

# SANKHYA VIGNAN

## संख्या विज्ञान

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DECEMBER 2015

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# **SANKHYA VIGNAN**

## **संख्या विज्ञान**

(e-mail ID : [svgsa2015@gmail.com](mailto:svgsa2015@gmail.com))

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C/o. Statistics Department,  
Gujarat University, Ahmedabad-380 009. (India)  
Ph. (R) 27476770, (M) 9824057902  
E-mail : [bbjani2012@gmail.com](mailto:bbjani2012@gmail.com)

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Near Shyamal Cross Road, Sattellite, AHMEDABAD-380015

Contact No. : +91 9909900799

e-mail : [drjayesh.purohit@gmail.com](mailto:drjayesh.purohit@gmail.com)

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**Note :** For any queries you may please contact (1) Dr. B. B. Jani, e-mail : bbjani2012@gmail.com; (2) Dr. J. R. Purohit, drjayesh.purohit@gmail.com, svgasa2015@gmail.com

## EDITORIAL

We are extremely happy to present this issue of our journal (NSV XI, Dec. 2015, No.2) to our readers. Incidentally we have completed 11 years of this journey, for which we thank our valued contributors, learned evaluators and readers for their co-operation and consistent support. This issue is a **special issue** in this chain of 11 years issues.

From this issue we begin a new section entitled **Statistics and Management**. Under this heading a very lucid and interesting article on **Multivariate Analysis - A Reality** has been furnished by **A. C. Brahmhatt**.

Next is an **Article** entitled **Implementation of statistics in Business and Industry** which is presented by **Jayesh R. Purohit**.

There are five **Research Articles** in this issue. **First Research Article** is related to the design of experiments and it is given by **C. C. Gujarathi** and **Vikas G. Doshi**.

**Second Research Article** pertaining to environmental statistics is furnished by Rakesh Srivastav.

**Thirst Research Article** is a specialised study in Inventory control and it is given by **Ankit Bhojak** and **U. B. Gothi**.

**Fourth Research Article** is an interesting study in Regression analysis for Cadila Health Care Ltd. It is given by **D. J. Shah** and **H. M. Dixit**.

**Fifth Research Article** is a specialised study in seismological field given by Rupal **H. Budhbhatti** and **B. P. Rathore**.

There are two **Technical Notes**.

**First Note** concerns about some specific properties of T.P. This is given by **J. P. Shah** and **A. J. Patel**.

**Second Note** is an introduction for social sector indicators which is presented by **S. G. Raval** and **M. P. Vaghela**.

There are two interesting **Biographies** in this issue.

**First Biographical Sketch** is for very eminent Indian Statistician **Prof. P. V. Sukhatme** which is presented by **H. D. Budhbhatti**.

**Second Biographical Sketch** is for British Economist Angus Deaton who was given **Nobel Prize** for study on poverty. This has been furnished by **Jayesh R. Purohit**.

There is a **Book Review** carried out on a very famous book on Statistics. This is presented by **H. M. Dixit**.

As usual, the other sections on **S V NEWS LETTER, READERS FORUM** etc are included in this issue.

We are highly indebted to our following **Referees** who did excellent job of evaluating the material submitted for this issue.

(Names are taken one by one in order of their appearance in the issue)

- |                         |                            |                        |                             |
|-------------------------|----------------------------|------------------------|-----------------------------|
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In the last issue we had attached a proforma for membership update and feedback from our readers. We have not received good response. Once again we resubmit this proforma. All the readers/members are advised to kindly fill up the complete details and submit the same at the communication address or you may send it on our email ID. (**Email**: svgsa2015@gmail.com)

You are also requested to kindly have your email so that we can have the ecommunication data.

We are making efforts to make this journal **DIGITAL**. Most probably from the year 2016 onwards you may be able to get your digital copy through your email ID submitted to us.

We wish you all a very happy and prosperous New Year 2016.

Ahmedabad

Dated: 31-12-2015.

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e-mail : drrgb@yahoo.com  
Nationality : Indian  
Address : Department of Statistics,  
Gujarat University, Ahmedabad-380 009.
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Nationality : Indian  
Address : C/o. Department of Statistics,  
Gujarat University,  
Ahmedabad-380 009.  
(M) 9824057902 (R) 27476770  
e-mail : bbjani2012@gmail.com
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e-mial : darjidinesh4675@gmail.com
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**MULTIVARIATE ANALYSIS - A REALITY**

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**A. C. Brahmhatt**

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**MULTIVARIATE ANALYSIS – A REALITY**

The experienced and statistically oriented researcher, having gone through the stages of a research process viz. defining a research problem, choosing the appropriate research design, collecting data etc. reaches the crucial stage where he has to select an appropriate Data Analysis Strategy. It would depend, along with several other considerations, on the number of variables of interest. If it were only one ,it would call for Univariate analysis, if there were two ,the Bivariate analysis would suffice; but had they been more than two and are to be analyzed simultaneously it calls for the use of the Multivariate Analysis.

Limiting the analysis only to Univariate or Bivariate might probably serve the purpose for justifying the course outline prescribed for the undergraduate or graduate classes. For example, in order to develop the sales estimates for the product of a business unit based on the advertising expenditure incurred by that unit ; a simple Bivariate regression analysis can serve the purpose. But do the sales depend on advertising expenditure only? Any rational sales manager would admit that there are several variables, in addition to advertising expenditure impacting sales. Therefore a reasonably true sales estimate could be generated only by the use of Multiple Regression Analysis.

The moment you step out the classroom, and enter into the world outside, it is multidimensional and multifarious. In order to tackle the multi dimensionality, the multivariate analysis and use of the multivariate techniques is only the answer.

There are clearly two categories of Multivariate techniques—i) the techniques for analysis of dependence and ii) the techniques for the analysis of interdependence amongst variables. The Multiple Regression Analysis that we talked about belongs to category (i). Also there are few more techniques like Discriminant Analysis where the dependent variable has to be

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1. Chairperson, Ph.D. (Ext.) Program, Institute of Managemnt  
Nirma University

categorical. For example researcher is facing a problem as to how to discriminate the Heavy users, Medium users and Light users of a product; the Discriminant analysis would be more pertinent. There is also one more technique in this category, and it is a Binary Regression or Logit Regression, where the dependent variable is binary in nature . For example, if the product manager is interested in knowing as to how likely a consumer is to belong to loyal customer group, Binary regression could be of help to him. These techniques are functional in nature.

The category (ii) of Multivariate techniques aims at the analysis of interdependence amongst variables. They are structural in nature. Factor Analysis belongs to this category. It is a versatile technique to condense the original large set of variables into a smaller , meaningful subset of factors. For example, the potential buyers of a small car may have expressed 20 attributes of a small car that attract them to buy it; Factor Analysis condenses these 20 attribute into say only four meaningful factors like fuel efficiency, convenience, comfort and luxury .On the basis of these factors, the manufacturer of the car can do even market segmentation.

Whereas the Factor Analysis clubs the related variables and generates factors, the Cluster Analysis generates the clusters of people. For example, the customers shopping in Reliance Mall could be clustered as the customers who are Fun lovers, Economic shoppers, those who combine shopping with eating, the Indifferent customers etc.

The technique called '**Multidimensional Scaling (MDS)**' exhibits the perceptions and preferences of respondents for some product or service generating the perceptual and preference maps. For example, the product manager of a particular brand of shampoo can learn as to how the customers perceive his brand and the other 5 competitive brands i.e. which brands are perceived more similar, similar or less similar on some attributes of the shampoo like its whiteness, shininess, prevention of hair loss etc. MDS would help him to develop his Brand-positioning strategy.

There is no dearth of such advanced multivariate techniques whose access has now become so easy and simple in the wake of availability of the latest softwares such as SPSS, SAS, MDSCAL, MDPREF, MINITAB AMOS etc. Gone are the days of the tedious calculations and derivations. The researchers should not be apprehensive of these techniques thinking that they are esoteric and complex, because the Multivariate Analysis is the Reality.

#### **ACKNOWLEDGEMENTS**

I thank Dr. B. B. Jani for his invitation to present this article under the caption, 'Statistics and Management'. I also thank the referee for his review of my article.



## IMPLEMENTATION OF STATISTICS IN BUSINESS AND INDUSTRY

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Jayesh R. Purohit\*

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### ABSTRACT

Statisticians have devised many tools for application and these are available to be utilized for general business improvement and industrial problem solving. However, there is a wide gap between the available tools and what are practiced in business and industrial organizations. Thus it is important for statisticians to direct serious attention to bridging this gap; if statistics is to be relevant in business and industry and to the society at large. In this article we look at some ideas for implementation of statistical methods in business and industry.

#### 1. Introduction

Implementation means different things in different contexts and to different people. What do we mean by implementation? **H G Wells** said ‘**Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write**’. **Harry & Schroeder** remarked ‘**We believe that statistical knowledge is to the information and technical age what fossil fuel was to the industrial age. In fact, the future of industry depends on an understanding of Statistics**’.

These statements imply that Statistical Thinking and Methods should become part of the knowledge; base of an organization and part of doing business. This is the kind of implementation we are discussing. In other words, a business or industrial organization institutes **Statistical Thinking** in all its functions and the use of statistical tools and data based decisions becomes a part of the everyday business.

In this context, we are not thinking just about a statistician consulting with a scientist or an engineer for a project. Although such activities are very important, our goal is much broader. Our vision is that **Statistical Thinking** and tools should be entrenched in the organization so that they play a prominent role in its daily activities. In such organizations the roles of statisticians are quite different from the traditional one of helping a client with some data analysis.

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\* Mantra Consultants : Social and Industrial Support, Ahmedabad  
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## 2. Statistics in Business and Industry

In general, a typical organization has three levels:

**Strategic, Managerial and Operational.** This classification is somewhat general and arbitrary. However, this can help identify and emphasize different tools to be directed at the different levels.

We envisage Statistics to play important roles at all these levels.

### 2.1 Strategic Level (Top of an Organization)

At the strategic level the most emphasis should be on Statistical Thinking which has the following components:

- a) Notion of a Process
- b) Notion of measurement and data based decisions
- c) Understanding and dealing with variation
- d) Statistical tools and
- e) Systematic approach.

The notion of process thinking is fundamental to any organizational change. Every action has some inputs and outputs which need to be identified in every context. Decisions at the strategic level should be based on facts supported by appropriate data and this requires an understanding of variation.

Business and Industry have seen the arrival and demise of many programs such as Total Quality Management (TQM). Embracing any program that comes along without firm commitment and understanding is doomed to failure.

### 2.2 Managerial Level (Middle Level)

This is the level at which systems are devised for implementation of the directions taken by upper management. In particular, systems for robust product and process design, process control and improvement and training are the responsibility of middle management. Understanding of some statistical tools and statistical thinking are prerequisites for those who are designing these systems.

### 2.3 Operational Level

At this level, the methods are implemented through the system built at the managerial level. Understanding of statistical tools such as control charting, capability analysis, design of experiments, measurement system analysis, regression analysis etc. are essential. Appropriate statistical tools need to be used by operational people as part of their daily work. People in some areas need to know the details only of certain statistical tools. For instance, an operator responsible for maintaining stability of a process by charting should know the workings of a control chart but need not know a lot about design of experiments. On the other hand an engineer responsible for process improvement should be knowledgeable in several aspects of statistical process control and design of experiments.

### **3 Implementation: General Issues**

#### **3.1 Commitment of Top Management**

For the success of any program that affects an organization as a whole, full and highly visible commitment of senior management is essential. Thus the vision and values of the top level management is highly important. Employees must perceive active leadership and involvement of senior people in implementation.

Top management has to assess the situation early and to decide to allocate the needed resources. They have to decide in advance what role they can and will play. For example, the success of the Six Sigma program at General Electric and Motorola is due to the commitment of its senior management.

Recognition of the potential benefits of implementation in the beginning can help focus on what is needed. Top management must recognize that, in addition to help solving problems and improving processes, statistical tools can help increase customer satisfaction and help measure the performance of the organization. Implementation of statistical tools is an ongoing process and it helps the organization to be a learning organization and a knowledge based enterprise. **Knowledge based organizations will be the successful ones in the long run.**

#### **3.2 Role of Statisticians**

The traditional role of a statistician in business or industry has been to act as a consultant to projects, or to train some workers in certain tools such as statistical process control and design of experiments. This role has to be broadened. Statisticians need to teach at all levels of an organization (senior managers, scientists and engineers, middle managers and operational people). Statisticians have to be leaders, facilitators, aide to management etc. They have to identify the role of statistics in various business functions and also have to interact with the outside world. These require broadening statistician's skills set.

Statisticians need to acquire communication skills and have to be good communicators. We have to keep the statistical tools appropriate and simple. We also need to make sure that the implementation adds value to the organization.

#### **3.3 Systems Thinking**

As in any other implementation, there are several components involved and these need to be considered as a part of a system for successful implementation. Some of these components include people, technology, organizational structure and culture, methodology, etc. and these components have to work jointly. There will be interaction between these components. Systems need to be built such that the interaction effects are positive so that the total effect from the components is more than the sum of the individual effects.

For instance if there are two components A and B; then  
Effect (A + B) = Effect (A) + Effect (B) + Effect (AB);

Where, Effect (AB) is the 'interaction' between the components A and B. Statistical tools should be implemented such that Effect (AB) is positive, so that Effect (A + B) is more than Effect (A) + Effect (B).

A negative 'interaction' can not only lower the total effect but also can cause long term problems.

### **3.4 Planning for Implementation**

Before embarking on the implementation of statistical methods one has to have a plan with answers to the following questions

- a) How does it start?
- b) Who are responsible for the tasks?
- c) When do the activities take place?
- d) What is the scope of the system?
- e) Is there a calendar of activities?
- f) Is there a measurement system to track progress of activities?
- g) What are the review points?
- h) Are resources assigned for the planned activities?

The important fact is that it is essential to have a systematic and disciplined approach to implementation. Implementation of statistical tools needs to be interfaced with such systems.

Most of the systems are very similar and it does not matter which one is used. It is better to go with what is already available in the organization rather than looking for something new. It is important to be consistent and stable with it. It is not a good idea to change to some other system in the middle. It is also possible to interface with quality systems such as ISO.

### **4 Implementation via Training and/or Consulting**

All organizations have a certain set of existing knowledge base and acquiring new knowledge may require changes in thinking and culture. An assessment of the existing knowledge and the skills are necessary before embarking on training to introduce new skills and knowledge into any organization. Also any plans for training should reflect an understanding of the existing knowledge.

Training needs for different levels of an organization can be very diverse. The distinctive needs should be recognized and the training programs should be designed in such a way to suit each of the Strategic, Managerial, and Operational levels.

Trainers should have good statistical knowledge and good business and/or industrial experience. They should be aware of the structure and culture of the organization, the background of the trainees and the training context. It is important to make sure that the training material is relevant, appropriate, of good quality and at the correct level. Schedule and duration of each module is also important. In addition, presentation of the material in an understandable and enjoyable way requires careful planning. The use of appropriate technology is also important. For a certain level one technology may be

relevant while another level requires the use of a modified technology. Communication between the trainer and the trainee and that between software and participants should be smooth. The interaction between people and technology should be positive.

Statistical training can be interfaced with other training in the organization such as ISO-9000, ISO -14000, Six Sigma. In this context sequencing need to be carefully planned.

## **5 Implementation via Education**

Today's students are tomorrow's employees and employers. Graduates with technical and non-technical skills are needed in business and industrial organizations.

These students have to get their education from the universities. Hence it is important for universities to give more attention to the preparation of undergraduate and graduate students for business and industry. Students need technical and nontechnical skills to be successful in the workplace and it is difficult for universities to provide all the skills to function in the workplace. Universities are providing some of these already. However, university statistics curriculum can be improved so that potential employees have enough statistics and communication skills.

### **5.1 Undergraduate Program**

It is important to design programs in such a way that a student has some minimum statistical and non-statistical background before he/she enters the work place. For instance students need to understand the Scientific Method , Problem Solving Systems, Measurement System Analysis, Sampling, Experimental Design, Regression, some Mathematical statistics, Process Control, and Computing and Mathematics etc. In addition, the students should get experience in solving industrial or business problems and communicating the results to people in other areas. There are different ways of achieving this goal.

### **5.2 Graduate Education**

In the graduate program also there is to be a recognition of the need for non-statistical training. In addition to the typical graduate courses in Statistics (Mathematical Statistics, Linear Models or Regression, Forecasting, Sampling, Design, Generalized Linear Models, Multivariate Analysis, Statistical Computing, etc.) a student needs to have some training in communication or interpersonal relationships.

One useful model to consider is to have a graduate statistics program after an undergraduate degree in another area like Engineering. Or another idea is to have a joint graduate program in Statistics and Business or Engineering. These require co-operation between departments which is not that easy. It is also good to have internships in business or industry during a graduate program. This can enhance familiarity with working environments, hands-on-experience and communication skills. Joint projects such as joint seminars between academia and business or industry will also be very helpful. In such cases business leaders should be invited to give talks in these seminar series.

## 6 University - Business - Industry Collaboration

Universities seek academic excellence and business and industrial enterprises require their employees to work on issues relevant to their organizations. These two goals need not be on a collision course. With proper insight universities can provide academic excellence with relevance. Typically a university provides education to students. It can also provide training in the workplace through its faculty. It is difficult for a university, by itself, to provide the well-rounded education required for students to function well in the workplace. Business and industry can help by providing contexts for relevance. They can also provide input to the universities. Collaboration between university and business or industry is essential to produce graduates for the future who can handle the challenges associated with the new workplace.

Such collaboration requires carefully designed systems for implementation. The needs of the university, and business or industry must be clearly defined in a university-business-industry partnership. The roles of the partners should be clearly understood. The system need to be flexible so that the students and faculty can spend time in business or industry to enhance non-technical skills and to gain some hands-on-experience. University courses can be modified to include project oriented teaching. Business and/or industry should provide opportunities to gain experience in problem formulation, planning of approach and data collection and problem solving. Different models can be used for undergraduate and graduate students. The systems of collaboration must make sure that the transition between university and business or industry is smooth for students as well as faculty. Also should accommodate the interests of students and faculty. We note that long term commitments are required of the partners to make this partnership successful.

## 7 Concluding Remarks

Statistical Thinking and Methods need to become a part of the knowledge base of an organization. We outlined several issues related to the implementation of statistical methods in business and industry. Well planned systems and training are necessary for implementation. Enhancement of university education is also necessary. We also discussed the need for University -Business- Industry collaboration. Such collaboration will provide better opportunities for students. Some may argue that universities should be places for education and should not be in the business of training. It is important to keep the balance, and all such endeavours' need to be motivated by “**academic excellence with relevance**”.

