_4 can be obtained under the usual assumptions and hence-

$$\hat{Y}_{9xj} = \hat{\beta}_0 + \hat{\beta}_1 Z_{1xj} + \hat{\beta}_2 Z_{2xj} + \hat{\beta}_3 Z_{3xj} + \hat{\beta}_4 Z_j$$

And

$$\hat{Y}_{9xj} = \hat{\beta}_0 + \hat{\beta}_1 Z_{1xj} + \hat{\beta}_2 Z_{2xj} + \hat{\beta}_3 Z_j$$

give the estimated values of aggregate consumption component of electricity for the required year. For all the above models while analysing care is taken to tackle the problems of hetroscadasticity and autocorrelation.

3. Analysis and Conclusions

Model-A

Per capita consumption of electricity vs time

$$PC_{xj} = 601.9615 + 78.2692\ X_j$$
$$t = (7.184)^{**} \qquad (7.414)^{**}$$
$$R^2=0.8332, \quad F=54.97^{**}, \quad n=13$$

For the above regression model pertaining to per capita consumption of electricity, regression model is found to be statistically significant. 83.32% variation is explained by the model. Elasticity of per capita consumption with unit change in the time (year) is 78.26 i.e, every year per capita consumption of electricity increases by about 78.26 units.

Per capita Power Consumption indicator (PPCI) for electricity is calculated on the basis of 1989-90 as base year (see appendix). This indicator reveals the fact that PPCI has increased substantially since 2004-05.

Model B-1

Consumption in domestic sector vs time

$$Y_{ixj} = 8.0928 + 0.09351X_j$$
$$t = (261.182)^{**} \qquad (20.469)^{**}$$
$$R^2=0.9790, \quad F=418.97^{**}, \quad n=11$$

For the above regression model pertaining to domestic sector, it is found that regression model is statistically significant. 97.96% variation is explained by the model. Elasticity of domestic sector with unit change in time is 0.9351, which shows that due to unit change in time , consumption of electricity in domestic sector increases by about 9.35%.

Model B-2

Consumption in commercial sector vs time

$$Y_{ixj} = 6.8439 + 0.1388\ X_j$$
$$t = (99.321)^{**} \qquad (13.663)^{**}$$
$$R^2=0.9540, \quad F=186.67^{**}, \quad n=11$$

Regression model for commercial Sector suggests that 95.40% variation is explained by the model. By this model it appears that for unit change in time ,about 13.88% increment is found in consumption by commercial sector.

Model B-3

Consumption in Industrial sector vs time

$$Y_{ixj} = 9.0004 + 0.09573X_j$$
$$t = (239.686)^{**} \quad (17.291)^{**}$$
$$R^2=0.9708, \quad F=298.98^{**}, \quad n=11$$

The above model for industrial sector is found to be fitted. The value of R^2 suggests that 97.08% variation is explained by the model and the unit change in time brings about 9.57% upward variation in consumption of electricity by industrial sector.

Model B-4

Consumption in Public lighting vs time

$$Y_{ixj} = 4.9566 + 0.05530\ X_j$$
$$t = (152.070)^{**} \quad (11.508)^{**}$$
$$R^2=0.9364, \quad F=132.44^{**}, \quad n=11$$

From the regression analysis for public lighting sector in Gujarat, it is found that the model is adequately fitted. Elasticity of consumption of electricity by public lighting sector is 0.05530, which shows for unit change in time the percentage of consumption for public lighting sector increases about 5.53%

Model B-5

Consumption in Agriculture vs time

$$Y_{ixj} = 9.5163 - 0.01678X_j$$
$$t = (102.794)^{**} \quad (-1.230)$$
$$R^2=0.1439, \quad F=1.51, \quad n=11$$

The above model does not seem to be statistically appropriate.

Model B-6

Consumption in Public water works vs time

$$Y_{txj} = 6.2836 + 0.07599X_j$$
$$t = (284.710)^{**} \quad (23.353)^{**}$$
$$R^2=0.9838, \quad F=545.37^{**}, \quad n=11$$

Regression model for electricity consumption for public water works suggests that 98.38%, variation is explained by the model. In this model, unit change in time accounts for about 7.59% increment in consumption of electricity by public water works segment.