It is important that the professional statistician is equipped with good technical and non-technical skills. This is a challenge universities have to face and one model for successes is to form partnership with business and industry as we have suggested. There is no need to compromise on academic excellence, however building in ‘relevance’ to the program enhances its value.

## 8 Acknowledgements

I thank **Dr B B Jani** for his inspiration and encouragement to prepare this article. I also thank the referee for his helpful suggestions to revise the earlier draft of this article.

**A NEW CRITERION FOR MEASURING ORTHOGONALITY  
OF GIVEN MIXED LEVEL ARRAY**

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**C.C.GUJARATHI<sup>(1)</sup> and VIKAS G. DOSHI <sup>(2)</sup>**

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**ABSTRACT**

We developed  $\delta$ -criterion as a new optimality criterion for measuring orthogonality of given mixed level arrays and also have described simple and effective procedure in the form of an algorithm for constructing mixed level orthogonal and nearly orthogonal arrays with different small sized designs. Further, we have compared our criterion with the rest existing criteria.

**Key words :** Orthogonal/Nearly Orthogonal Arrays, Optimality Criteria,  $\delta$ -criterion,  $J_2$ -criterion.

**1. Introduction**

Quality of a product is a major concern now-a-days. Orthogonal Arrays have been used widely in manufacturing and high-technology industries for quality and productivity improvement experiments. The concept of Orthogonal Arrays dates back to Rao (1947). Orthogonal Arrays (OAs), Nearly Orthogonal Arrays (NOAs) and mixed - level orthogonal arrays are defined below.

**Definition 1.1 : (Orthogonal Array) :** Let  $S$  be a set of  $s$  symbols denoted by  $0,1,2,\dots,s-1$ . An orthogonal array  $d$  OA( $N,k,s,t$ ) of index  $\lambda$  with  $s$  symbols, strength  $t$  is an  $N \times k$  array with entries from  $S$  such that every  $N \times t$  sub-array of  $d$  contains each  $t$ -tuple from  $S$  in exactly  $\lambda$  rows.

The OAs discussed above are such that each factor has same number of levels. It may not be desirable or demand of real life situations to use an OA in some situations (e.g. in machine setting, one component can be set at 2 levels, second at 4 levels and so on). Thus the concept of OAs can be extended to situations where different factors have different number of levels. This types of OAs are known as Mixed OAs and are defined as follows.

**Definition 1.2 : (Mixed Orthogonal Array)** Let  $S_i$  be a set of  $s_i$  levels denoted by

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(1) Rtd. Prof. and Head, Statistics Dept., S. P. Univeristy Vallabh Vidyanagar

(2) Asst. Professor, Dept. of Preventive and Social Medicine, Govt. Medical College, Vadodara  
(rcd. Aug. 2015 / rvd. Nov. 2015)

$0, 1, 2, \dots, s_i - 1$  for  $1 \leq i \leq v$  for some positive integer  $v$  ( $s_i \geq 2$ ). We define a mixed orthogonal array OA  $(N, s_1^{k_1}, s_2^{k_2}, \dots, s_v^{k_v}, t)$  to be an array of size  $N \times k$  such that  $k = k_1 + k_2 + \dots + k_v$  and the first  $k_1$  columns have symbols from  $S_1$ , the next  $k_2$  columns have symbols from  $S_2$ , and so on, such that given any  $N \times t$  sub-array, each possible  $t$ -tuple appears in the same number of rows.

When OA of required run size does not exist, an ideal compromise is to construct an array that is in some way as close to an OA as possible. This concept was considered by Wang and Wu (1992) as “Nearly Orthogonal Arrays (NOAs)” and defined as follows.

**Definition 1.3 : (Nearly Orthogonal Array) :** Let  $S_i$  be a set of  $s_i$  levels denoted by  $0, 1, 2, \dots, s_i - 1$  for  $1 \leq i \leq v$  for some positive integer  $v$  ( $s_i \geq 2$ ). We define a nearly orthogonal array NOA  $(N, s_1^{k_1}, s_2^{k_2}, \dots, s_v^{k_v})$  to be an array of size  $N \times k$  such that  $k = k_1 + k_2 + \dots + k_v$  and the first  $k_1$  columns have symbols from  $S_1$ , the next  $k_2$  columns have symbols from  $S_2$ , and so on such that the array is optimal according to some criterion.

In the past regular fractions remained an important class. Traditionally, nonregular fractions were not advocated because of their complex aliasing structure. However, in the last decade, they have received increasing attention in the literature. The interested readers may refer Hamada and Wu (1992), Wang and Wu (1995), Lin and Draper (1992), etc. There are few algorithms giving construction of mixed level OAs / NOAs. Wang and Wu (1992) proposed the D criterion to measure the overall efficiency of a nearly orthogonal array, Xu and Wu (2001) defined  $A_j(d)$  as a measure of the aliasing between the  $j$ -factor interactions and the general mean. Further, Ye and Sudjianto (2003) used  $c^2$  - criterion for measuring non-orthogonality as well constructing OAs / NOAs of strength two with small run sizes. Later Xu (2002) introduced  $J_2$ -optimality criterion giving its statistical justification and explaining advantages of its use over D, (M,S),  $A_2$  and  $\text{ave}(s^2)$  criteria.  $J_2$  - criterion is a combinatorial criterion based on design matrix and it is shown to be more efficient than the criteria based on model matrix.  $J_2$  - criterion contributes richly, however it's not time efficient as similarities are to be obtained. Therefore an algorithm is needed which is time efficient as well users' friendly. In the present paper we provide  $d$  - criterion, a combinatorial criterion, as an alternative to  $J_2$  - criterion as well  $\chi^2$  - criterion and illustrate that it is more efficient and users' friendly.

In Section 2 we introduce the delta criterion for mixed-level OAs/NOAs of strength two. Section 3 describes an algorithm for constructing mixed-level OAs and NOAs. Section 4 illustrates the criterion through examples. Section 5 deals with comparison and concluding remarks.

## 2. Concept of $u$ - criterion and its applications

A column of  $n \times m$  matrix  $d$  is said to be balanced if each level of the



corresponding factor occurs equally often in that column and hence design matrix  $d$  is said to be balanced if it has all balanced columns.

Let  $d = [x_{ij}]$  be a design matrix of order  $n \times m$  in which all columns are balanced where  $x_{ij}$ 's ( $i=1,2,\dots,n$ ) are elements from  $1,2,\dots,s_j$ ,  $j=1,2,\dots,m$ .

Consider a design  $d$  having all factors at two levels denoted by  $(+,-)$ . By orthogonality we mean that the number of  $+$ 's and  $-$ 's in each design column is same and for every two design columns, the four level combinations  $(++)$ ,  $(+-)$ ,  $(-+)$  and  $(--)$  occur with the same frequency.

Let us denote that 
$$a_{jk}^2 = \sum_{i=1}^n (x_{ij} \cdot x_{ik}) \cdot w_{ijk} \quad , \quad 1 \leq j < k \leq m$$

Where,  $w_{ijk}$  is the frequency of occurrence of pair  $(x_{ij}, x_{ik})$  in the given matrix, which corresponds to the index " $\lambda$ " in definition 1.1. Note that for orthogonal design,  $w_{ijk}$  is always constant.

We prove below necessary and sufficient conditions for  $d = [x_{ij}]$  to be orthogonal.  
**Theorem 2.1.** A balanced design matrix  $d = [x_{ij}] : n \times m$  is said to be an orthogonal array of strength two iff for every pair of columns  $j$  and  $k$ ,

I.  $n$  is a constant integer multiple of  $(s_j \cdot s_k)$  and

II. 
$$a_{jk}^2 = \sum_{i=1}^n (x_{ij} \cdot x_{ik}) \cdot w_{ijk} = \frac{n^2(s_j + 1)(s_k + 1)}{4s_j s_k}$$

**Proof : If Part**

Let  $d = [x_{ij}] : n \times m$  be an OA of strength 2 i.e. all pairs of columns  $j$  &  $k$  of  $d$  are orthogonal. Hence from definition 1.1, frequency of occurrence of pair  $(x_{ij}, x_{ik})$ ,

for all  $i$ , of  $d$  is  $\frac{n}{s_j s_k}$  i.e.  $w_{ijk} = \frac{n}{s_j s_k}$ . Thus, for given OA  $d = [x_{ij}] : n \times m$  of strength 2,  $n$  is multiple of  $(s_j \cdot s_k)$  for every pair of columns  $j$  and  $k$ .

Now to prove that  $a_{jk}^2 = \sum_{i=1}^n (x_{ij} \cdot x_{ik}) \cdot w_{ijk} = \frac{n^2(s_j + 1)(s_k + 1)}{4s_j s_k}$ , let's take L.H.S. of the equation.

$$a_{jk}^2 = \sum_{i=1}^n (x_{ij} \cdot x_{ik}) \cdot w_{ijk}$$

$$\begin{aligned}
&= w_{ijk} \sum_{i=1}^n x_{ij} \cdot x_{ik} \quad (\text{as } d \text{ is orthogonal}) \\
&= w_{ijk} \frac{s_j(s_j+1)}{2} \frac{s_k(s_k+1)}{2} \frac{n}{s_j s_k} \\
&= w_{ijk} \frac{n(s_j+1)(s_k+1)}{4} \\
&= \frac{n}{s_j s_k} \frac{n(s_j+1)(s_k+1)}{4} = \frac{n^2(s_j+1)(s_k+1)}{4s_j s_k} \quad \text{as } w_{ijk} = \frac{n}{s_j s_k}
\end{aligned}$$

This proves if part.

**Only If Part**

Given that  $d = [x_{ij}] : n \times m$  is a balanced matrix where  $n$  is constant integer multiple of  $s_j \cdot s_k$ ,  $1 \leq j, k \leq m$  and ... (i)

$$a_{jk}^2 = \sum_{i=1}^n (x_{ij} \cdot x_{ik}) \cdot w_{ijk} = \frac{n^2(s_j+1)(s_k+1)}{4s_j s_k}; \text{ for all } 1 \leq j < k \leq m \quad \dots \text{ (ii)}$$

From (i), it is given that  $n$  is constant integer multiple of  $s_j \cdot s_k$ .

Hence  $\frac{n}{s_j s_k}$  is constant for all pairs of columns  $j$  &  $k$  of  $d$ . ... (a)

$$\begin{aligned}
\text{From (ii), } a_{jk}^2 &= \sum_{i=1}^n (x_{ij} \cdot x_{ik}) \cdot w_{ijk} = \frac{n^2(s_j+1)(s_k+1)}{4s_j s_k} \\
&= \frac{n}{s_j s_k} \left( \frac{s_j(s_j+1)}{2} \frac{s_k(s_k+1)}{2} \frac{n}{s_j s_k} \right) \\
&= \frac{n}{s_j s_k} \sum_{i=1}^n x_{ij} \cdot x_{ik} \quad \text{as } d \text{ is balanced design} \\
&= \sum_{i=1}^n x_{ij} \cdot x_{ik} \frac{n}{s_j s_k} \quad (\text{because } \frac{n}{s_j s_k} \text{ is independent of } i)
\end{aligned}$$

Thus,  $w_{ijk} = \frac{n}{s_j s_k}$ . i.e. every level combination in  $j^{\text{th}}$  and  $k^{\text{th}}$  column occurs  $\frac{n}{s_j s_k}$  times which is constant by (a). Hence from definition 1.1, given design  $d$  is an

orthogonal array of strength two.

Hence the proof.

Using results of from Theorem 2.1, we develop the following.

- 1)  $\delta$  - criterion for verifying whether the given array is orthogonal or not,
- 2) If given array is not orthogonal, we develop a procedure to make it orthogonal or at least nearly orthogonal,
- 3) Also we develop a procedure for studying more number of orthogonal estimates by adding new balance columns in the given orthogonal / nearly orthogonal matrix (i.e. with the same run size).

From Theorem 2.1, we have for an OA of strength 2,

$$a_{jk}^2 = \sum_{i=1}^n (x_{ij} \cdot x_{ik}) \cdot w_{ijk} = \frac{n^2(s_j+1)(s_k+1)}{4s_j s_k}, \quad \forall 1 \leq j < k \leq m.$$

$$\text{We denote by } T_{jk}^2(d) = \frac{n^2(s_j+1)(s_k+1)}{4s_j s_k}$$

$$\text{Define } \delta_{jk}(d) = a_{jk}^2 - T_{jk}^2(d), \quad 1 \leq j < k \leq m \quad \dots (1)$$

Thus any two balanced columns  $j$  &  $k$  are said to be orthogonal if  $\delta_{jk}(d) = 0$ .

$$\text{Define } \delta(d) = \sum_{1 \leq j < k \leq m} |u_{jk}(d)| \quad \dots (2)$$

We define below  $d$  - criterion for  $d$  to be an orthogonal.

**Definition 2.1 :** ( $u$  - criterion) Design  $d = [x_{ij}] : n \times m$  is said to be an orthogonal array of strength two if

$$\delta(d) = 0.$$

Thus an  $n \times m$  array  $d = [x_{ij}]$  is

- a) Orthogonal if  $\delta(d) = 0$ ,
- b) Nearly orthogonal if  $\delta(d)$  is near to zero,
- c) Non orthogonal if neither a) nor b) is true.

Our main goal is to obtain OA's with small run size giving orthogonal estimates of maximum number of main and 2-factor interaction effects. We search new balanced columns which are orthogonal to all columns of given OA to have more orthogonal / nearly orthogonal estimates.

We add a balanced column  $c$  with levels  $s_c$  ( $1, 2, \dots, s_c$ ) in  $d$ . The new design matrix is denoted by  $d_1 = [x_{ij}] : n \times (m+1)$ . Certainly  $d_1$  is an orthogonal array of strength two if  $\delta(d_1) = 0$ .

Using (2),  $\delta(d_1)$  is defined as 
$$\delta(d_1) = \sum_{1 \leq j < k \leq m+1} |u_{jk}(d_1)| = \sum_{1 \leq j < k \leq m} |u_{jk}(d)| + \sum_{j=1}^m |u_{jc}(d_1)|$$

$$= \delta(d) + \sum_{j=1}^m |u_{jc}(d_1)| \quad \dots (3)$$

$d_1$  is orthogonal if  $\delta(d_1) = 0$ . i.e.  $d_1$  is an orthogonal array, if  $\sum_{j=1}^m |u_{jc}(d_1)| = 0$ , as  $\delta(d) = 0$ . If  $d_1$  is non-orthogonal, then we try to reduce non-orthogonality by taking  $\sum_{j=1}^m |u_{jc}(d_1)|$  equal or near to zero. This can be done by switching two appropriate symbols from newly added column. Now we study the change in each  $d_{jc}(d_1)$  if two symbols are switched in a newly added column.

Let  $x_{r_j}$  &  $x_{s_j}$  be elements in  $r$ th &  $s$ th rows of  $j$ th column and  $x_{r_c}$  &  $x_{s_c}$  be

elements in  $r$ th &  $s$ th rows of column  $c$ . Making a switch in column  $c$  between  $r$ th &  $s$ th rows we get new matrix  $d_1^*$ .

By switching  $r$ th &  $s$ th rows in column  $c$ ,  $\delta_{jc}(d_1)$  is reduced by

$$(x_{r_j} x_{r_c}) * (2w_{jc} - 1) + (x_{s_j} x_{s_c}) * (2w_{jc} - 1) \text{ and increased by}$$

$$(x_{r_j} x_{s_c}) * (2w_{jc} - 1) + (x_{s_j} x_{r_c}) * (2w_{jc} - 1),$$

where  $w_{jc}$ ,  $w_{jc}$ ,  $w_{jc}$  and  $w_{jc}$  are frequencies of  $(x_{r_j} x_{r_c})$ ,  $(x_{s_j} x_{s_c})$ ,  $(x_{r_j} x_{s_c})$  and  $(x_{s_j} x_{r_c})$  respectively.

Therefore new  $\delta_{jc}(d_1)$  is given by

$$\delta_{jc}(d_1^*) = \delta_{jc}(d_1) - (x_{r_j} x_{r_c}) * (2w_{jc} - 1) - (x_{s_j} x_{s_c}) * (2w_{jc} - 1) + (x_{r_j} x_{s_c}) * (2w_{jc} - 1) + (x_{s_j} x_{r_c}) * (2w_{jc} - 1).$$

Thus, for all  $j = 1, 2, \dots, m$

$$\delta_{jc}(d_1^*) = \delta_{jc}(d_1) - \Delta_{jc}(d_1^*), \text{ where,}$$

$$\Delta_{jc}(d_1^*) = (x_{\alpha_j} x_{\alpha_c}) * (2w_{\alpha jc} - 1) + (x_{\beta_j} x_{\beta_c}) * (2w_{\beta jc} - 1) - (x_{\alpha_j} x_{\beta_c}) * (2w_{\alpha \beta c} - 1) - (x_{\beta_j} x_{\alpha_c}) * (2w_{\beta \alpha c} - 1) \quad \dots (4)$$

$$\text{Define, } u(d_1') = \sum_{j=1}^m u_{jc}(d_1^*) \quad \dots (5)$$

$$\text{Thus, } u(d_1^*) = u(d_1) - u(d_1') \quad \dots (6)$$

Note : If  $x_{rj} = x_{sj}$  and / or  $x_{rc} = x_{sc}$ ,  $\Delta_{jc}(d_1^*)$  reduces to zero.

To achieve our target of reducing the  $u_{jc}(d_1^*)$  we compute  $\Delta_{jc}(d_1^*)$  for all possible pairs of rows. Find the best pair of rows which reduce  $u_{jc}(d_1^*)$  the most. Repeat the above procedure until we get  $u(d_1^*)$  equal to zero or near to zero. Now,  $d_1$  becomes  $d$  and we try to add another balanced column in  $d$ . Thus we keep adding balanced columns one by one to construct an OA/NOA of strength 2 to study desired number of effects keeping run size constant.

### 3. An Algorithm

The algorithm for constructing an OA/NOA of strength three from given OA  $d$  of strength two using  $\delta$ -criteria is as follows :

- a) Read the value of  $t$ ,  $T_1$  and  $T_2$  and first two given columns. Let,  $n_0=2$  and  $T = T_1$ .
- b) For  $k = 3, 4, \dots, m$  do the following.
  - i) Randomly generate a column  $c$  with  $s_c$  level. Compute  $\delta(d_1)$  using (3). If it is zero, go to iv), otherwise go to ii).
  - ii) For all pairs of rows  $\alpha$  and  $\beta$  with distinct symbols, compute  $\Delta_{jc}(d_1^*)$  for all  $j = 1(1)m$  columns using (4). Exchange the symbols in pair of rows  $\alpha$  and  $\beta$  of column  $c$  for which  $u(d_1^*)$  is minimum. Reduce  $\delta(d_1)$  by  $u(d_1^*)$  and compute  $u(d_1^*)$  using (6). If  $u(d_1^*) = 0$ , go to iv) otherwise repeat ii) until no further improvement is made.
  - iii) Repeat i) and ii)  $T$  times and choose a column  $c$  that produces the smallest  $u(d_1^*)$ .
  - iv) Add column  $c$  as the  $k^{\text{th}}$  column of  $d$ . Let  $\delta(d) = u(d_1^*)$  and update  $\delta_{jk}(d)$ .  
If  $\delta(d) = 0$ ,  $n_0 = k$  otherwise  $T = T_2$ .
- c) Return the  $n \times m$  design  $d$ , of which first  $n_0$  columns form an OA and the whole design will be nearly orthogonal.

In Xu (2002), for constructing OAs, a recommended moderate value of  $T_1$  is 100 and  $T_2$  can be 0. For  $\delta$ -criterion, values of  $T_1$  and  $T_2$  are predefined by an experimenter depending on the required efficiency. However, as per our guess, these values are

smaller than those in Xu (2002).

#### 4. Illustration

Consider the following design matrix as given in an example-1 in Xu (2002).

OA' (12,3<sup>1</sup>,2<sup>9</sup>)

Run	1	2	3	4	5	6	7	8	9	10
1	0	0	1	0	1	1	1	0	0	0
2	0	1	0	0	1	1	0	0	1	0
3	0	0	1	1	0	1	0	1	1	1
4	0	1	0	1	0	0	1	1	0	0
5	1	0	0	0	0	0	0	0	0	1
6	1	1	1	0	0	0	1	0	1	1
7	1	0	1	1	1	0	0	1	1	0
8	1	1	0	1	1	1	1	1	0	1
9	2	0	0	1	0	1	1	0	1	0
10	2	1	1	0	0	1	0	1	0	0
11	2	0	0	0	1	0	1	1	1	1
12	2	1	1	1	1	0	0	0	0	1

It is shown by Xu (2002) that the subset ( $d_1$ ) of first 5 columns of this matrix forming an OA(12,3<sup>1</sup>2<sup>4</sup>,2) has  $J_2(d_1) = L_2(n) = 330$ . Moreover, the entire matrix is not an orthogonal array of strength 2 as  $J_2(d) = 1284$  while  $L_2(n) = 1260$ .

Using  $\delta$ -criterion we get  $\delta(d_1) = \sum_{1 \leq j < k \leq m} |u_{jk}(d_1)| = 0$ . Thus we conclude that sub-

matrix  $d_1$  is an orthogonal array of strength 2. But the value  $\delta(d) = \sum_{1 \leq j < k \leq m} |u_{jk}(d)| \neq 0$

and hence  $d$  is not an orthogonal array of strength 2. The detailed comparison of  $J_2$ - and  $\delta$ -criteria for the above design is discussed in the next section.

#### 5. Comparison and Conclusion

$J_2$  - optimality criterion for OAs/NOAs of strength two is a combinatorial criterion introduced by Xu (2002) which is defined as follows :

For an  $N \times n$  matrix  $d = [x_{ij}]$ , with weight  $w_k > 0$  is assigned for column  $k$

having  $s_k$  levels. For  $1 \leq i, j \leq N$ , let  $u_{i,j}(d) = \sum_{k=1}^n w_k u(x_{ik}, x_{jk})$ , where  $\delta(x, y) = 1$  if  $x = y$  and 0 otherwise.

$J_2(d) = \sum_{1 \leq i < j \leq N} [u_{i,j}(d)]^2$  and lower bound of  $J_2(d)$  is given by,

$$L(n) = 2^{-1} \left[ \left( \sum_{k=1}^n w_k N / s_k \right)^2 + \sum_{k=1}^n (s_k - 1) (N s_k^{-1} w_k)^2 - N \left( \sum_{k=1}^n w_k \right)^2 \right]$$

When given  $d$  is OA,  $J_2(d)$  attains lower bound.

Note : Throughout our discussion, we assume that weight  $w_k$  is 1 for every  $k$ .

For the above criterion, one needs to compute number of similarities between every pair of rows for different numbers of columns as shown below.

$$u_{i,j}(d) = \sum_{k=1}^n u(u_{ik}, u_{jk}), \quad 1 \leq i, j \leq N.$$

- 1) The comparison of  $\delta$  - criterion for strength 2 with  $J_2$  - criterion is as follows
- Both the criteria are based on design matrix.
  - Both the criteria are capable of assessing orthogonality of a given  $n \times m$  array, if exists. The number of algebraic calculations required by both of these criteria for assessing orthogonality and obtaining possible number of OAs of strength 2 from given OA  $d : n \times m$  are as follows :

**Table 1**  
**Head-to-Head comparison of algebraic operations between two criteria**

Criterion	Multiplication	Addition
$J_2$ - criterion di of above given illustration ( $n=12, m=5$ )	$\binom{m}{2} \binom{n}{2} 2 + \left\{ \sum_{i=1}^2 \binom{m}{i} (i+1) \right\} + 3$  1750	$\binom{m}{2} \binom{n}{2} (2+1) + \sum_{i=1}^2 \left[ \binom{m}{i} - 1 \right]$  2120
$\delta_2$ - criterion di of above given illustration ( $n=12, m=5$ )	$\binom{m}{2} [2n+6]$  300	$\binom{m}{2} (n+1)$  130

- All logical and similarities finding computations of  $J_2$  - criterion are converted into efficient algebraic computation.

- It is very clear that  $\delta$  – criterion is much more efficient as well very convenient than  $J_2$  – criterion, since in  $J_2$  – criterion whole array is required to be examined.
- Time saving due to  $\delta$  – criterion is tremendous as is clear by above given exact number of algebraic calculations.
- Below given table – 2 gives clear idea about the computational increased load due to  $J_2$  – criterion in comparison to  $\delta$  - criterion when there is increase in n or m or both.

**Table 2**  
**Comparison of algebraic operations between two criteria for a strength 2 design with different n and m**

Criterion	Multiplications	Additions	Total operations
$J_t$ - criterion (n=12, m=7)	4452	4725	9177
$\delta_t$ – criterion (n=12, m=7)	630	273	903
$J_t$ - criterion (n=15, m=5)	2530	3290	5820
$\delta_t$ – criterion (n=15, m=5)	360	160	520

- 1) To construct OAs/NOAs of strength 2,  $\delta$  – criterion is much more efficient than  $J_2$  – criterion as is made clear in tables 1&2. Besides these calculations for every newly added column c, the task of finding similarities in  $J_2$  – criterion demands more time which is not mentioned in the tables.
- 2) In  $J_2$  – criterion similarities are required to be obtained for each pair of rows which is a tedious and time consuming task, whereas in  $\delta$  – criterion, product of entries for each pair of columns is to be obtained which is much more simple than finding similarities. Hence  $\delta$  – criterion is also user’s friendly.

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*With Best Compliments from*

**Dr. Mrs. R. G. Bhatt**

**Prof. and Head  
Statistics Dept.  
Gujarat University  
AHMEDABAD-380 009**



**Wish you all  
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**Dr. Ashwin J. Patel**

Dept. of Statistics  
R. H. Patel College of Arts and Commerce  
Nava Vadaj  
AHMEDABAD-380 013



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Dept. of Statistics  
GLSIC  
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**“LIVES OF GREAT MEN  
ALL REMIND US THAT  
WE CAN MAKE OUR LIVES SUBLIME  
WHILE DEPARTING LEAVE BEHIND US  
FOOTPRINTS ON THE SANDS OF TIME.”**

**\*LONGFELLOW**

**ENVIRONMENTAL SUSTAINABILITY AND INCLUSIVE  
GROWTH MODEL: SOME STATISTICAL ASPECTS**

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**Rakesh Srivastava\***

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**ABSTRACT**

Enhancement of human well being is the primary objective of Government policy in particular and Society at large. Evaluating and measuring the progress of Society from this perspective requires identification of appropriate indicators for the purpose. Traditionally the level of Economic activities measured by the GDP

and GNP and its various components has been adopted as proxy for measuring the development .Several key indicators are in the process to identify different aspects of Social progress.GDP being the best known measure of Economic activity is subjected to the limitation of non inclusion of the issues of Envio-renmental sustainability and social inclusion.

The present paper addresses the emergence of new development paradigm like growth with redistribution, human welfare, sustainable development which in turn Introduced a whole lot of Social, Economic, political and Environmental dimensions of development and corresponding measurable indicators. The paper also includes a brief review of MDG<sup>s</sup>(Millennium Development Goals) particularly the **goal-7** which is devoted to Environment sustainability. Some recent findings on the **progress of indicators of Environmental sustainability only** using some standard Statistical tracking procedureds up to **2014** have also been presented.

**KEY WORDS AND PHRASES:** Environmental sustainability, Tracking, Targets, Regression.

**INTRODUCTION**

Sustainability is the ability to continue a defined behavior for fairly long time.

Sustainable development is that which meets the needs of the present without compromising the ability of future generations to meet their own needs.

The Environmental sustainability could be defined as a condition of balance, resilience and interconnectedness that allows human society to satisfy its needs while

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\* Department of Statistics, The M. S. University of Baroda, Vadodara-390002  
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neither exceeding the capacity of its supporting ecosystems to continue to regenerate the services necessary to meet those needs nor by our actions diminishing biological diversity.

The problem of Environmental sustainability requires aggressive wide spread action **NOW** to avoid catastrophic consequences later.

The **KEY** concept called for redefining and expansion of the concept Environmental sustainability is to link it directly to Social and Economic developmental process, without Environmental sustainability it is almost impossible to achieve Sustainable development.

Most of the available literatures have agreed generally on the following broad areas to measure the Social progress.

- State of economy: May include cost of living conditions, GDP, GNP, inflation rate etc.
- Education status: Enrollment ratios, Dropout rates, Literacy rates.
- Infrastructure: Teledensity, Electrification, Roads etc.
- Freedom: Economic, Political, Social, Cultural.
- Environment: It includes climatic factors as well.
- Security: Economic, social, psychological.

The present paper briefly mentions only Female literacy under the educational status as this has been envisaged as an important indicator for Socio-economic development.

A closer look at the Net enrollment Ratio (NER) a more appropriate indicator than Gross Enrollment Ratio (GER) reveals that the country is tending to achieve 2015 target of Universal primary education to all the children aged 6-11 years. A trend based on DISE (District Information System on Education) shows that NER has improved from 83% in the year 2000 to over 95% in 2007-08. The NER for girls in primary schools tends to have sharper rise compared to that of boys.

Security is a vast issue as it relates to economic, social and Psychological issues. In the present paper we are just mentioning them as the indicators to measure the Social progress, however we briefly introduce the **psychological security** as “we may be secure even though don’t feel secure

And we may not be secure though we feel so.” Security is also a feeling based not on probabilities and mathematical calculations, but on psychological reactions to both risks and countermeasures.

Some of these dimensions have cross linkages while some are mutually exclusive.

Nevertheless, it is acceptable to argue that assessing cross linkages across these dimensions is critical. One of elements that need attention is the fact that measuring QOL (Quality Of Life) includes wide range of factors that make it worth living. Some

of the indexes like: Gross National Happiness (GNH), Happy Planet Index (HPI) include environment as one of the components. Thus, while the prevalent Economic and Statistical system provides good range of information on some of these aspects, it certainly does not cover the exhaustive list of variables required for such assessments. The possible way out is to include more variables which are subjective in nature such environmental factors, lower crime rates etc. while collecting the current Statistics.

Within above mentioned broad groups, enormous possible combinations may exist to measure the progress from different perspective depending upon underlying assumption of an index. However more challenging task is to propose proper weights of these indicators. Measuring the progress of a Nation is important for any country regardless of its development. Different countries have tried various approaches to measure the progress of a Society.

#### **INDIAN SCENARIO (INCLUSIVE GROWTH MODEL)**

India has followed the inclusive growth strategy in its 11<sup>th</sup> five year plan. The broad vision includes several inter-related components: rapid growth that reduces poverty and creates employment opportunity, access to essential services in health, empowerment through education and skill development, Environmental Sustainability and Good governance.

#### **THE MODEL**

To ensure that the objectives are achieved and progress could be measured, 27 Moniterable targets have been identified at the National level of which 13 are disaggregated at the State level. The 27 targets fall in to six major categories:

(i) Income and Poverty (ii) Education (iii) Health (iv) Women and children (v)Infrastructure and (vi) Environment.

The targets in each category have been specified.

Here we present specific targets fixed under **Environmental head only**. These comprise: (i) To increase forest tree cover by 5% (ii) To attain WHO standards of air quality in all major cities (iii) To treat urban waste by cleaning river waters and (iv) To increase energy efficiency by 20 %.

#### **WHERE ARE WE?**

Under this heading it is attempted to gauge the overall progress made by the country during the MDG period and where do we stand in terms of achieving those targets.

Despite of the fact that the population pressure amounts to almost 16% of the World population, with lot of floods, droughts and deforestation the access to safe drinking water tends to reach all, as India is on track in acheveing the MDG target for sustainable access to safe drinking water. The overall proportion of House Holds having access to improved water resources increased from about 68.2% in 1992-93 to 84.4% in 2007-08.The Urban coverage has increased from 87.6% to 95% during the

same period. The growth in rural areas is also significant being 79.6% in 2007-08 as against 61% in 1992-93.

**MOSPI** (Ministry Of Statistics and Programme Implementation) is responsible for tracking and monitoring the progress achieved on the MDGs and publishes the report reports at regular intervals starting from Midterm appraisal: India country report in 2007 to the recent one in 2014.

**MDG 7 is devoted to ensure environmental sustainability** with some specific targets like: **Target 9, 10 and 11.** Following are some recent findings (Ref. MDGs and targets –Summary of progress achieved by India 2014).

**TARGET 9:** Integrate the principle of sustainable development into country policies and programmes and reverse the loss of environmental resources.

For this target it is found that it is **MODERATELY ON TRACK** considering all the indicators.

**TARGET 10:** halve by 2015, the proportion of people without sustainable access to safe drinking water and basic sanitation. For this target it is reported to be: **ON TRACK FOR THE INDICATOR OF DRINKING WATER BUT SLOW FOR THE INDICATOR OF SANITATION.** The reason being that the MDG target was to make it half by 2015 and in our country the achievement on sanitation is slow.

**TARGET 11:** By 2020, to have achieved a significant improvement in the lives of at least 100 million slum dwellers. It is reported that **THE PATTERN OF CHANGE IS NOT STATISTICALLY DISCERNIBLE** due to lack of sufficient data.

The methodology for tracking of the MDGs is prescribed by **UNSD** (United Nations Statistics Division) for developing countries. The methodology is selected for its simplicity and ease of interpreting the results obtained. As the targets are based on several related indicators for which the progress is to be measured the indicator selection criterion becomes very important. The two categories of indicators having quantitative targets to be reached by 2015 are covered for tracking purpose.

#### **INDICATOR SELECTION CRITERION**

- (i) Indicators that are directly related to a target.
- (ii) Indicators for which progress is measured for developing countries
  - a. i.e. not for developed countries
  - b. i.e. not for target meant for least developed or Island countries.
- (iii) Indicators that have quantitative targets to be reached by 2015. Under this the two categories are:
  - A. Explicit target values for 2015
    - i. Relative (like reduce by 1/3 rd, 1/2 etc.)
    - ii. Absolute (Full enrolment, Gender parity).
  - B. Reversal of trends
    - i. Halt and begun to reverse

ii. Reverse the loss of Environmental resources (Goal 7/target 9).

As these are some time series data hence most of the MDG indicators based on such data move relatively slowly over time.

**TRACKING PROGRESS PRINCIPLE**

As we wish that the analysis should be performed easily so it suggested to keep the following points in consideration for the tracking purpose.

- Keep it simple

As most of the MDG targets move relatively slowly over time data gaps and number of observations don't allow for sophisticated Time Series analysis.

- Use all the information available.

Provide efficient estimates

Provide Confidence Intervals (it is **not** included in the tracking principle but it may be considered)

**INDICATOR TRACKING TECHNIQUE**

The Indicator tracking technique involves calculating the 'required' rate of change from the last data last available value, for the target to be met on time.

For this we calculate the historical rate of change between 1990 and the latest year for which an indicator value is available then compare the required rate with historical rate of change.

The historical rate of change is estimated by using the Exponential model given by:  $X_t = ae^{bt}$  where  $X_t$  is the indicator value for year t. For  $t=0$  ,  $X_0=a$  . Taking natural logarithm on both the sides we have

$$\ln X_t = \ln a + bt = \ln X_0 + bt, \quad \text{or } \hat{b} = (\ln X_t - \ln X_0)/t \dots\dots\dots (1)$$

In terms of historical rate of change, 'r'

$$X_t = X_0(1+r)^t \quad \text{which after some simplification can be written as}$$

$$r = \exp(\ln X_t - \ln X_0)/t - 1. \dots\dots\dots (2)$$

Using relation (1) and (2) we get

$$r = \exp(\hat{b}) - 1, \text{ where 'r' is the historical rate of change.}$$

**REGRESSION RUN**

UNPDP guidelines on concepts, rationale and methodology of MDG indicators recognize all the 53 indicators that UN framework standardized for global monitoring of MDG<sup>s</sup> as rationally valid and statistically indicative. So, the regression is run on all the observations from 1990 onwards and in case the sign of  $\hat{b}$  differs, the value of second run is used to calculate 'r', but excluding first few observations.

**Reasons:** catch the outliers at the beginning of the series. Particular issue for countries in which social sector (health and Education) collapsed in early 1990<sup>s</sup>

The **required** rate is calculated with an explicit target i.e. those selected for monitoring and it is given by:  $r^* = (X^*/X^T)^{1/(2015-T)} - 1$

Where  $X^*$  is the target value for 2015 and  $X_T$  is the indicator value for the last available year.

$r^* = 0$  if the target has been reached, implies that a certain pre-defined absolute value called cut-off value is reached. For the environmental indicator of percentage of population without access to drinking water the MDG target is to 'reduce it by half' for this the cut-off applicable was decided to be 5%.

$X_T \leq X^*$  for indicators of which values have to decrease

$X_T \geq X^*$  for indicators of which values have to increase

The classification of the decision has to be based on historical rate of change alone.