Model B-7

Consumption in Railway vs time

$$Y_{txj} = 5.8494 + 0.06278X_j$$
$$t = (268.464)^{**} \quad (19.544)^{**}$$
$$R^2=0.9770, \quad F=381.95^{**}, \quad n=11$$

The regression model for consumption in railway sector is found to be highly significant. On an average logarithmic value of consumption of electricity by railway sector increases at the rate of about 6.27% with unit change in time. 97.70% variation is explained by the model itself. Thus an upward trend is found by this model.

Model B-8

Consumption in others vs time

$$Y_{txj} = 7.7986 + 0.1207\ X_j$$
$$t = (41.534)^{**} \quad (4.361)^{**}$$
$$R^2=0.6788, \quad F=19.02^{**}, \quad n=11$$

From the above regression model for consumption in other sectors, it is found that model is statistically significant. About 67.88% variation is explained by the model. Elasticity of consumption of electricity by other sectors with respect to time is 0.1207, which shows that due to unit change in time the consumption of electricity by other sectors increases by about 12.07%

Model B-9

Aggregate consumption vs time

$$Y_{9xj} = 10.2601 + 0.06440\ X_j$$
$$t = (169.681)^{**} \qquad (7.224)^{**}$$
$$R^2 = 0.8529, \quad F = 52.19^{**}, \quad n = 11$$

Model pertaining to aggregate consumption of electricity in Gujarat state is found to be statistically significant. The value of R^2 is 0.8529 which shows explained variation is about 85% due to time. Regressend (aggregated consumption) increases by 6.44% as regressor (time) increases by one unit (year).

Model C-1

Aggregate Consumption vs Domestic and Commercial Sectors Consumption with time as a trend variable

$$Y_{9xj} = 5.7273 - 0.2040Z_{1xj} + 0.9035Z_{2xj} - 0.04193X_j$$
$$t = (1.618) \quad (-0.312) \quad (3.074)^{*} \quad (-1.443)$$
$$R^2 = 0.9814, \quad F = 123.45^{**}, \quad n = 11$$

From the above multiple regression model it is revealed that 98.14% variation is explained by the model. Among partial regression coefficients only one coefficient pertaining to commercial sector is found to be statistically significant. That means commercial sector plays a significant role in the aggregate consumption of electricity.

Model C-2

Aggregate Consumption vs Domestic and Commercial Sectors Consumption with time as a trend variable

$$Y_{9xj} = 3.7453 - 0.5554Z_{1xj} + 0.7282Z_{2xj} - 0.6694Z_{3xj} - 0.4883X_j$$
$$t = (1.066) \quad (-0.861) \quad (2.474)^{*} \quad (1.507) \quad (-1.801)$$
$$R^2 = 0.9865, \quad F = 109.97^{**}, \quad n = 11$$

Above regression output suggests that 98.65% variation is explained by the model. Among partial regression coefficients only one regression coefficient pertaining to commercial sector is found to be statistically significant which means that commercial sector plays a significant role in the overall consumption of electricity.

4. Projections for electricity consumption in Gujarat state based upon model- B

Sector	Year	Projected values of electricity consumption (Million Units)
Domestic	2011-12	10046.621
	2012-13	11031.431
	2013-14	12112.775
	2014-15	13300.117
Commercial	2011-12	4962.7900
	2012-13	5701.7385
	2013-14	6550.8182
	2014-15	7526.2800
Industrial	2011-12	25572.169
	2012-13	28141.298
	2013-14	30968.538
	2014-15	34079.818
Public Lighting	2011-12	275.9749
	2012-13	291.6679
	2013-14	308.2533
	2014-15	325.7818
Agricultural	2011-12	11102.082
	2012-13	10917.284
	2013-14	10735.562
	2014-15	10556.865
Public Water works	2011-12	1333.5053
	2012-13	1438.7928
	2013-14	1552.3933
	2014-15	1674.9632
Railway	2011-12	737.23168
	2012-13	785.00272
	2013-14	835.86923
	2014-15	890.03178
Others	2011-12	10377.918
	2012-13	11709.637
	2013-14	13212.245
	2014-15	14907.672
Aggregate	2011-12	61882.237
	2012-13	65999.044
	2013-14	70389.727
	2014-15	75072.508

Projections for per capita Consumption of electricity and PPCI for Gujarat state based upon Model A

Year	Projected Value (in units)
2011-12	1697.730 (467.69)
2012-13	1776.000 (489.26)
2013-14	1854.269 (510.82)
2014-15	1932.538 (532.38)

(Figures in bracket give projected values of PPC indicator for the year 2011-2012 to 2014-2015 as compared to the years 1989-90.)