## CONCLUSIONS

Environmental sustainability and its linkage to Socio-economic development is a multidimensional phenomenon and hence require development of various appropriate indicators, it may be a challenging task for the Statisticians to develop such indicators and then Indices based on them, so that some meaningful analysis can be performed. It would be of interest and utility to study the linkages of these indicators with the development, using some standard Statistical techniques.

**Data as such tells nothing but application of Statistical tools and techniques makes the data meaningful.**

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## **INVENTORY MODEL FOR AMELIORATING AND DETERIORATING ITEMS WITH TIME DEPENDENT DEMAND**

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**Ankit Bhojak<sup>1</sup>, U. B. Gothi<sup>2</sup>**

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### **ABSTRACT**

When the items such as fruits, green vegetables, flowers and some dairy products are kept in farms or in supermarkets or in flower shops or in cold storages, the stock increases due to growth and it also decreases due to spoilage or staleness. Hence the effect of amelioration and deterioration are observed simultaneously. For this type of inventory system, an inventory model is developed for ameliorating and deteriorating items. In this paper we have assumed that time to ameliorate follows Weibull distribution and deterioration rate is considered to be constant. Demand rate is time dependent and shortages are not allowed to occur. The problem is to minimise total inventory cost. The developed model is illustrated by a numerical example with its partial sensitivity analysis.

*Keywords* – Amelioration, Deterioration, Weibull distribution

### **1. INTRODUCTION**

Currently many researches being carried out in the field of inventory management. New inventory models are defined by changing deterioration, shortages, demand rates and costs depending on time to get optimum order quantity or to get optimum production quantity. Here a new dimension is added in such inventory models by introducing amelioration. The changes occurring due to ameliorating items are being studied in defining new inventory models.

Ghare and Schrader [6] have defined a model for an exponentially decaying inventory. Many researches are being done by researchers in inventory control to get EOQ under different circumstances. Giri, Goswami and Chaudhuri [7] formulated an EOQ model for deteriorating

- 
1. Department of Statistics, GLS Institute of Commerce, GLS University, Ahmedabad, Gujarat, INDIA.
  2. Head, Department of Statistics, St. Xavier's College (Autonomous), Ahmedabad, Gujarat, India.

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items with time varying demand and costs. Wu and Lee [22] developed an EOQ inventory model for items with Weibull distributed deterioration under shortages and time-varying demand. A review on deteriorating inventory study has been done by Ruxian Li, Hongjie Lan and John R. Mawhinney [15]. Tripathy and Pradhan [20] presented an EOQ model for Weibull deteriorating items with power demand and partial backlogging. Ankit Bhojak and U. B. Gothi [1] formulated an EOQ model with time dependent demand and Weibull distributed deterioration. Devyani Chatterji and U. B. Gothi [5] have developed an EOQ model for deteriorating items under two and three parameter Weibull distribution and constant IHC with partially backlogged shortages. Kirtan Parmar, Indu Aggarwal and U. B. Gothi [13] have given order level inventory model for deteriorating item under varying demand condition.

To get optimum production quantity in inventory management system EPQ models are developed. Many researchers have developed EPQ models by considering different deterioration rates, imposing conditions on shortage and demand. A note on the EPQ model with shortages and variable lead time is given by Chang [4]. Sarkar and Chakrabarti [17] have defined an EPQ model under Weibull deterioration with exponential demand and production with shortages under permissible delay in payments. EPQ model for imperfect quality items under constant demand rate and varying IHC is presented by U.B. Gothi and Devyani Chatterji [21]. Kirtan Parmar and U. B. Gothi [12] have defined an EPQ model of deteriorating items using three parameter Weibull distribution with constant production rate and time varying holding cost. Ankit Bhojak and U. B. Gothi [2] have developed an EPQ model with time dependent IHC and Weibull distributed deterioration under shortages.

Items like fruits, green vegetables, flowers, some dairy products are kept in farms or in supermarkets or in cold storages, their stock increases due to growth and stock can decrease also due to spoilage or staleness. By providing appropriate treatment at farm level, keeping appropriate temperature at supermarkets or at storage place and applying suitable techniques the spoilage or wastage of such items can be reduced. In such cases the combined effect of deterioration and amelioration are observed. Hwang [8] , [9] & [10] has studied inventory models for items with Weibull ameliorating. Mondal, Bhunia and Maiti [3] have defined an inventory system of ameliorating items for price dependent demand rate. EOQ models for ameliorating/deteriorating items under inflation and time discounting are framed by Moon, Giri and Ko [11]. Law and Wee [16] have derived an integrated production inventory model for ameliorating and deteriorating items considering time discounting. Tadj, Sarhan and Gohary [14] presented an optimal control of an inventory system with ameliorating and deteriorating items. Mishra, Misra, Mallick and Barik [19] have derived inventory model with Weibull amelioration under the influence of inflation and time-value of money.

Mishra, Raju, U.K. Misra and G. Misra [18] have published their work on optimal control of an inventory system with variable demand and ameliorating / deteriorating items. We have modified the same model by considering replenishment time  $T$  as a decision variable and inventory holding cost as a linear function of time. We have minimised the total average

cost and illustrated this developed model by numerical example with its partial sensitivity analysis.

## 2. ASSUMPTIONS

The model is developed under the following assumptions

1. The inventory system involves only one item and one stocking point.
2. Replenishment rate is infinite.
3. Lead-time is zero.
4. The deterioration and amelioration occur when the item is effectively in stock.
5. No replacement of the deteriorated items takes place during a given cycle.
6. Infinite time horizon period is considered.
7. Shortages do not occur.
8. Holding cost  $C_h = h + r t$  ( $h, r > 0$ ) is a linear function of time.
9. The deterioration rate is constant.
10. Demand rate is  $R(t)$  defined by

$$R(t) = R_0 + a(t - t_2) H(t - t_2) \quad \text{where} \quad H(t - t_2) = \begin{cases} 1 & t \geq t_2 \\ 0 & t < t_2 \end{cases}$$

11. Amelioration cost, deterioration cost, production cost and ordering cost are known and constants.
12.  $A(t) = t^{-1}$  is the two parameter Weibull amelioration rate in the time interval  $[0, T]$  where  $\lambda$  is a scale parameter ( $0 < \lambda < 1$ ) and  $k$  is a shape parameter ( $k > 0$ )
13. Total inventory cost is a continuous real function which is convex to the origin.

## 3. NOTATIONS

The following notations are used to develop the mathematical model

1.  $Q(t)$  : Inventory level of the product at time  $t$ .
2.  $R(t)$  : Demand rate varying over time.
3.  $A(t)$  : Amelioration rate at any time  $t$ .
4.  $\lambda$  : Deterioration rate. ( $0 < \lambda < 1$ )
5.  $A$  : Ordering cost per order during the cycle period.
6.  $C_h$  : Inventory holding cost per unit per unit time.
7.  $C_a$  : Amelioration cost per unit
8.  $C_d$  : Deterioration cost per unit per unit time.
9.  $C_p$  : Production cost per unit. ( $C_p > C_a$ )
10.  $S$  : Initial inventory level at time  $t = 0$ .
11.  $S_1$  : Inventory level at  $t = \mu$ .
12.  $S_2$  : Inventory level at time  $t = t_1$ .

- 13.  $a$  : Rate of change of demand with respect to time  $t$ .
- 14.  $R_0$  : Initial demand.
- 15.  $T$  : Duration of a cycle.
- 16.  $TC$  : Total cost per unit time.

**4. MATHEMATICAL FORMULATION AND SOLUTION**

Cycle starts with inventory level of  $S$  units. The total time is distributed into three time intervals. In the first time interval  $[0, \mu]$  the inventory level increases due to the combined effect of amelioration and deterioration and reaches to  $S_1$ . In the second time interval  $[\mu, t_1]$  the inventory level goes down and reaches to  $S_2$  due to the effects of deterioration and demand even though amelioration exists. The situation remains the same in the third time interval  $[t_1, T]$  and inventory level becomes zero at time  $T$ . The above mentioned inventory system is presented graphically in Fig. 1.

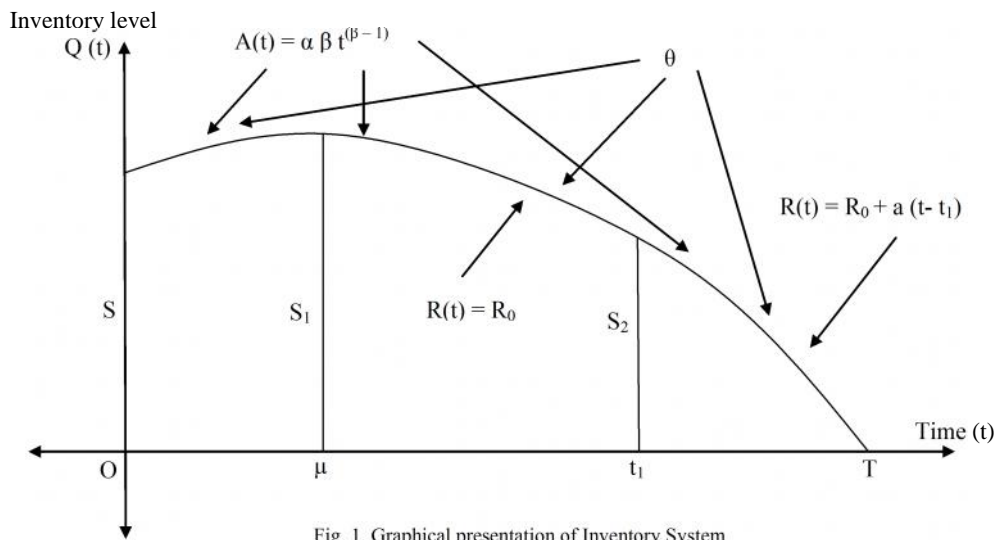


Fig. 1. Graphical presentation of Inventory System

Differential Equations pertaining to the situations as explained above are given by

$$\frac{d Q(t)}{d t} = r s t^{s-1} Q(t) - \Lambda(t) Q(t) \quad 0 \leq t \leq \mu \quad \dots (1)$$

$$\frac{d Q(t)}{d t} = r s t^{s-1} Q(t) - \Lambda(t) Q(t) - R_0 \quad \mu \leq t \leq t_1 \quad \dots (2)$$

$$\frac{d Q(t)}{d t} = r s t^{s-1} Q(t) - \Lambda(t) Q(t) - R_0 - a(t - t_1) \quad t_1 \leq t \leq T \quad \dots (3)$$

Using the boundary conditions

$$Q(0) = S, Q(\sim) = S_1, Q(t_1) = S_2, Q(T) = 0$$

the solutions of equation (1), (2) and (3) are given by

$$Q(t) = S(1 + r t^s - \theta t) \quad 0 \leq t \leq \sim \quad \dots (4)$$

$$Q(t) = \left[ S - R_0 \left\{ (t - \sim) - \frac{r}{s+1} (t^{s+1} - \sim^{s+1}) + \frac{\theta}{2} (t^2 - \sim^2) \right\} \right] (1 + r t^s - \theta t) \quad \sim \leq t \leq t_1 \quad \dots (5)$$

and

$$Q(t) = \left[ \begin{aligned} & (R_0 - a t_1)(T - t) + \frac{(a + \theta)(R_0 - a t_1)}{2} (T^2 - t^2) + \frac{a \theta}{3} (T^3 - t^3) \\ & - \frac{r(R_0 - a t_1)}{s+1} (T^{s+1} - t^{s+1}) - \frac{a r}{s+2} (T^{s+2} - t^{s+2}) \end{aligned} \right] (1 + r t^s - \theta t) \quad t_1 \leq t \leq T \quad \dots (6)$$

Substituting  $Q(\sim) = S_1$  in equation (4), we get

$$S_1 = S(1 + r \sim^s - \theta \sim) \quad \dots (7)$$

By placing  $Q(t_1) = S_2$  in equation (5) and (6), we get

$$S_2 = \left[ S - R_0 \left\{ (t_1 - \sim) - \frac{r}{s+1} (t_1^{s+1} - \sim^{s+1}) + \frac{\theta}{2} (t_1^2 - \sim^2) \right\} \right] (1 + r t_1^s - \theta t_1) \quad \dots (8)$$

$$\Rightarrow S_2 = \left[ \begin{aligned} & (R_0 - a t_1)(T - t_1) + \frac{(a + \theta)(R_0 - a t_1)}{2} (T^2 - t_1^2) + \frac{a \theta}{3} (T^3 - t_1^3) \\ & - \frac{\alpha(R_0 - a t_1)}{\beta + 1} (T^{\beta+1} - t_1^{\beta+1}) - \frac{a \alpha}{\beta + 2} (T^{\beta+2} - t_1^{\beta+2}) \end{aligned} \right] (1 + \alpha t_1^\beta - \theta t_1) \quad \dots (9)$$

Eliminating  $S_2$  from equation (8) and (9), we get

$$S = \left[ \begin{aligned} & (R_0 - a t_1)(T - t_1) + \frac{(a + r)(R_0 - a t_1)}{2}(T^2 - t_1^2) + \frac{a r}{3}(T^3 - t_1^3) \\ & - \frac{r(R_0 - a t_1)}{s + 1}(T^{s+1} - t_1^{s+1}) - \frac{a r}{s + 2}(T^{s+2} - t_1^{s+2}) \\ & + R_0 \left\{ (t_1 - \tilde{t}) - \frac{r}{s + 1}(t_1^{s+1} - \tilde{t}^{s+1}) + \frac{r}{2}(t_1^2 - \tilde{t}^2) \right\} \end{aligned} \right] \dots (10)$$

### Cost Components

The total cost per replenishment cycle consists of the following cost components

#### I. Operating Cost (OC)

The operating cost over the period  $[0, T]$  is

$$OC = A \dots (11)$$

#### II. Amelioration Cost (AMC)

The amelioration cost over the period  $[0, T]$  is

$$AMC = C_a \int_0^T r s t^{s-1} Q(t) dt$$

$$\Rightarrow AMC = C_a r s \left[ \begin{aligned} & \frac{s \tilde{t}^s + (s + R_0 \tilde{t})}{s}(t_1^s - \tilde{t}^s) - \frac{R_0}{s + 1}(t_1^{s+1} - \tilde{t}^{s+1}) \\ & + \frac{\left\{ (R_0 - a t_1)T + \frac{a}{2}T^2 \right\}}{s}(T^s - t_1^s) - \frac{(R_0 - a t_1)}{s + 1}(T^{s+1} - t_1^{s+1}) \\ & - \frac{a}{2(s + 2)}(T^{s+2} - t_1^{s+2}) \end{aligned} \right]$$

... (12)

#### III. Deterioration Cost (DC)

The deterioration cost over the period  $[0, T]$  is

$$DC = C_d \int_0^T r Q(t) dt$$

$$\Rightarrow DC = C_d \left[ \begin{aligned} & \left[ S + (S + R_0)(t_1 - t_0) - \frac{R_0}{2}(t_1^2 - t_0^2) + \left\{ (R_0 - a t_1)T + \frac{a}{2}T^2 \right\} (T - t_1) \right] \\ & - \left[ \frac{(R_0 - a t_1)}{2}(T^2 - t_1^2) - \frac{a}{6}(T^3 - t_1^3) \right] \end{aligned} \right] \quad \dots (13)$$

#### IV. Inventory Holding Cost (IHC)

The holding cost for carrying inventory over the period [0, T] is

$$\begin{aligned} \text{IHC} &= \int_0^T (h + r t) Q(t) dt \\ \Rightarrow \text{IHC} &= \left[ \begin{aligned} & h \left\{ S \left( t_0 + \frac{r}{s+1} t_0^2 - \frac{r}{2} t_0^2 \right) \right\} + r \left\{ S \left( \frac{t_0^2}{2} + \frac{r}{s+2} t_0^3 - \frac{r}{3} t_0^3 \right) \right\} + \\ & h \left\{ \left( S + R_0 \left( t_0 - \frac{r}{s+1} t_0^2 + \frac{r}{2} t_0^2 \right) \right) (t_1 - t_0) - \frac{(R_0(1+r) + S)}{2} (t_1^2 - t_0^2) \right\} + \\ & \left. + \frac{R_0 r}{6} (t_1^3 - t_0^3) + \frac{r(S + R_0)}{s+1} (t_1^{s+1} - t_0^{s+1}) - \frac{r s R_0}{(s+1)(s+2)} (t_1^{s+2} - t_0^{s+2}) \right\} + \\ & r \left\{ \left( \frac{S + R_0}{2} \left( t_0 - \frac{r}{s+1} t_0^2 + \frac{r}{2} t_0^2 \right) \right) (t_1^2 - t_0^2) - \frac{(R_0(1+r) + S)}{3} (t_1^3 - t_0^3) \right\} + \\ & \left. + \frac{R_0 r}{8} (t_1^4 - t_0^4) + \frac{r(S + R_0)}{s+2} (t_1^{s+2} - t_0^{s+2}) - \frac{r s R_0}{(s+1)(s+3)} (t_1^{s+3} - t_0^{s+3}) \right\} + \\ & h \left\{ \left( (R_0 - a t_1)T + \frac{(a + r(R_0 - a t_1))}{2} T^2 + \frac{a r}{3} T^3 \right) (T - t_1) \right. \\ & \left. - \left( \frac{r(R_0 - a t_1)}{s+1} T^{s+1} - \frac{a r}{s+2} T^{s+2} \right) \right\} + \\ & h \left\{ - \left( (R_0 - a t_1)(1 + r T) + \frac{a r T^2}{2} \right) \left( \frac{T^2 - t_1^2}{2} \right) + \left( \frac{r(R_0 - a t_1) - a}{2} \right) \left( \frac{T^3 - t_1^3}{3} \right) \right\} + \\ & + \frac{a r}{6} \left( \frac{T^4 - t_1^4}{4} \right) + \left( r(R_0 - a t_1)T + \frac{a r T^2}{2} \right) \left( \frac{T^{s+1} - t_1^{s+1}}{s+1} \right) \\ & - \left( \frac{r s (R_0 - a t_1)}{s+1} \right) \left( \frac{T^{s+2} - t_1^{s+2}}{s+2} \right) - \left( \frac{a r s}{2(s+2)} \right) \left( \frac{T^{s+3} - t_1^{s+3}}{s+3} \right) \\ & r \left\{ \left( (R_0 - a t_1)T + \frac{(a + r(R_0 - a t_1))}{2} T^2 + \frac{a r}{3} T^3 \right) \left( \frac{T^2 - t_1^2}{2} \right) \right. \\ & \left. - \left( \frac{r(R_0 - a t_1)}{s+1} T^{s+1} - \frac{a r}{s+2} T^{s+2} \right) \right\} + \\ & r \left\{ - \left( (R_0 - a t_1)(1 + r T) + \frac{a r T^2}{2} \right) \left( \frac{T^3 - t_1^3}{3} \right) + \left( \frac{r(R_0 - a t_1) - a}{2} \right) \left( \frac{T^4 - t_1^4}{4} \right) \right\} + \\ & + \frac{a r}{6} \left( \frac{T^5 - t_1^5}{5} \right) + \left( r(R_0 - a t_1)T + \frac{a r T^2}{2} \right) \left( \frac{T^{s+2} - t_1^{s+2}}{s+2} \right) \\ & - \left( \frac{r s (R_0 - a t_1)}{s+1} \right) \left( \frac{T^{s+3} - t_1^{s+3}}{s+3} \right) - \left( \frac{a r s}{2(s+2)} \right) \left( \frac{T^{s+4} - t_1^{s+4}}{s+4} \right) \end{aligned} \right] \quad \dots (14)$$

**V. Production Cost (PC)**

The production cost during the period is

$$PC = C_p S$$

... (15)

Hence, Total cost per unit time is

$$TC = \frac{1}{T} [ OC + AMC + DC + IHC + PC ]$$

$$\Rightarrow TC = \frac{1}{T} \left[ \begin{aligned} & A + \left\{ C_a r s \left[ \frac{S - s}{s} + \frac{(S + R_0 -)}{s} (t_1^s - s) - \frac{R_0}{s + 1} (t_1^{s+1} - s^{s+1}) + \frac{\{(R_0 - a t_1)T + \frac{a}{2}T^2\}}{s} (T^s - t_1^s) \right. \right. \\ & \left. \left. - \frac{(R_0 - a t_1)}{s + 1} (T^{s+1} - t_1^{s+1}) - \frac{a}{2(s + 2)} (T^{s+2} - t_1^{s+2}) \right] \right\} \\ & + \left\{ C_d s \left[ S - (S + R_0 -) (t_1 - s) - \frac{R_0}{2} (t_1^2 - s^2) + \{(R_0 - a t_1)T + \frac{a}{2}T^2\} (T - t_1) \right. \right. \\ & \left. \left. - \frac{(R_0 - a t_1)}{2} (T^2 - t_1^2) - \frac{a}{6} (T^3 - t_1^3) \right] \right\} \\ & + \left\{ h \left[ S \left( - + \frac{r - s^{s+1}}{s + 1} - \frac{s - s^2}{2} \right) \right] + r \left[ S \left( \frac{s^2}{2} + \frac{r - s^{s+2}}{s + 2} - \frac{s - s^3}{3} \right) \right] + \right. \\ & \left. \left\{ \left[ S + R_0 \left( - - \frac{r - s^{s+1}}{s + 1} + \frac{s - s^2}{2} \right) \right] (t_1 - s) - \frac{(R_0(1 + s -) + S s)}{2} (t_1^2 - s^2) \right\} + \right. \\ & \left. + \frac{R_0 s}{6} (t_1^3 - s^3) + \frac{r(S + R_0 -)}{s + 1} (t_1^{s+1} - s^{s+1}) - \frac{r s R_0}{(s + 1)(s + 2)} (t_1^{s+2} - s^{s+2}) \right\} + \right. \\ & \left. + \left\{ \left[ S + R_0 \left( - - \frac{r - s^{s+1}}{s + 1} + \frac{s - s^2}{2} \right) \right] (t_1^2 - s^2) - \frac{(R_0(1 + s -) + S s)}{3} (t_1^3 - s^3) \right\} + \right. \\ & \left. + \frac{R_0 s}{8} (t_1^4 - s^4) + \frac{r(S + R_0 -)}{s + 2} (t_1^{s+2} - s^{s+2}) - \frac{r s R_0}{(s + 1)(s + 3)} (t_1^{s+3} - s^{s+3}) \right\} + \right. \\ & \left. \left\{ \left( (R_0 - a t_1)T + \frac{(a + s(R_0 - a t_1))}{2} T^2 + \frac{a s}{3} T^3 - \frac{r(R_0 - a t_1)}{s + 1} T^{s+1} - \frac{a r}{s + 2} T^{s+2} \right) (T - t_1) \right. \right. \\ & \left. \left. - \left( (R_0 - a t_1)(1 + s T) + \frac{a s T^2}{2} \right) \left( \frac{T^2 - t_1^2}{2} \right) + \left( \frac{s(R_0 - a t_1) - a}{2} \right) \left( \frac{T^3 - t_1^3}{3} \right) \right. \right. \\ & \left. \left. + \frac{a s}{6} \left( \frac{T^4 - t_1^4}{4} \right) + \left( r(R_0 - a t_1)T + \frac{a r T^2}{2} \right) \left( \frac{T^{s+1} - t_1^{s+1}}{s + 1} \right) \right. \right. \\ & \left. \left. - \left( \frac{r s (R_0 - a t_1)}{s + 1} \right) \left( \frac{T^{s+2} - t_1^{s+2}}{s + 2} \right) - \left( \frac{a r s}{2(s + 2)} \right) \left( \frac{T^{s+3} - t_1^{s+3}}{s + 3} \right) \right\} + \right. \\ & \left. \left\{ \left( (R_0 - a t_1)T + \frac{(a + s(R_0 - a t_1))}{2} T^2 + \frac{a s}{3} T^3 \right) \left( \frac{T^2 - t_1^2}{2} \right) \right. \right. \\ & \left. \left. - \frac{r(R_0 - a t_1)}{s + 1} T^{s+1} - \frac{a r}{s + 2} T^{s+2} \right\} \left( \frac{T^2 - t_1^2}{2} \right) \right. \\ & \left. + \left\{ \left( (R_0 - a t_1)(1 + s T) + \frac{a s T^2}{2} \right) \left( \frac{T^3 - t_1^3}{3} \right) + \left( \frac{s(R_0 - a t_1) - a}{2} \right) \left( \frac{T^4 - t_1^4}{4} \right) \right. \right. \\ & \left. \left. + \frac{a s}{6} \left( \frac{T^5 - t_1^5}{5} \right) + \left( r(R_0 - a t_1)T + \frac{a r T^2}{2} \right) \left( \frac{T^{s+2} - t_1^{s+2}}{s + 2} \right) \right. \right. \\ & \left. \left. - \left( \frac{r s (R_0 - a t_1)}{s + 1} \right) \left( \frac{T^{s+3} - t_1^{s+3}}{s + 3} \right) - \left( \frac{a r s}{2(s + 2)} \right) \left( \frac{T^{s+4} - t_1^{s+4}}{s + 4} \right) \right\} \right. \\ & \left. + C_p S \right] \end{aligned}$$

... (16)



Our objective is to determine optimum values  $\mu^*$ ,  $t_1^*$  and  $T^*$  of  $\mu$ ,  $t_1$  and  $T$  respectively so that  $TC$  is minimum. Note that values  $\mu^*$ ,  $t_1^*$  and  $T^*$  can be obtained by solving the equations

$$\frac{\partial(TC)}{\partial \mu} = 0, \frac{\partial(TC)}{\partial t_1} = 0 \text{ \& } \frac{\partial(TC)}{\partial T} = 0 \quad \dots (17)$$

such that

$$\left. \begin{array}{l} \left| \begin{array}{ccc} \frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu \partial t_1} & \frac{\partial^2 TC}{\partial \mu \partial T} \\ \frac{\partial^2 TC}{\partial t_1 \partial \mu} & \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial T} \\ \frac{\partial^2 TC}{\partial T \partial \mu} & \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T^2} \end{array} \right|_{\mu=\mu^*, t_1=t_1^*, T=T^*} > 0 \\ \\ \left| \begin{array}{cc} \frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu \partial t_1} \\ \frac{\partial^2 TC}{\partial t_1 \partial \mu} & \frac{\partial^2 TC}{\partial t_1^2} \end{array} \right|_{\mu=\mu^*, t_1=t_1^*, T=T^*} > 0 \\ \\ \left. \frac{\partial^2 TC}{\partial \mu^2} \right|_{\mu=\mu^*, t_1=t_1^*, T=T^*} > 0 \end{array} \right\} \dots (18)$$

The optimal solution of the equations in (17) can be obtained by using Mathematica software. The above developed model is illustrated by means of the following numerical example.

### 5. NUMERICAL EXAMPLE

Let us consider the following numerical example to illustrate the above developed model. We take the values of the parameters  $A = 200$ ,  $\alpha = 0.0001$ ,  $\beta = 2$ ,  $h = 12$ ,  $r = 5$ ,  $C_a = 10$ ,  $C_d = 7$ ,  $C_p = 14$ ,  $R_0 = 12$ ,  $\theta = 0.02$  &  $a = 12$  (with appropriate units of measurement). We obtain the values  $\mu = 129.11$  units,  $t_1 = 238.71$  units,  $T = 241.79$  units and total cost  $TC = 125878.98$  units and order quantity  $S = 1575$  units by using Mathematica software.

### 6. PARTIAL SENSITIVITY ANALYSIS

Partial sensitivity analysis is very important technique to identify the effect of optimal solution of the model by the changes in its parameter values. In this section, we study the partial sensitivity of total cost per time unit  $TC$  with respect to the changes in the values of the parameters  $A$ ,  $h$ ,  $r$ ,  $C_a$ ,  $C_d$ ,  $C_p$ ,  $R_0$  and  $a$ .

This analysis is performed by considering 10% and 20% increase and decrease in each

one of the above parameters keeping all other remaining parameter as fixed. The results are shown in the Table below. The last column of table shows the % change in TC as compared to the original solution corresponding to the change in parameters values, taken one by one.

### Partial Sensitivity Analysis

Parameter	% Change	Values	$\mu$	$t_1$	T	TC	% change in TC
A	-20	160	129.1144034	238.7073241	241.7932892	125878.8081	-0.000152429
	-10	180	129.114404	238.7073237	241.7932882	125878.9015	-0.000078222
	10	220	129.1144053	238.7073228	241.7932863	125879.0777	0.000061694
	20	240	129.1144059	238.7073224	241.7932854	125879.1497	0.000118908
h	-20	9.6	129.2617677	238.7155387	241.790363	125044.6347	-0.66283118
	-10	10.8	129.1879783	238.7114241	241.7918336	125499.3186	-0.301624126
	10	13.2	129.0410461	238.7032361	241.7947242	126258.8389	0.301749206
	20	14.4	128.9679017	238.6991626	241.7961445	126638.8976	0.603673076
R	-20	4	128.8824292	238.6943989	241.7987094	101586.971	-19.29792021
	-10	4.5	129.0110975	238.7015651	241.7957166	114799.7064	-8.801542444
	10	5.5	129.1991762	238.7120511	241.7912762	139925.0154	11.15834682
	20	6	129.2699898	238.7160025	241.789584	150951.3096	19.91778584
$C_a$	-20	8	129.1329818	238.7082344	241.7925299	125828.287	-0.040287073
	-10	9	129.1236934	238.7077788	241.7929087	125853.8829	-0.019953392
	10	11	129.1051156	238.7068677	241.7936658	125904.0781	0.019922368
	20	12	129.0958261	238.7064121	241.7940441	125929.1804	0.039863995
$C_d$	-20	5.6	129.1279286	238.7082411	241.7927761	125824.1096	-0.043605716
	-10	6.3	129.1211665	238.7077822	241.7930318	125851.5434	-0.021811898
	10	7.7	129.1076431	238.7068644	241.7935427	125906.4161	0.021779695
	20	8.4	129.1008818	238.7064057	241.7937979	125933.8549	0.04357747
$C_p$	-20	11.2	129.1216246	238.7076903	241.7930428	125860.7423	-0.01450417
	-10	12.6	129.1180147	238.7075068	241.7931651	125869.8604	-0.007260655
	10	15.4	129.1107945	238.7071397	241.7934095	125888.0979	0.007227512
	20	16.8	129.1071843	238.7069562	241.7935316	125897.2174	0.014472164
$R_0$	-20	9.6	128.7325618	238.964071	241.7532156	100874.1713	-19.86417808
	-10	10.8	128.9306668	238.8313431	241.7737765	113383.6262	-9.926495923
	10	13.2	129.2859739	238.5907082	241.8119011	138361.2914	9.91610309
	20	14.4	129.4470758	238.480487	241.8297366	150831.4672	19.82258135
a	-20	9.6	129.524135	238.4275159	241.8383905	125649.9666	-0.181947251
	-10	10.8	129.3043644	238.5781615	241.8139184	125772.7883	-0.084376058
	10	13.2	128.9479259	238.8197313	241.7755919	125972.0634	0.073930818
	20	14.4	128.8003542	238.9187655	241.760199	126054.5905	0.139491479

Table: Partial sensitivity analysis of parameters considered in defining inventory model

## 7. GRAPHICAL PRESENTATION

Graphical presentation of the above partial sensitivity analysis is shown in Fig. 2.

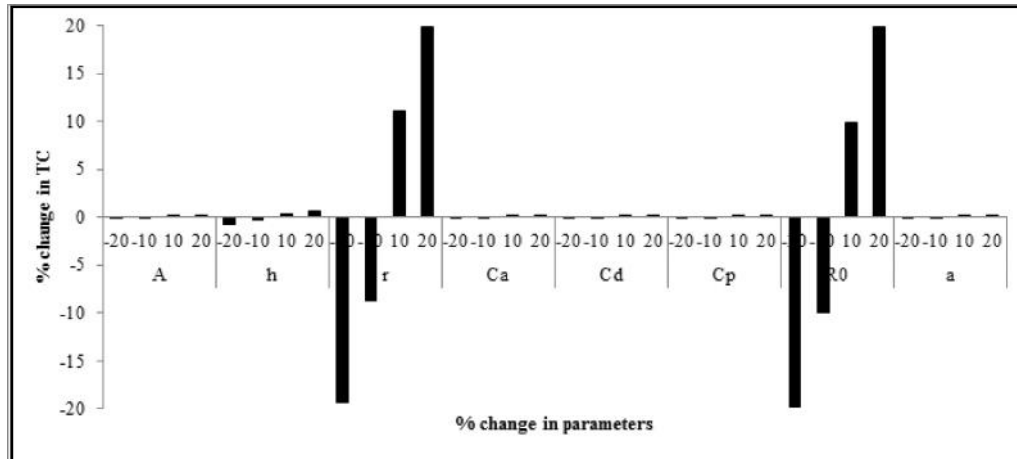


Fig. 2. Graphical presentation of the above partial sensitivity analysis

## 8. CONCLUDING REMARKS

From the above partial sensitivity analysis and graphical presentation we may conclude that the total cost TC per time unit is highly sensitive to changes in the values of the parameters  $R_0$  and  $r$  and it is less sensitive to changes in the values of the other parameters.

## 9. ACKNOWLEDGEMENT

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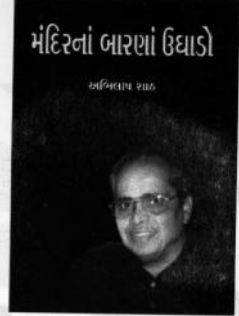
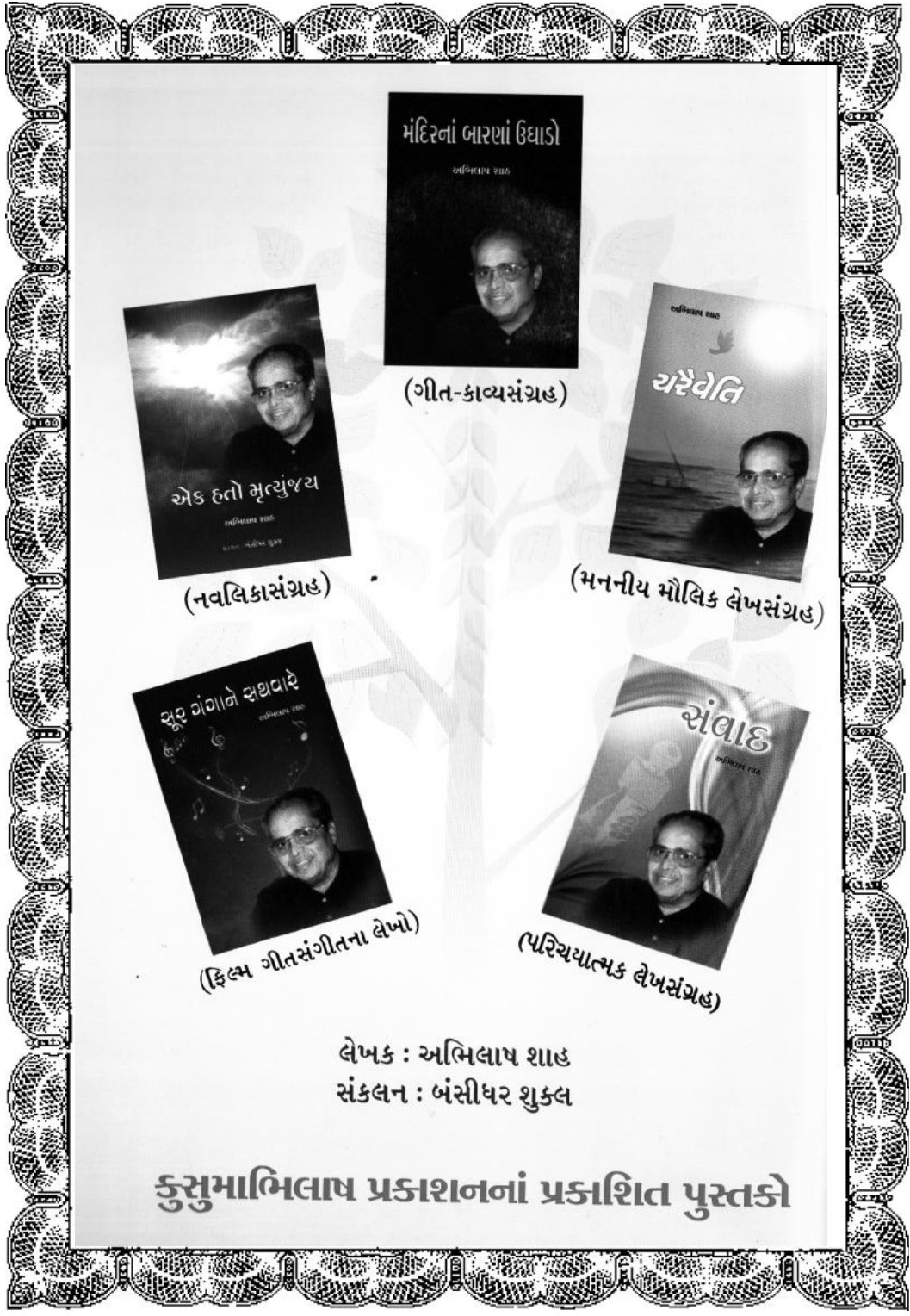
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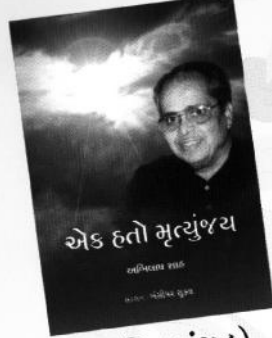
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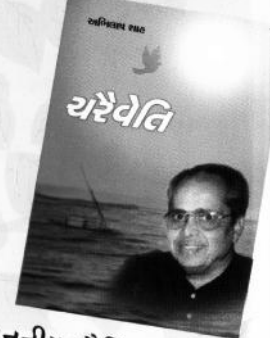




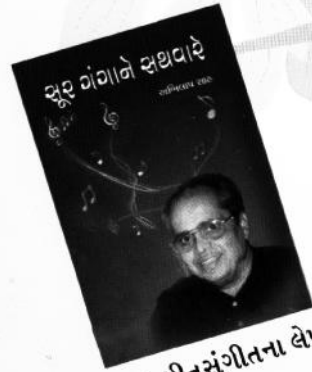
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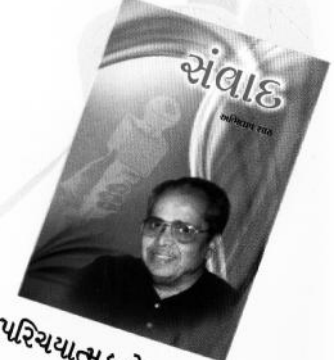
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(મનનીય મૌલિક લેખસંગ્રહ)



(ફિલ્મ ગીતસંગીતના લેખો)



(પરિચયાત્મક લેખસંગ્રહ)

લેખક : અમ્બિલાષ શાહ  
સંકલન : બંસીધર શુક્લ

કુસુમામ્બિલાષ પ્રકાશનનાં પ્રકાશિત પુસ્તકો

**REGRESSION ANALYSIS FOR SOME MAJOR  
FINANCIAL INDICATORS OF CADILA HEALTH CARE LTD.**

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**D. J. Shah\* and H. M. Dixit\*\***

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**ABSTRACT**

Prime objective of any business entity is to protect interests of its stakeholders. The fulfillment of this objective may be examined by using financial statement analysis techniques. It comprises of several methods, but among them the technique of ratio analysis is widely used. In this paper an attempt is made to develop some regression models in order to identify the trends of certain major financial indicators. Moreover multiple regression models have been developed on the basis of financial informations used in obtaining well known debt-equity ratio and RONW ratio.