5. CONCLUDING REMARKS

(1) PPCI indicator measure calculated for the years 1989-90 to 2010-11 and their projections based upon the fitted model envisages the fact that per capita power consumption for the coming years is increasing very remarkably as compared to 1989-90 year cenerio. This fact also suggests for establishing enhancement of electric power supply to meet individual demand in the coming years.

(2) Out of the models studied in this paper the agricultural sector is not that responsive, as statistical modeling for this does not respond properly. This may be due to the fact that there may be theft in electric power usage mostly among villages and barren areas. This suggests to take drastic steps and be more vigilant at such places whenever it is found to be under suspicion.

(3) Though overall picture for consumption of electricity in Gujarat state appears to be little bit satisfactory, the policy designers, executives, government can be very much alert to keep the things moving in the coming years to establish sustained economic growth in the state.

6. ACKNOWLEDGEMENTS

We thank DR. B.B.Jani for his encouragement to prepare this research article. We also thank the referee (anonymous) for his review of our paper and suggestions to improve the earlier draft of this article.

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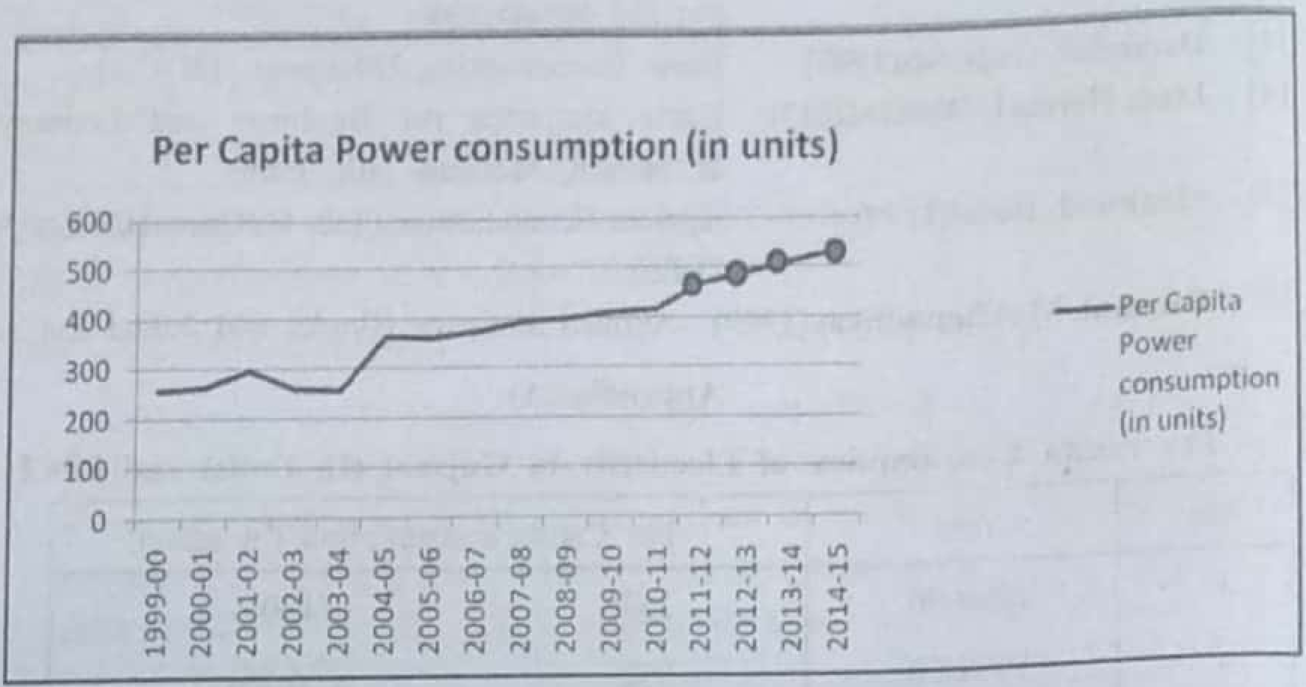
Appendix-(A)

Per capita Consumption of Electricity in Gujarat (In Units) and PPCI

No	Year	Per Capita consumption (in units)	
1	1989-90	363	(100)
2	1999-00	932	(256.75)
3	2000-01	953	(262.53)
4	2001-02	963	(295.29)
5	2002-03	944	(260.06)
6	2003-04	932	(256.75)
7	2004-05	1321	(363.91)
8	2005-06	1313	(361.71)
9	2006-07	1354	(373.00)
10	2007-08	1424	(392.29)
11	2008-09	1446	(398.35)
12	2009-10	1491	(410.74)
13	2010-11	1512	(416.53)

(Source: Socio- Economic Review, Gujarat state) Figures in bracket gives PPC Indicator for relevant years with 1989-90 as the base year.

Graphical Representation of PPCI(Plotted graph is for PPCI)



Appendix-(B)

Sectorwise Electricity Consumption in Gujarat

X	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9
2000-01	3981	1279	9813	166	15489	611	383	2603	34325
2001-02	3922	1278	9817	160	15695	612	406	2907	34797
2002-03	4136	1353	10708	165	12965	685	409	3439	33860
2003-04	4613	1543	11270	168	11625	721	420	3785	34145
2004-05	5026	1713	12340	177	9958	762	477	3965	34418
2005-06	5490	1905	13244	189	10617	816	501	5396	38358
2006-07	6097	2154	15680	202	11016	863	578	4983	41513
2007-08	7160	3183	18987	226	11209	1001	590	11117	53473
2008-09	7825	3572	19610	240	11733	1064	625	10941	55610
2009-10	8366	3925	21226	257	12826	1179	666	6560	55005
2010-11	9353	4412	23415	265	13285	1264	683	5993	58670

(Source: Socio- Economic Review, Gujarat state) *Figures in Million Units

Symbols used in Appendix (B)

X : Year
Y_1 : Domestic
Y_2 : Commercial
Y_3 : Industrial
Y_4 : PublicLighting
Y_5 : Agricultural
Y_6 : WaterWork
Y_7 : Railway
Y_8 : Others
Y_9 : Total

Statistics Day : 29th June 2015

P.C. MAHALANOBIS

WILLIAM GEMMEL COCHRAN+

H. D. BUDHBHATTI*

Very commonly and popularly known as W. G. Cochran was born at Ruthglen, Scotland on 15 July 1909. He was son of Thomas and Jeannie Cochran. His father, Thomas the eldest of seven children, had begun his lifetime employment with the raiload at the age of thirteen. Thus william (nick name Willie pronounced wully) was born in moderate circumstances. The Cochrans moved several times, finally setting in Glasgow, where in 1927, william was the first in the Glasgow University Bursary competition. This award enabled him to finance his studies at the University, from which he received M.A. degree with first class honours in Mathematics and Physics in 1931. He shared the Logan medal for being the most distinguished graduate of the Arts Faculty. As a result he had secured a scholarship for graduate work in Mathematics at Cambridge.

John Wishart had transffered from the Rothamsted Experimental station to Cambridge in 1931. Fortunately Cochran elected to take Wishart's course in mathematical stastistics, followed by his practical statistics course in the school of Agriculture. In 1934 Cochran wrote his important paper presenting **Cochran's theorem** under Wishart. In the same year he was offered a position at Rothamsted that had become available when R. A. Fisher left to accept Galton chair in Eugenics at University College, London and Frank Yates had moved up to become head of statistics department. Cochran had to decide whether to complete his docotrate at Cambridge or accept the Rothamsted position.

Cochran stayed at Rothamsted for five years. During this time he worked closely with Yates on experiemental designs and sample survey techniques and had many opportunities to discuss problems with Fisher, who continued to spend much time at

+ This article is adapted from wikipedia (The free encyclopedia), as well as Complete Dictionary of Scientific Biography through net collection.

* Ex. CSO, Head, Statistics Dept., GSRTC, Ahmedabad.

Rothamsted. By the time he left, Cochran had published 23 papers and had become a well known statistician. One of his most exhaustive projects was a review of the long term series of field experiment as at the Woburn Experimental Station.