**1. Introduction**

The basic requirement for financial planning analysis and decision making is the financial information. Financial information is required to forecast, compare and evaluate the firm's earnings ability. It is also needed for economic decision making, investment and financial decision making. The financial information of an enterprise is contained in the financial statements or accounting reports.

Financial statements refer to profit and loss account and the balance sheet drawn by every business at the end of each year. These statements are of great importance and significance to various classes of persons i.e. management, shareholders, prospective investors, creditors, bankers, financial institutions, government, income tax authorities, market researches etc. The profit and loss account shows the amount of profit earned or the loss incurred during the year and the balance sheet reveals the financial position of a business entity and about its owner's interests in the business. In nutshell it communicates information about assets, liabilities and owner's equity for a business firm as on a specific date.

It is a well known fact that the financial statements i.e. the profit and loss account and the balance sheet indicates the profitability and financial strength of a business concern. The detailed methodological study of the data contained in these

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\* and \*\* (Arts, Science & Commerce College Pilvai (N.G.)  
(rcd. Aug. 2015 / rvd. Dec. 2015)

statements is called its analysis. Interpretation means studying the data and commenting on it. Thus both analysis and interpretation are supplements to each other. For interpretation analysis is required and analysis without interpretation is useless and does not serve any purpose.

According to John Myer “**Financial Statement Analysis is largely a study of relationship among the financial parameters in a business as disclosed by a single set of statements and a study of the trends of these factors as shown in a series of statements**”.

## 2. Aims and objectives of Financial Analysis

The main aims and objectives of analysis of financial statements are elaborated below.

1. To judge the earning capacity of the enterprise.
2. To judge the solvency of the firm, both short term and long term.
3. To help the management to judge the operational efficiency of the concern and to make forecasts about future course of action.
4. To make comparison with other similar firms.

In this paper we want to consider some famous financial indicators associated with a business concern. In the light of this, regression analysis is carried out and an econometric application pertaining to the causal relationship between reported profit, net worth and total debt, for Cadila Health Care Ltd. which is a flagship of Zydus Cadila Group(Ahmedabd) and the fifth largest pharmaceutical company in India. It is a significant manufacturer of generic drugs.

## 3. Database

The following analysis is carried out on the basis of audited & published financial reports of Cadila Health Care Ltd., during financial years 2000 – 2001 to 2014 – 2015.

## 4.. Methodology

The following models are considered for major financial indicators of Cadila Health Care Ltd,

**Model A :- Regression models pertaining to reported profit, net worth and total debt.**

**Model A – 1 : -**

Let us define  $(RP)_{x_i}$  = Reported profit for the year  $x_i$  (where  $i = 1,2,3,\dots,15$ )  
Then we consider  $(RP)_{x_i} = \alpha_1 \exp [\beta_1 X_i + u_{x_i}]$  as the regression model pertaining to reported profit, where  $\alpha_1$  and  $\beta_1$  are the parameters of the model and  $u_{x_i}$  denotes, the disturbance term for the year  $x_i$

We want to fit this model for the relevant data regarding reported profit of the Cadila Health Care Ltd. Let us consider log-linear transformation for the above model, then equation takes the form-



$$\text{Ln}(\text{RP})_{x_i} = \text{Ln}\alpha_1 + \beta_1 X_i + u_{x_i}$$

We define  $\text{Ln}(\text{RP})_{x_i} = y_{x_i}$

$$\text{Ln} \alpha_1 = A_1$$

$$\beta_1 = B_1 \text{ and}$$

$$u_{x_i} = Z_{x_i} \text{ then}$$

$y_{x_i} = A_1 + B_1 X_i + u_{x_i}$  is the log-linear form for the above regression model. Under the usual assumptions we can obtain OLSE of  $A_1$  and  $B_1$  given by  $\hat{A}_1$  and  $\hat{B}_1$  by using the standard formulae. The estimated values of  $y_{x_i}$  for given  $X_i$  are computed from

$$\hat{Y}_{x_i} = \hat{A}_1 + \hat{B}_1 X_i$$

We can also examine the significance of the estimated regression coefficients as well as do ANOVA for the fitted model.

#### MODEL A - 2 :-

Let us define  $(\text{NW})_{x_i}$  = Net worth (i.e. equity + reserves and surplus) for the year  $X_i$  (where  $i = 1, 2, 3, \dots, 15$ ). Then we consider  $(\text{NW})_{x_i} = \alpha_2 \exp [\beta_2 X_i + u_{x_i}]$  as the regression model pertaining to net worth, where  $\alpha_2$  and  $\beta_2$  are the parameters of the model and  $u_{x_i}$  denotes the disturbance term for the year  $X_i$ .

We want to fit this model for the relevant data regarding the net worth of the Cadila Health Care Ltd. Let us consider log- linear transformation for the above model then equation takes the form.

$$\text{Ln}(\text{NW})_{x_i} = \text{Ln}\alpha_2 + \beta_2 X_i + u_{x_i}$$

We define  $\text{Ln}(\text{NW})_{x_i} = Y_{x_i}$

$$\text{Ln}\alpha_2 = A_2$$

$$\beta_2 = B_2 \text{ and}$$

$$u_{x_i} = Z_{x_i} \text{ then}$$

$Y_{x_i} = A_2 + B_2 X_i + Z_{x_i}$  is the log-linear form for the above regression model.

Under the usual assumptions we can obtain OLSE of  $A_2$  and  $B_2$  by  $\hat{A}_2$  and  $\hat{B}_2$  by using the standard formulae. The estimated values of  $Y_{x_i}$  for given  $x_i$  are computed from.

$$\hat{Y}_{x_i} = \hat{A}_2 + \hat{B}_2 X_i$$

We can also examine the significance of the estimated regression coefficients as well as do ANOVA for the fitted model.

#### MODEL A - 3 :-

Let us define  $(\text{TD})_{x_i}$  = Total Debt for the year  $x_i$  (where  $i=1,2,3,\dots,15$ ). Then we consider  $(\text{TD})_{x_i} = \alpha_3 \exp [\beta_3 X_i + u_{x_i}]$  as the regression model pertaining to total debt, where  $\alpha_3$  and  $\beta_3$  are the parameters and  $u_{x_i}$  is the disturbance term for the year  $x_i$ .

We want to fit this model for the relevant data regarding total debt of the Cadila

Health Care Ltd. Let us consider log-linear transformation for the above model, then equation takes the form.

$$\begin{aligned} \text{Ln (TD)}_{x_i} &= \text{Ln}\alpha_3 + \beta_3 x_i + u_{x_i} \\ \text{We define } \text{Ln(TD)}_{x_i} &= Y_{x_i} \\ \text{Ln}\alpha_3 &= A_3 \\ \beta_3 &= B_3 \text{ and} \\ u_{x_i} &= Z_{x_i} \text{ then} \end{aligned}$$

$Y_x = A_3 + B_3 X_i + Z_{x_i}$  is the log-linear form for the above regression model. Under the usual assumptions we can obtain OLSE of  $A_3$  and  $B_3$  given by  $\hat{A}_3$  and  $\hat{B}_3$  by using the standard formulae. The estimated values of  $y_{x_i}$  for given  $X_i$  are computed from

$$\hat{Y}_{x_i} = \hat{A}_3 + \hat{B}_3 X_i$$

We can also examine the significance of the estimated regression coefficients as well as do ANOVA for the fitted model.

**MODEL B :-** Multiple regression models pertaining to reported profit, net worth and total debt.

**MODEL B – 1 :-** Relationship between Reported Profit (RP). and Networth(NW) as per passage of time.

$$(\text{RP})_{x_i} = \alpha_0 (\text{NW})_{x_i}^{\alpha_1} \cdot \exp(\alpha_2 x_i + u_{x_i})$$

Taking natural logarithm on both the sides,

$$\text{Ln}(\text{RP})_{x_i} = \text{Ln}\alpha_0 + \alpha_1 \text{Ln}(\text{NW})_{x_i} + \alpha_2 X_i + u_{x_i}$$

Here  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$  are the structural parameters of the model and OLS estimates of these parameters can be obtained and they can be tested for their statistical significance.

**MODEL B – 2 :-** Relationship between Net worth (NW) and Total Debt (TD) as per passage of time.

$$(\text{TD})_{x_i} = \beta_0 (\text{NW})_{x_i}^{\beta_1} \cdot \exp(\beta_2 x_i + u_{x_i})$$

Taking natural logarithm on both sides, then

$$\text{Ln}(\text{TD})_{x_i} = \text{Ln} \beta_0 + \beta_1 \text{Ln}(\text{NW})_{x_i} + \beta_2 X_i + u_{x_i}$$

Here  $\beta_0$ ,  $\beta_1$  and  $\beta_2$  are the structural parameters of the model and OLS estimates of these parameters can be obtained which can be tested for their statistical significance.

## 5. Statistical Applications and Conclusions

Theoretical models discussed above can be applied in the following way and conclusions are drawn on the basis of them for Cadila Health Care Ltd;

**Model A – 1**  
**Reported Profit against time**

$$Y_x = 3.9418 + 0.2044X_i$$

$$t = (26.89)^{**} (14.09)^{**}$$

$$R^2 = 0.9386, F = 198.78^{**}, n = 15$$

For the above regression model for reported profit it is found that model is statistically significant. About 93.86% variation is explained by the model. Elasticity of reported profit is 0.2044, which indicates that due to unit change in time, the reported profit of Cadila Health Care Ltd. increases by about 20.44% per year.

**Model A – 2**  
**Net Worth against time**

$$Y_x = 5.4219 + 0.1980X_i$$

$$t = (156.46)^{**} (57.78)^{**}$$

$$R^2 = 0.9961, F = 3398.70^{**}, n = 15$$

Regression model for net worth suggests that 99.61% variation is explained by the model. It seems that unit change in time accounts for 19.80% increment in net worth of Cadila Health Care Ltd.

**Model A – 3**  
**Total Debt against time**

$$Y = 5.3436 + 0.1292X_i$$

$$t = (28.39)^{**} (6.94)^{**}$$

$$R^2 = 0.7876, F = 48.21^{**}, n = 15$$

The regression model of total debt is found to be statistically significant. On an average the logarithmic value of regressand (total debt) increases by 12.92% with unit change in regressor (i.e. time) and 78.76% variation is explained by the model.

After studying the above models, it appears that growth rate of total debt (fund borrowed) is lower than the reported profit and net worth with passage of time.

**Model B – 1**  
**Reported Profit against Net worth and time**

$$\text{Ln(RP)}_{xi} = - 11.5176 + 2.8512 \text{Ln(NW)}_{xi} - 0.3604X_i$$

$$t = (-2.3550)^{**} (3.1620)^{**} (-2.0140)^{**}$$

$$R^2 = 0.9665, F = 173.21^{**}, n = 15$$

Above model shows statistical significance for the relationship amongst reported profit, net worth and time component. All partial regression coefficients are found to be significant. The value of  $R^2$  shows that about 96.65% variation is explained by the model. Theoretically there should be positive relationship between net worth and reported profit, which is justified by the above model as regression coefficient attached to net worth is found to be positive.

**Model B – 2**  
**Total Debt against Net worth and time**

$\text{Ln(TD)}_x = 15.0537 - 1.7908 \text{Ln(NW)}_x + 0.4840X_t$ $t = (1.8760) \quad (-1.2100) \quad (1.6480)$ $R^2 = 0.8107, F = 25.70, n = 15$
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Even though  $R^2$  is high the above model is not found to be statistically significant. It seems that the company remains unsteady pertaining to borrowing funds from outside, which may be due to the steady growth of its net worth.

**6. Acknowledgement**

We thank the referee for his review of this paper which has helped us in its revision.

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**FIRST ORDER HAZARD ASSESSMENT OF GLACIAL LAKE  
OUTBURST FLOOD USING A STATISTICALMODEL**

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**Rupal H. Budhbhatti\* and B. P. Rathore\*\***

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**ABSTRACT**

Global climate change during the twentieth century had a significant impact on glaciers and glacial environments. This causes the formation and enlargement of glacial lakes in many mountain ranges. Due to increase in the rate of melting of the glaciers, the lakes are increasing in areal extent and water level. Sudden discharge of large volumes of water and debris from these lakes is termed as glacial lake outburst flood (GLOF). The outburst discharge from glacial lakes may cause catastrophic flooding and damage in downstream areas. In the present study Glacial Lake Outburst Flood (GLOF) hazard assessment, using remote sensing has been carried out in a part of Sikkim state of India. In this paper the statistical model given by Mc. Killop and Clague has been used to estimate the outburst Probability of a lake from Sikkim Himalayas. The result of the study can help in carrying out downstream risk assessment, spatial planning and better preparedness for future GLOF hazards.

**Keywords:** GLOF ; Moraine dammed ; NDWI ;Remote sensing;

**1. INTRODUCTION**

Glaciers are formed due to re-crystallization and metamorphism of naturally fallen snow on land surface. It is a permanent snow cover which gives rise to the formation of glaciers. Glaciers are formed when the rate of accumulation is more than the rate of ablation and falling snow gets enough time and space to get metamorphosed to form ice. Glaciers are continuous in motion due to gravity. Glaciers are made up of fallen snow that, over many years, compresses into large, thickened ice masses. Glaciers form when snow remains in one location long enough to transform into ice. During the Little Ice Age (1550–1850 AD) the glaciers reached their maximum state worldwide.

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\* M. E Student L.D. College of Engineering, Navrangpura, Ahmedabad-380015

\*\* Scientist, ISRO (SAC)  
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Since then, a general recession of the world's glaciers has occurred (Mayewski and Jaschke 1979).

The Hindu Kush-Himalaya (HKH) is one of the most sensitive regions to climate change, and particularly the trend of warming that was observed throughout the 20th century to the present day (IPCC 2007). The glaciers of the HKH have responded to the climate change. As a result of this climate change various glacial lakes are formed. A glacial lake is defined as a water mass existing in sufficient amount and extending with a free surface in, under, besides, and/or in front of a glacier and originated by glacier activities. Moraine-dammed lakes are common in glacierized regions around the world (Liboutry et al., 1977; Haeberli, 1983; Costa and Schuster, 1988; Clague and Evans, 2000; Richardson and Reynolds, 2000). They form between the snout of a glacier and its own end moraine. Outburst floods from moraine-dammed lakes have caused tens of millions of dollars of damage to infrastructure and killed thousands of people worldwide (Richardson and Reynolds, 2000). With the increase in glacier retreat on the one hand and the growing population living close to high mountains on the other, the outburst of such moraine dammed lake causes lots of catastrophic flood downstream.

The objective for this study is to estimate the areal expansion or reduction of moraine dammed lakes in past 3 decades, and also to estimate Lake volume of the moraine dammed lakes in order to make the First order Hazard Assessment.

## **2. LITERATURE REVIEW**

In the 1980s, different scientists worked on analysing the outburst flooding events in the Himalayan terrain. Goudie (1981) and Miller (ed. 1984) reported that a total of 339 disasters occurred in 1980 along the Karakorum Highway and the most destructive of these were GLOF events. Scientists Costa and Schuster(1988 was developed model for the peak discharge estimation based on eight moraine-dammed lake outburst events in the Three Sisters and Mount Jefferson wilderness areas, Oregon. The International Centre for Integrated Mountain Development (ICIMOD) and various UN organizations have been actively carrying out extensive studies on the probable hazard site of GLOFs in the Himalayas and the communities that are at a higher risk of suffering the impacts of these events.

These organizations, along with the support of governments and local communities, have been organizing community awareness and training activities to enhance their capacity for the future hazard events in their vicinity. Other recent research into this

field includes a study by Richardson and Reynolds (2000), producing an overview of glacial lakes in the Himalayas. Bajracharya et al. (2007) researched the impacts of climate change on Himalayan glaciers and glacial lakes through a study conducted in joint collaboration with ICIMOD and the UN. McKillop and Clague, J. J.(2007) gave statistical, remote sensing-based approach for estimating the probability of catastrophic drainage from moraine-dammed lakes in south-western British Columbia.