Cochran and Yates collaborated on research on the analysis of long term experiments and groups of experiments. Here Cochran initiated his illustrious research on the Chi-Squared distribution and the analysis of count data. On July 1937, he married Betty Mitchell, who had Ph.D. in Entomology. They had two daughters and a son Cochran visited Iowa state statistical Laboratory in 1938 and accepted a position there in 1939 to develop a Graduate program in statistics (it was part of mathematics department in 1947). There he and Gertrude cox initiated their collaboration that culminated in their famous book **Experimental Designs (1950)**

Late in 1943 Cochran took leave from Iowa State to join S.S. Wilks statistical Research Group at Princeton University as a research mathematician working on army-nevy research problems for the office of scientific Research and Development. Much of his work there was devoted to on analysis of his probabilities in naval combat that utilised little of his statistical background.

In 1945, he was asked to serve on a select team of statisticians to evaluate efficacy of the world war II bombing raids.

In 1946 Cochran joined the newly Created North Carolina Institute of Statistics (Directed by G. Cox) to develop a graduate program in experimental statistics at North Carolina State College. (now a University)

Harold Hotelling was to develop a graduate program in Mathematical Statistics at the North Carolina at Chapel Hill. Conchrn was a member of the organising committee for the International Biometric Society, which was founded in 1947 at woods Hole, Massachusetts. His major contribution at North Carolina State was setting a few foundation for a Graduate Programe balanced in theory and practice and well co-ordinated with the more theoretical program at Chapel Hill.

In January 1949, the Cochrans moved to Baltimore, where he chaired the Biostatistics department in the school of Hygiene and public health at the John Hopkins University. He was faced with medical rather than agricultural problems there. He had to develop procedures to obtain recitable information from observations rather than from experimental data, an area that became his dominant interest for the rest of his life.

In 1963, he published **Sampling Techniques.** Cochran remained at John Hopkins until 1957, when he joined the faculty at Harvard University to help Fred Mosteller and others to develop the department of Statistics. He continued to work closely with research workers at the Medical School and School of Public Health but also did his own reserarch on a variety of topics. In 1967 Cochran was co-author with G.W. Snedecore of the sixth edition of Latter's Statistical methods. He retired from Harvard in 1976.

Despite a dozen years of serious health problems, Cochran continued a wide range of professional activities and working on seventh edition of **Statistical Methods** and a book on **Observational Studies** until shortly before his death.

Cochran was Spresident of the Institute of Mathematical Statistics (IMS) in 1946 and the **American Statistical Association** (ASA) in 1953. He served as editor of the Journal of the ASA (1945-1950.) He was president of the Biometric Society (1954-1955) and of the International Statistical Institute (1967-1971). He was Vice President of the American Association of the advancement of Science in 1966. Cochran was elected to the National Academy of Science in 1974. He was fellow of ASA, the IMS, the AAAS and also fellow of Royal Statistical Society. He was also Guggenheim fellow (1964-1965).

He served on a number of scientific investigatory panels, including those concerned with the kinsey Reports, the efficacy of the Salk Poliovaccine, the effects of radiation at Hiroshima and the surgeon general's report on smoking. He wrote more than 100 Scientific Articles.

Probably Cochran's greatest contribution to scienctific community were his guidance of students (he guided more than 40 Ph.D. dissertations) and his textbooks. He had the ability to present complicated materials in a format that could be understood by anyone who had an interest in collecting and analysing data.

Cochran is best known for his two very precious books on sampling and design. He is also famous for his research work comprising (1) Cochran's C Test, (2) Cochran's Q Test, (3) Cochran's theorem etc.

C0chran died on 29 March 1980 in Orleans, Massachusetts, USA. Cochran was that rarity, a man with both a keen mind and the desire to use it for the benefit of mankind. His office was always open to the Struggling students, nonplussed scientist or inquiring citizen.

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EDITORIAL

We are pleased to present this issue of the Journal (NSV XI), June 2015, No. 1) to our readers. This issue contains 4 research articles, Biography, S V. News letter, Readers Forum etc.

First Research article is a study in construction of a specific design done by **D. K. Ghosh** and **S. Ahuja.**

Second Research article is a study for EPQ inventory model under given situations. This work is done by **U. B. Gothi** and **Devyani A. Chatterji.**

Third Research article is an inventory model study developed and analysed under given situations. The work is done by **Kirtan Parmar, Indu Aggarwal** and **U. B. Gothi.**

Fourth Research article is regarding statistical analysis for power consumption in Gujarat State. This study is carried out by **H. M. Dixit** and **S. G. Raval**.

Bographical Sketch for famous statistician **W. G. Cochran** is given by **H. D. Budhbhatti**.

We regreat for sad demise of honourable **Prof. A. G. Pathak** of M. S. University, Vadodara. (Condolnce meeting report is given in S. V. News Letter.)

We thank the following referees who have helped us in the evaluation work for this issue

R. G. Bhatt	**J. R. Purohit**
D. S. Dave	**P. H. Thaker.**

We sincerely thank our valued contributors, learned evaluators and readers for their support and co-operation.

* A specific proforma for membership update is attached with the journal. You are requested to fill it up alongwith your feedback and send us immediately as directed.

Ahmedabad

Date 15 June, 2015

SV NEWS LETTER

*K. Muralidharan

(We begin this section to give news update to the readers.)

- **Prof. P. G. Phatak** of M. S. Uni. Vadodara expired on 30th April, 2015. Condolence meeting message is presented below.

- As usual this time also, 29th June, 2015 was celebrated as **Statistics Day** in memory of Prof. Mahalanobis in almost all the universities in India. This time theme was on **Social Statistics.**

- One of the most prestigious International conference on **Statistics for 21st Century** will be held at Kerala university statistics department at Trivendrum during Dec. 17-19, 2015. You may contact Prof. Satheeshkumar for details. Email : icstc2015@gmail.com

- A new association is organised as AIAP (Association of Inventory Academicians and Practioners) by Prof. C. K. Jaggi at New Delhi. For details you may contact Dr. Nita H. Shah (Prof. Maths Dept., G. U., Ahmedabad) or email to nitahshah@gmail.com

- A study workshop is organised at GOA by CSO. on GDP base year revision in July-August 2015. Collect information from www. mospi.nic.ac.in

- 48th Annual convention of Operations Research Society of India (ORSI) and International Conference will be held at Bhubneshwar, Orissa during Dec. 17-19, 2015.
  In this conference, awards are given for best student paper, best application paper, best theoretical paper and best theis on O.R.
  For further details contact conveners Prof. Shrikant Pattnaik, srikantpatnaik@soauniveristy.ac.in; Prof J R. Nayak, jrniter@yahoo.co.in

- One recent book published on 'Six Sigma for Organisational Excellence' by Dr. K. Muralidharan (HOD, Stat. Dept., M. S. Uni., Vadodara 380002) Publisher is Springer & Co.

- On Statistics Day Celebration special video presentations were given from Dr. Babasaheb Ambedkar Open University. For many such other details also you may log into : URL:baou.edu.in/swadhyay.tv and also URL : baou.edu.in/swadhyay.radio

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* Head, Statistics Dept., M. S. Uni., VADODARA-380 002.

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Fax. : 91-265-795569

Date : 02.05.2015

CONDOLENCE MESSAGE

Today on 2nd May, 2015, the meeting of the members of the Faculty of Science was held in Room No. 4 of the Department of Statistics to mourn the sad demise of **Prof. A. G. Phatak.**

Prof. A. G. Phatak joined Department of statistics in 1953 as Demonstrator, became lecturer in 1957, Reader in 1970, and Professor in 1979. He retired on 31.10.1991. In his long 38 years of service in the Department, his contribution in the growth of the department is significant. He also groomed several generations of students who are doing / did very well in their carreer. **Prof. A. G. Phatak** also headed the Department during the period 1988-1989.

He passed away on 30th April, 2015. The faculty and specifically department has lost a very senior and important member who was a continuous source of inspiration.

We pray to the almighty that his soul may rest in peace. May God give enough courage and strength to the members of his family to bear the loss due to his sad demise.

Head,
Statistics Department,
Faculty of Science,
M. S. Univeristy, Baroda-2

READERS FORUM

*A. M. PATEL

(We begin this section to present views, crities, feedback from our readers.)