### 3. STUDY AREA

**Geography** The state of Sikkim is characterised by mountainous terrain. Almost the entire state is hilly, with an elevation ranging from 280 meters (920 ft) to 8,586 meters (28,169 ft). The summit of Kangchenjunga—the world’s third-highest peak—is the state’s highest point, situated on the border between Sikkim and Nepal. For the most part, the land is unfit for agriculture because of the rocky, precipitous slopes. These streams combine into the major Teesta River and its tributary, the Rangeet, which flow through the state from north to south. About a third of the state is heavily forested. The Himalayan mountains surround the northern, eastern and western borders of Sikkim. The Lower Himalayas, lying in the southern reaches of the state, are the most densely populated. The state has 28 mountain peaks, more than 80 glaciers, 227 high-altitude lakes (including the Tsongmo, Gurudongmar and Khecheopalri Lakes), five major hot springs, and more than 100 rivers and streams.

**Geology** The hills of Sikkim mainly consist of gneissose and half-schistose rocks, producing generally poor and shallow brown clay soils. The soil is coarse, with large concentrations of iron oxide; it ranges from neutral to acidic and is lacking in organic and mineral nutrients. This type of soil tends to support evergreen and deciduous forests. Most of Sikkim is covered by Precambrian rock, which is much younger in age than the hills. The rock consists of phyllites and schists, and is highly susceptible to weathering and erosion. This, combined with the state’s heavy rainfall, causes extensive soil erosion and the loss of soil nutrients through leaching. As a result, landslides are frequent, often isolating rural towns and villages from the major urban centres.

**Climate** The state has five seasons: winter, summer, spring, autumn, and a monsoon season between June and September. Sikkim’s climate ranges from sub-tropical in the south to tundra in the north. Most of the inhabited regions of Sikkim experience a temperate climate, with temperatures seldom exceeding 28 °C (82 °F) in summer. The average annual temperature for most of Sikkim is around 18 °C (64 °F).

The Lake is situated at 27 31 44.69 N 88 05 11.67 E

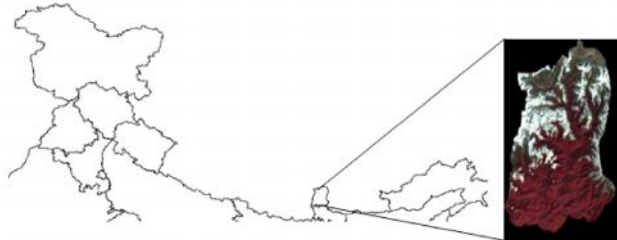


Fig.1. Location map of study area

#### 4. NOTATIONS

Q=Peak Discharge in  $\text{m}^3/\text{s}$ ;  
V=Volume in  $\text{m}^3$ ;  
NDWI= Normalised Difference Water Index;  
GLOF = Glacial Lake Outburst Flood Index;  
GLCF = Global Land Cover Facility;  
USGS =United States Geological Survey.

#### 5. DATA AND METHODOLOGY

The basic materials utilized for this study are satellite images and ASTER DEM. The remote sensing datasets used for the study are coarse to high resolution satellite imagery and Digital Elevation Model (DEM). Since LANDSAT data is available from 1972, therefore to carry out the time series analysis of glacial lakes from 1989 to 2014, various LANDSAT data were used in the research. By comparing all the available images for the study area, images with max 10 percent cloud cover and least snow cover from the month of September to December were selected for the study. LANDSAT (MSS/TM/ETM+/ ALOS) images were downloaded from Earth Explorer of United States Geological Survey (USGS) and GLCF. The combination of digital satellite data and the Digital elevation Model (DEM) is used for Probability estimation. ERDAS Imagine9.1 was used for pre-processing and processing the satellite images such as layer stacking, subset, image enhancement, NDWI generation etc.

The data set images of 1989 and 2014 of Sikkim Himalaya were used. Mapping



of the lake was carried out manually by visual Interpretation. Surface area of the lake could be extracted from remotely sensed data. However, glacier lake volume rather than area is essential to estimate potential peak discharge for a possible outburst. Following Costa and Schuster empirical relation was used.

$$Q_p = 0.00013(PE)^{0.60}$$

where  $Q_p$  is the peak discharge ( $m^3/s$ ) and PE the potential energy of the lake water which is the product of dam height (m), lake volume ( $m^3$ ) and the specific weight of water ( $9800 N/m^3$ ). This model was developed based on eight moraine-dammed lake outburst events and the results are reliable for peak discharge. We used following Akiko Sakai equation for estimation of volume. The lake volume ( $V$  in  $\times 10^6 m^3$ ) and lake area ( $A$  in  $km^2$ ) have following relationship:

$$V = 43.24 \times A^{1.5307}$$

## 6. STATISTICAL BASED APPROACH

McKillop and Clague's probability model quantify outburst probability using a statistical model based on predictor variables for moraine-dammed lakes in south western British Columbia.

To use this model for the Himalayan region an assumption is made that the landscape of coastal British Columbia is analogous to that of the Indian Himalayan region.

$$P(\text{Outburst}) = \{1 + \exp[-\beta_0 + \beta_1(M\_hw) + \sum \beta_j (Ice\_core_j) + \beta_2(Lk\_area) + \sum \beta_k (Geology_k)]\}^{-1}$$

Here  $\beta_0$  is the intercept, and  $\beta_1$  is the regression coefficient for moraine height to width ratio;  $\beta_2$  is the regression coefficient for Lake area;  $\beta_j$  is the regression coefficient for Ice core; and  $\beta_k$  is the regression coefficient for geology.

The calibrated values of the regression coefficients are shown in the table below.

Variable	Category	Coefficient
Intercept	-	-7.1074 ( $\beta_0$ )
M_hw	-	9.4581 ( $\beta_1$ )
Ice_core <sub>j</sub>	Ice-free	1.2321 ( $\beta_{ice-free}$ )
	Ice-cored	-1.2321 ( $\beta_{ice-cored}$ )
Lk_area	-	0.0159 ( $\beta_2$ )
Geology <sub>k</sub>	Granitic	1.5764 ( $\beta_{Intrusive}$ )
	Volcanic	3.1461 ( $\beta_{Volcanic}$ )
	Sedimentary	3.7742 ( $\beta_{Sedimentary}$ )
	Metamorphic	-8.4968 ( $\beta_{Metamorphic}$ )

By substituting the values collected for Moraine dammed lake of Sikkim Himalayan region using the model proposed by McKillop and Clague, the probability of outburst is calculated for the Glacial Lakes Outburst Flood.

Following table shows the estimated value of volume of water and the magnitude of outbursts.

Sr.	Latitude	Longitude	Volume (m <sup>3</sup> )	Peak discharge (m <sup>3</sup> /s)
1	27 31 44.69 N	88 05 11.67 E	8.384276*10 <sup>6</sup>	2725.228522

**Probability estimation:**

No.	Intercept	Lake area in 2014(Ha).	Lake area* S <sub>2</sub>	Moraine h/w	M h/w* S <sub>1</sub>	Probability
1	“7.1074	34.24	0.544444	0.37763	3.57169	0.001025198

**7. CONCLUSION**

It is concluded that the lake can be easily detected and monitored by using remotely sensing data. As per the statistical model for probability estimation the outburst probability is low. It is mainly due because the moraines in the Himalayan regions are made of metamorphic rocks, moraines have low width-to-height ratios and consist of non-cohesive granular materials, hence are strong enough from breaching without any other external factors. They can definitely burst if they encountered an earthquake, avalanche, landslides, cloud bursting, etc.

As a moraine dam failure is a complex phenomenon the dangerous lakes should be studied in detail so that various mitigation measures can be taken .

**8. ACKNOWLEDGEMENTS:**

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## MATHEMAGIC

### Numbers Sequence 1 to 9

$$\begin{aligned}123456789 \times 9 \times 1 &= 111111111 \\123456789 \times 9 \times 2 &= 222222222 \\123456789 \times 9 \times 3 &= 333333333 \\123456789 \times 9 \times 4 &= 444444444 \\123456789 \times 9 \times 5 &= 555555555 \\123456789 \times 9 \times 6 &= 666666666 \\123456789 \times 9 \times 7 &= 777777777 \\123456789 \times 9 \times 8 &= 888888888 \\123456789 \times 9 \times 9 &= 999999999\end{aligned}$$

$$\begin{aligned}152207 \times 73 &= 111111111 \\152207 \times 146 &= 222222222 \\152207 \times 219 &= 333333333 \\152207 \times 292 &= 444444444 \\152207 \times 365 &= 555555555 \\152207 \times 438 &= 666666666 \\152207 \times 511 &= 777777777 \\152207 \times 584 &= 888888888 \\152207 \times 657 &= 999999999\end{aligned}$$

### Pyramid of 8

$$\begin{aligned}9 \times 0 &+ 8 = 8 \\9 \times 9 &+ 7 = 88 \\9 \times 98 &+ 6 = 888 \\9 \times 987 &+ 5 = 8888 \\9 \times 9876 &+ 4 = 88888 \\9 \times 98765 &+ 3 = 888888 \\9 \times 987654 &+ 2 = 8888888 \\9 \times 9876543 &+ 1 = 88888888 \\9 \times 98765432 &+ 0 = 888888888\end{aligned}$$

### Pyramid of 9

$$\begin{aligned}0 \times 81 + 9 &= 9 \\1 \times 81 + 18 &= 99 \\12 \times 81 + 27 &= 999 \\123 \times 81 + 36 &= 9999 \\1234 \times 81 + 45 &= 99999 \\12345 \times 81 + 54 &= 999999 \\123456 \times 81 + 63 &= 9999999 \\1234567 \times 81 + 72 &= 99999999 \\12345678 \times 81 + 81 &= 999999999\end{aligned}$$

### Wonder Number 37

$$\begin{aligned}37 \times 0 &= 000 \\37 \times 3 &= 111 \\37 \times 6 &= 222 \\37 \times 9 &= 333 \\37 \times 12 &= 444 \\37 \times 15 &= 555 \\37 \times 18 &= 666 \\37 \times 21 &= 777 \\37 \times 24 &= 222 \\37 \times 27 &= 999\end{aligned}$$

### Wonder Number 3367

$$\begin{aligned}3367 \times 33 &= 111111 \\3367 \times 66 &= 222222 \\3367 \times 99 &= 333333 \\3367 \times 132 &= 444444 \\3367 \times 165 &= 555555 \\3367 \times 198 &= 666666 \\3367 \times 231 &= 777777 \\3367 \times 264 &= 888888 \\3367 \times 297 &= 999999\end{aligned}$$

\* Dinesh Darji

## SOME SALIENT FEATURES OF TRANSPORTATION PROBLEMS

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\*Jignesh P. Shah and \*\*Ashvinkumar J. Patel

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### ABSTRACT

In this paper brief account about certain significant properties of T.P. (Transportation Problem) are stated. Indication about some related areas of research work is also mentioned with selected references.

### 1. Introduction

We are familiar with the standard form of T.P. as under. Let there be  $m$  origins  $O_i$  ( $i=1,2,\dots,m$ ) and  $n$  destinations  $D_j$  ( $j=1,2,\dots,n$ ), with unit transportation cost matrix  $C = (c_{ij}):m \times n$ . Let  $x_{ij}$  denote the units to be transported from  $O_i$  to  $D_j$ , ( $i=1,2,\dots,m$ ;  $j=1,2,\dots,n$ ), and  $a_i =$  Available units at  $O_i$ , with  $b_j =$  Required units at  $D_j$  ( $i=1,2,\dots,m$ ;  $j=1,2,\dots,n$ ), then standard T.P. is as under

Determine  $x_{ij}$  ( $i=1,2,\dots,m$ ;  $j=1,2,\dots,n$ ) such that

Total T.C. =  $f = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$  is minimum subject to

$$\sum_{j=1}^n x_{ij} = a_i \quad (i=1,2,\dots,m)$$

$$\sum_{i=1}^m x_{ij} = b_j \quad (j=1,2,\dots,n) \quad \text{and all } x_{ij} \geq 0.$$

If  $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$ , it is called Balanced T.P., otherwise it is unbalanced T.P.

**Matrix Form** for the above T.P. is as under:

We define

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\* N. C. Bodiwala and Prin. M. C. Desai Commerce College, Ahmedabad-380001.

\*\* R. H. Patel Arts and Commerce College, Ahmedabad-380013.

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$$\begin{aligned} \underline{x}' &= [x_{11}, x_{12}, \dots, x_{1n}, x_{21}, x_{22}, \dots, x_{2n}, \dots, x_{m1}, x_{m2}, \dots, x_{mn}] : 1 \times mn \\ \underline{c}' &= [c_{11}, c_{12}, \dots, c_{1n}, c_{21}, c_{22}, \dots, c_{2n}, \dots, c_{m1}, c_{m2}, \dots, c_{mn}] : 1 \times mn \\ \underline{a}' &= (a_1, a_2, \dots, a_m) : 1 \times m \\ \underline{b}' &= (b_1, b_2, \dots, b_n) : 1 \times n \\ \underline{d}' &= (\underline{a}', \underline{b}') : 1 \times (m+n) \end{aligned}$$

$$A = \begin{bmatrix} 1'_n & 0'_n & 0'_n & \dots & 0'_n \\ 0'_n & 1'_n & 0'_n & \dots & 0'_n \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0'_n & 0'_n & 0'_n & \dots & 1'_n \\ I_n & I_n & I_n & \dots & I_n \end{bmatrix} : (m+n) \times mn$$

Then Minimise  $f = f(\underline{x}') = \underline{c}' \underline{x}$   
s.t.  $A \underline{x} = \underline{d}$   
and  $\underline{x} \geq \underline{0}$

## 2. Properties:

- [1] The above T.P. has  $mn$  unknown variables with  $m+n$  constraints.
- [2] Rank of matrix  $A$  is  $(A) = m+n-1$ .
- [3] Every minor of  $A$  can have only one of the three values  $-1, 0, 1$ . This is called uni- modular property of matrix  $A$ .
- [4] A necessary and sufficient condition for the existence of a feasible solution is

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = G$$

(This suggests that a feasible solution is  $(a_i, b_j) / G$ )

- [5] There are exactly  $m+n-1$  basic variables in any basic feasible solution.
- [6] An unbalanced T.P. can be converted into a balanced T.P. by introducing dummy origins or dummy variables (as needed) with zero cost coefficients.
- [7] Initial b.f.s can be obtained by methods such as NWC rule, Row Minima, Column Minima, Matrix Minima, VAM etc.
- [8] Optimal solution can be obtained by using MODI method or Stepping Stone algorithm etc.
- [9] A maximization problem can be converted into a minimization problem by converting  $c_{ij}$  elements into  $-c_{ij}$  values.

- [10] If all the cost coefficients of T.C. matrix are multiplied by a constant  $c$ , the optimum solution does not change, but min. T.C. changes accordingly.
- [11] If a constant is added or subtracted from all the cost elements of T.C. matrix, then the optimum solution does not change, but min. T.C changes accordingly.
- [12] Transportation Problem has a triangular basis.
- [13] An ordered set of 4 or more cells in a Transportation table is said to form a Loop if:
- (i.) Any 2 adjacent cells in the ordered set lie either in the same row or same column and
  - (ii.) Any 3 or more adjacent cells in the ordered set do not lie in the same row or same column.
- [14] Every Loop has an even number of cells.
- [15] Let  $x$  be a set of column vectors of the coefficient matrix of a T.P. then the vectors in  $x$  are linearly dependent if and if only, the set of their corresponding cells in transportation table contains a loop.
- [16] In a T.P. with  $m \times n$  transportation table, the maximum number of bases will be  ${}^m C_{m+n-1}$  (e.g: if  $m=2$ ,  $n=3$ , there will be 15 different bases.)
- [17] However, due to the unimodular property of matrix  $A$ , the maximum number of iterations will be  $m^{n-1} \cdot n^{m-1}$ . (e.g with  $m=3$ ,  $n=4$ , we get 432). It can be expected that approximately  $\frac{1}{2} m^{n-1} \cdot n^{m-1}$  of the basic solutions will be feasible.
- [18] If we consider the objective function of T.P. with a fixed cost coefficient

$$\text{e.g. } f = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} + \sum_{i=1}^m \sum_{j=1}^n f_{ij} y_{ij}$$

$$\text{with } y_{ij} = \begin{cases} 1 & \text{if } x_{ij} > 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

then it is a specific type of fixed cost coefficient T.P.

An exact solution can be obtained by using technique similar to Gomory's **cutting plane algorithm** as applied to **integer L.P.P.** Even though it gives the best solution, it may be a time consuming and expensive process.

However M.L.Balinski has suggested an approximate method which is very simple and it gives a solution that is close to the optimum solution.

- [19] Availability and requirement constraints can further be modified into linear or non-linear forms. Similarly objective function need not always be linear. This yields non-linear type problems needing specific solution methods.

- [20] Assignment problem can be viewed as a specific type of T.P. with all availability and requirement values as one. This needs specific methods. There is also a category of assignment problems with specific preferential conditions.

### 3. Concluding Remarks

There is a large area of researches connected with T.P. multiobjective programming problems such as cost as well as time minimization can be considered with specific methods. Transshipment problems are another associated problems etc. There are some issues also related with paradox in Transportation problem.

### 4. Acknowledgement

We are highly indebted to Late Dr. Y. K. Shah whose inspiration and efforts could make this article fruitful. We also thank the referee for revising the earlier draft of this paper.

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## A BRIEF NOTE ON SOCIAL SECTOR DEVELOPMENT INDICATORS

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**\*S. G. Raval and \*\*Mahesh H. Vaghela**

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### ABSTRACT

Social Sector development of a region reflects the growth perspectives pertaining to that region. The state government of Gujarat has given topmost priority for development of social sector. In this context it may be necessary to visualize these indicators for development. An attempt is made in this paper to identify such indicators based upon the available data pertaining to the respective fields.

### 1. INTRODUCTION

Social statistics is the use of statistical measurement systems to study human behavior in a social environment. This can be accomplished through polling a group of people, evaluating a subset of data observations and statistical analysis of a set of data that relates to people and their behaviors.

Social scientists use social statistics for many purposes including

- (1) The evaluation of the quality of services available to a group of people.
- (2) Analyzing behaviors of groups of people in their environment and special situations.
- (3) Determining the needs of people through statistical sampling approach. etc.

Statistics and statistical analysis have become a key feature of social science. Statistics is of basic importance in all the behavioral sciences like Economics, Psychology, Political science, Sociology, Education etc. The use of statistics is so wide spread in the social science that many institutes such as Harvard also have developed statistical institutes which can focus on the quantitative aspects of social science. In some places they have developed departments of decision sciences for study and research works to be executed in different disciplines.