[1] APPRAISAL

- **K. R. Patel :** Journal is noteworthy approach for establishing rapport between theoreticians and practitioners in the field.
- **D. S. Swaminarayan :** Congrats to board members for their efforts of the journal.
- **Hiren Doshi :** The journal enlightens us by means of transfer of knowledge and information.
- **Mukesh B. Shah :** This work is indeed an outstanding approach. It is a creditable work.
- **V. H. Bajaj :** Articles published on O. R. applications, Econometric studies and database analysis are really good. Biographical sketch is a new pattern.
- **N. M. Patel :** It is a good teamwork. Life sketches about dignitories like Fisher, C. R. Rao, Mahalanobis etc. are inspiring. Perhaps class room notes, technical notes and statistical quizz should be made a permanent feature of the journal.

[2] CRITICS / CORRIGENDUM (December 2014 issue)

Page 14-26 Paper by Bhadauria and Gothi

(1) Read equation (1.1) as $f(x,p) = (1-p)^{x-1} \cdot p$

(2) Read equation (1.2) as $f(x,p) = (1-p)^{x-1} \cdot p$, $V(x) = \frac{1}{\bar{X}}$, $F(x) = 1 - q^x$

(3) $\text{MOME} = \text{MLE} = \frac{1}{\bar{X}} = n \Big/ \sum_{i}^{n} X_i$

(4) Last line read T_I as T_1.
Page 16 second line read C_I as C_1.
(Authors are requested to refer to Ref. 10, 14 and 15 for details.)

*R. T. Ratani

* Rtd. Principal, H. K. Commerce College, Ahmedabad and Ex. Secretary, Gujarat Vidyasabha and Brahmchariwadi Trust, Ahmedabad.

- (December 2014 issue)

  Page 1-13 Paper by Kirtan Parmar and Gothi

  (1) Page 5. After equation (1.3) ... higher powers of θ

  (2) Page 6. Equation (1.4) ... higher powers of θ

  (3) Page 8. Sec. 5 Example. Hidden values $\alpha = 1$, $\beta = 0.02$, $\theta = 0.01$

  (4) Page 9. Table. First column, after α, Next is β.

  (5) Title Page ---> Last Line ---> Miscellaneous.

***H. D. Budhbhatti**

- (December 2014 issue)

  Page 33-40 Paper by R. Pandya and R. G. Bhatt

  (1) Page 36 Table 1

  Add in the paragraph after Table 1 : An increase in % was however observed during 1991-2001 by about 3% point.

  (2) Page 36. Method 1 and 2.

  Rn = Hn/Pn is the number of houses per person in the nth year. For computation of households the result can be corrected appropriately.

  (3) Method 4 last line correct q as θ.

  (4) It would be better if G.P. Curve can be fitted with statistical conclusions.

*** P. J. Jhala**

INSTRUCTIONS TO AUTHORS

Editorial board invites research articles, brief summary of research project reports, review articles, informative articles, research and project reports, review articles, informative articles, research notes, class room notes, statistical querries and other problems of interest as well as any relevant informations that can significantly highlight the applications part pertaining to statistics subject.

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Please also send your article on email of Sankhya Vignan **svgsa2015@gmail.com** for quick response.

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W. G. COCHRAN*

William Gemmel Cochran (15 July 1909 to 29 March1980). Very commonly and popularly known as **W. G. Cochran** was a very prominent statistician. He was born in Rutherglen at Scotland. He studied Mathematics at the University of Glasgow and also at the University of Cambridge. He worked at Rothamsted Experimental Station from 1934 to 1939 when he moved to the United States. There he helped to establish several departments of statistics. His longest spell in any one university was at Harvard which he joined in 1957 and from which he retired in 1976. He had the advantage of working with some other statisficians like **R. A. Fisher, Frank Yates, Fred Mostellar** etc. He died on 29 March, 1980 at Massachusetts, USA. He wrote more than 100 research articles which are universally known. He wrote many books also and he is very famous for his two books - **Sampling Techniques** and **Design of Experiments** (with G. M. Cox), which are very standard text books in universities.

W. G. Cochran is also very well known for his research work like *Cochran Theorem, Conchran's C Test, Cochran's Q Test* etc.

(* Brief Biographical sketch is given in the Journal)

This page is specially donated by **Prof. Shailesh Teredesai** (Ex. Head) Statistics Dept., S. M. Patel Institute of Commerce, GLS, Ahmedabad.

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