### 2. SOCIAL SECTOR

All social sector statistical activities for collection, presentation and interpretation

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\* Head, statistics dept., som lalit college of commerce, Ahmedabad.  
Email : drsgraval@gmail.com

\*\* Head, statistics dept., N. C. Bodiwala & Prin. M. C. Desai Commerce College,  
Ahmedabad. Email : mahesh.vaghela77@yahoo.com  
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of data at central and state levels are grouped generally under the following categories of classification. (1) Human Development (2) Education (3) Health services (4) Women and child Development (5) Water supply and Sanitation (6) Rural development (7) Rural Housing (8) Urban development and urban Housing (9) Employment and training (10) Welfare of weaker sections (11) Women Welfare (12) Environment (13) Ecological education etc.

The statistical information under each of the above heads are collected, compiled and published at state as well as at central levels. Each of the components in the above list represents specific situations and corresponding policies taken by the government and they are clearly mentioned in the publication year wise.

### **3. WHAT IS AN INDICATOR?**

**Social indicator (S.I.)** is an evidence that helps in assessment of present position and future directions. S.I. is a direct and valid statistical measure which monitors levels and changes over time in a fundamental and social concern.

Indicators may be mentioned such as number related to economic growth and/or immaterial such as values or goods. Most indicators are developed in order to describe important features of a larger system. OECD defines an indicator as parameter or a value derived from parameters information about determining the state of a phenomena/ environment/event with a significance that extends beyond and is directly associated with the parameter value.

#### **Objective and Subjective S.I.**

3.1 **Objective (S.I.)** are statistics which represent social facts independently of personal evaluations. e.g. unemployment rate, production rate, working hours per week, perinatal mortality etc. Objective S.I. needs to make assumption that living conditions can be judged to be favorable or unfavorable from the outside which succums comparing real conditions with normative criteria like values, goals or objectives.

3.2 **Subjective Indicator (S.I.)** are based upon individuals perception and evaluation of social conditions e.g. life satisfaction, job satisfaction, welfare measurements, Average life expectancy etc.

#### **3.3 Functions of S.I.**

Generally S.I. performs one or more of three functions – providing a basis for information for decision making, Monitoring and evaluating policies and/or searching for a common good and deciding how to reach to it.

Indicators should be phrased in such a manner that they can be interpreted by the public, so that community members can provide feedback to promote the development of the respective organization.

Indicators should also be designed so that they can also show progress when social circumstances have really changed. They should not be easily manipulated by political initiatives that do not have a real impact on peoples' lives.

#### 4. Overview of Economy

##### 4.1 For All India

GDP at current prices for the year 2012-13 is estimated at Rs.99.9 Lakh crores and that for 2013-14 is estimated at Rs.113.5 Lakh crore. This shows a growth of 13.1% and 13.6% during these years. Real GDP (i.e., GDP at constant prices stands at Rs. 92.8 Lakh crores for 2012-13 and Rs. 99.2 Lakh crores for 2013-14). This shows growth of 5.1% during 2012-13 and 6.9% during 2014-15 is estimated as Rs. 106.57 Lakh crores, showing a growth of 7.4 percent.

Per capita at current prices is Rs. 71,593 and Rs. 80,388 for years 2012-13 and 2013-14 respectively. For the year 2014-15 it is estimated to be Rs.88, 538 thus showing a rise of 10.1% as compared to previous year. Per capita income at constant prices (2011-12) is estimated at Rs. 66,344 and Rs. 69,959 for the years 2012-13 and 2013-14 respectively. During the year 2014-15, it is estimated to be Rs. 74,193 thus showing a rise of 6.05% as compared to previous year.

##### 4.2 For Gujarat state

Gross state Domestic Product (GSDP) at factor cost at constant prices in 2013-14 has been estimated at Rs.4, 52,625 crore as against Rs.4, 16,163 for 2012-13 showing a growth of 8.8% GSDP at factor cost at current prices in 2013-14 has been estimated at Rs.7, 65,638 crores as against Rs.6, 58,540 crores in 2012-13 thus showing a growth of 16.3% during the year. The share of primary, secondary and tertiary sectors has been reported as 22.1%, 43.1% and 43.8% respectively to total GSDP in 2013-14 at current prices.

Per capita NSDP at factor cost at constant prices is estimated at Rs. 63,168 in 2013-14 as against Rs. 59,157 in 2012-13 thus showing a growth of 6.8% during the year. Per capita NSDP at factor cost at current prices has been estimated at Rs. 106831 in 2013-14 as against Rs.93046 in 2012-13 thus showing an increase of 14.8% during the year.

#### 5. DIFFERENT TYPES OF INDICATORS

Keeping in view the social sector some major indicators can be listed briefly as under:

1. Human Development Index (HDI) and its ranking (It is for country wise comparison and the same at state level SHDI can be helpful for state wise comparison).
2. Education (Quality of Education, Education Expenditures, Drop out ratios, student/teacher ratio etc.)
3. Health (Different Health Indicators, Health Expenditures, Expenses on health to GDP, public health expenditure to total budget etc.)
4. IMR, MMR, CBR, CDR, Life expectancy at birth etc.
5. Housing (Urban and Rural) (Average household size, total number of houses, House occupancy, Housing finance ,Affordable housing needs etc.)

6. Employment
7. Water supply and Sanitation etc
8. Environment etc.

## 6. ILLUSTRATIONS FOR S.I.

Here we want to illustrate three specific types of indicators for Education, Health and Employment respectively.

(1) Table 1 given below shows QEI (Quality of Education Indicator) based upon student/teacher ratio in Gujarat state during 2007-08 to 2013-14. Indicator is calculated considering 2007-08 as the base year.

Table: 1

Quality of Education Indicator (QEI) Based upon student/ Teacher Ratio of Gujarat state

Year	Primary Education		Secondary Education		Higher Education	
	Ratio	Indicator	Ratio	Indicator	Ratio	Indicator
2007-08	33	(100)	35	(100)	42	(100)
2008-09	32	(96.96)	35	(100)	28	(66.67)
2009-10	32	(96.96)	35	(100)	32	(76.19)
2010-11	31	(93.94)	41	(117.14)	28	(66.67)
2011-12	31	(93.94)	39	(111.42)	33	(78.57)
2012-13	30	(90.91)	31	(88.57)	39	(92.86)
2013-14	30	(90.91)	34	(97.14)	42	(100)

Source : SER, G.S. 2014-15

(2) Table-2 given below shows some Health Indicators (Gujarat State) based upon Birth Rate ,Death Rate ,IMR & Community Health Centers respectively. This Indicator is calculated with 2007-08 as the base year.

Table-2

Year	Birth Rate		Death Rate		IMR		Community Health Centers	
	(In'000)	Indicator	(In'000)	Indicator	(In'000)	Indicator	Units	Indicator
2007-08	23	100	7.2	100	52	100	273	100
2008-09	22.6	98.26	6.9	95.83	50	96.15	283	103.67
2009-10	22.3	96.96	6.9	95.83	48	92.31	291	106.60
2010-11	21.8	94.78	6.7	93.06	44	84.62	305	111.72
2011-12	21.3	92.61	6.7	93.06	41	78.85	318	116.48
2012-13	21.1	91.74	6.6	91.67	38	73.08	318	116.48
2013-14	20.8	90.44	6.5	90.28	36	69.23	318	116.48

Source :SER,GS.2014-15

(3) Table3 given below shows ESI (Employment status indicator) based upon the data for employment exchanges in terms of vacancies, placements and jobseekers respectively. This Indicator is calculated with 2007-08 as the base year.

Table: 3

Employment status Indicator (ESI) (Gujarat state)

Year	Number of Employment Exchanges		Number of Vacancies		Number of Placements		Number of Job seekers	
	(In'000)	Indicator	(In'000)	Indicator	(In'000)	Indicator	(In'000)	Indicator
2007-08	37	100	242	100	192	100	799	100
2008-09	41	110.81	278	114.88	226	117.71	831	104
2009-10	41	110.81	180	74	154	80.21	905	113.27
2010-11	41	110.81	256	105.74	208	109.33	892	111.64
2011-12	41	110.81	304	125.62	226	117.70	899	112.52
2012-13	41	110.81	327	135.12	246	128.13	877	109.76
2013-14	41	110.81	330	131.36	272	148.67	778	97.37
<u>Source</u> SER, G.S. 2014-15								

The above indicators can provide a relative picture of comparison so that data can be readily interpreted.

## 7. DISCUSSION AND CONCLUDING REMARKS

There is an interesting remark considering Gujarat state S.I. data indicating that Gujarat state has more positive developing status as compared to some other states as well as All India .These remarks are based upon the relevant comparisons about SHDI and ranking, Literacy rate, QEI, ESI, IMR, MMR, Gender Bias, Housing situations etc.

This is mainly attributed due to the positive impact of the government policies in Gujarat state. Looking to the data analysis based upon social indicators it may also be concluded that views are fine and may be quoted satisfactorily at different levels, but truly speaking data should be hard and objective and comparison should be done honestly and unbiasedly .

## 8. ACKNOWLEDGEMENTS

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## Biography

### PROFESSOR PANDURANG VASUDEO SUKHATME\*

H. D. Budhbhatti\*\*



Pandurang Vasudeo Sukhatme was born of Vasudeo Hari Sukhatme and Satyabhama Sukhatme on 27th July 1911 in the village Budh, district Satara, 100 miles south of Pune. After completing his school education in Pune, he graduated in 1932 from Ferguson College of the same city with Mathematics as the principal subject and Physics as the subsidiary. During 1933-36, he studied at the University College, London and was awarded a Ph.D. in 1936 and a D.Sc. Degree in 1939 for his work on bipartitional functions. This work was published in the “Philosophical Transactions of the Royal Society of London, Series A”, June, 1938.

Whilst in London, Prof. Sukhatme came under the influence of such eminent authorities in Statistics as R. A. Fisher, Jerzy Neyman and E. S. Person and did valuable research in Statistical Theory of Sampling, his two most significant contribution being, one to bipartitional functions under the guidance of R. A. Fisher and the other to sampling theory entitled “Contributions to the Theory of the Representative Method” under the guidance of J. Neyman and E. S. Pearson. The latter paper laid solid foundations for his subsequent pioneering research in the sampling theory of surveys and improvement of agricultural statistics which ushered in what may be appropriately termed as the Sukhatme era in the development of agricultural statistics in India and the world.

During 1939-40, he was Professor at the All India Institute of Hygiene and Public Health, Calcutta. In 1940 he joined ICAR as a Statistician and was later on appointed as Statistical Advisor to the Council to head its Statistical Unit. On account of his dynamic leadership, following the path and tradition set by him, the statistical branch of ICAR eventually grew to become a full-fledged Institute (Indian Agricultural

\* This article is adapted by net collection from wikipedia and biographical sketch given by Dept. of Mathematics, I.I.T. Bombay.

\*\* Ex. CSO, Head Statistics Dept. GSRTC, Ahmedabad.  
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Statistics Research Institute) exclusively devoted to research in agricultural statistics. In the context of the green revolution the importance techniques in agricultural research hardly needs any emphasis.

Prof. Sukhatme, as a founder of the Indian Society of Agricultural Statistics, devoted a good deal of his time and energy to the popularization on statistical methods among the practitioners of agricultural, veterinary and related sciences. He served as the First Honorary Secretary of the Indian Society of Agricultural Statistics for a number of years. The Society owes him a lot for his continued valuable guidance as well as for shouldering the responsibility, as its President during 1991 and Executive President since 1970 till his demise. From its inception to 1963 he worked closely with Dr. Rajendra Prasad (Founder President of the Society), the then Minister for Food and Agriculture and later President of India.

In 1951, he was Visiting Professor at Iowa State University, Ames Iowa, USA where he completed his textbook on sampling. During 1952-70 he headed the Statistics Division of the Food & Agricultural Organisation (FAO) of the United Nations in Rome. After retiring from the UN in 1971 he served as Regents Professor, University of California at Berkely and then settled in Pune, carrying out valuable work on nutrition at the Maharashtra Association for the Cultivation of Science. He authored several books on the various scientific topics of interest and published more than 200 research papers in reputed national and international journals.

Prof. Sukhatme was well known in the field of nutrition for the Sukhatme-Margen hypothesis which in plain language implies the following. At low levels of calorie intake, energy is used with greater metabolic efficiency and efficiency decreases as the intake increases over the homeostatic range.

He was awarded the Guy Medal by the Royal Statistical Society for his paper on nutrition which he presented to the Society in 1963, the B.C.Guha Memorial Lecturership of the Indian Science Congress Association in 1965 and the B.D.Tilak Lecturership of the Indian National Science Academy in 1982. Among the numerous other honours he had received, mention must be made of the Fellowship of the Americal Statistical Association, National Academy of Sciences, Allahabad, Indian Academy of Sciences, Bangalore and Indian National Science Academy, New Delhi. He was elected member of the International Statistical Institute, Netherlands and its Vice President in 1969-70. For his outstanding contribution to Science and Human Welfare, he was conferred the by the President of India in 1973. He was awarded the Hari Om Ashram Trust Award by the University Grant Commission in 1983. For the distinguished service to the cause of Statistics and its application to agriculture and allied fields, he was conferred with the honour of **Sankhyiki Bhushan** in 1989 by the Indian Society



of Agricultural Statistics, New Delhi. **He also received the P. C. Mohalanobis Birth Centenary Award** at the Jaipur Session of the Indian Science Congress Association in 1994. Prof. Sukhatme expired on 28 January 1997. His life and work are exemplary for all of us. Indeed he was an eminent statistician of India. Government of India, Ministry of Statistics and Programme Implementation (MOSPI) is implementing a plan scheme entitled, **“Awards and Fellowships for outstanding and Meritorial Research Work in Statistics.”** Under the scheme the MOSPI has instituted two National Awards in the names of two renowned statisticians in the country for outstanding and meritorious research work in statistics.

\* **One in the MEMORY of Prof. P. V. Sukhatme and**

\* **Other in the HONOUR of Prof. C. R. Rao**

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## Biography

### NOBEL FOR ECONOMIST WHO STUDIED POVERTY.

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**Jayesh R. Purohit\***

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**Angus Deaton**, born in 1945, Edinburgh, United Kingdom; age 70, won \$ 975,000 prize from the Royal Swedish Academy of Sciences for work that the award committee says has had “immense importance for human welfare, not least in poor countries”.

Micro economist **Angus Deaton** has won the Nobel Prize in Economics for his work on consumption, poverty and welfare that helped governments improve their policies through household surveys and tax changes. The award was presented by the Royal Swedish Academy of Sciences. The organisation stated in its declaration note that the **Deaton’s** work had largely influenced policy making. It helped determine how different social classes are affected by specific tax changes.

The report read, “To design economic policy that promotes welfare and reduces poverty, we must first understand individual consumption choices, more than anyone else, **Angus Deaton** has enhanced this understanding.”

Tyler Cowen, economics professor at George Mason University and blogger, said that Deaton had found out a way to look at economic development from the starting point of consumption rather than income. “Think of Deaton as an economist who looks more closely at what poor households consume to get a better sense of their living standards and possible paths for economic development,” Cowen wrote in a blog.

**Here are three points you must know about Deaton’s Nobel-winning work:**

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\* Mantra Consultants - Social & Industrial Support, Ahmedabad  
(rcd. Nov. 2015 / rvd. Dec. 2015)

· In his work titled *The Great Escape; Health, Wealth and the Origins of Inequality*, **Deaton** shows how the world has witnessed a great economic boom in last two centuries. However, he also tells us about the inequality of wealth that has grown at a faster rate

· He showed how the demand for one good depends on the prices for all other items and on how much money people are making individually. This concept helped countries make policies according to public demand.

The policy of taxation affects all. **Deaton's** theory helped governments understand the actual needs of people and put taxes accordingly.

The secretary of the award committee Torsten Persson said **Deaton's** research has “really shown other researchers and international organizations like the World Bank how to go about understanding poverty at the very basic level so that’s perhaps the finest and most important contribution he has made”.

Persson singled out **Deaton's** work in showing how individual behaviour affects the wider economy and that “we cannot understand the whole without understanding what is happening in the miniature economy of our daily choices”.

**Deaton**, who was born in Edinburgh, Scotland and hold US and British dual citizenship , said he was pleased with the honour. He said he expects extreme poverty in the world to continue decreasing but that he isn't “blindly optimistic”.

He said there are “tremendous health problems among adults and children in India, where there has been a lot of progress”. He noted that half of the children in the country are “still malnourished” and “for many people in the world, things are very bad indeed”.

Ingvild Almas, associate professor at the Norwegian School of Economics, said the Indian government has changed it's methodology for measuring poverty, thanks to research from the likes of **Deaton** and that has affected the poverty-reduction policies.

“For instance, **Deaton** found that there were a lot more poor people in rural area of India than previously thought”. She said. “In practice, that has affected India's subsidy system for the poor, which allows them to buy necessities. Household that were not defined as poor before can now be reached with these policies, and that is a direct result of **Deaton's** research”.

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**H. S. Mediwala\***

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- Sciences without statistics bear no fruit, statistics without science have no root.
- Statistics are the straw out of which I like every economist, have to make bricks  
**-Marshall**
- There is no shortcut to truth, no way to gain a knowledge of the universe except through the gateway of scientific method.  
**-Karl Pearson**
- When you can measure what you are speaking about and express it in numbers, you know something about it. When can not express it in numbers, your knowledge is of meagre and unsatisfactory kind.  
**-Lord Kelvin**
- To consult the statistician after an experiment is finished is often merely to ask him to conduct a post-mortem examination. He can perhaps say what the experiment died of.  
**- R. A. Fisher**
- Everybody believes in the Law of Error, the experimenters because they think it is a mathematical theorem, the mathematicians because they think it is an experimental fact.  
**-Lippman**
- The fact that a general impression is more or less universal can not in itself be a guarrantee of its validity  
**- P.C. Mahalanobis**
- The modern concepts of statistics is the science, the philosophy, the art and the technique of making inferences from particular to general.  
**-J. W. Tukey**
- Scientific data are not taken for museum purposes, they are taken as a basis for doing something. If nothing is to be done with the data, then there is no use collecting any. The ultimate purpose of taking data is to provide a basis for action or a recommendation for action. The step intermediate between the collection of data and action is prediction.  
**-W. E. Deming**
- Statistical analysis conducted properly is a delicate dissection of uncertainties, a surgery of suppositions.
- He, who accepts statistics indiscrimantly will often be duped unnecessarily. But he who distrusts statistis indiscriminatly will often be ignorant unnecessarily.

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\* Rtd. Prof. and Ex. President, Gujarat Ganit Mandal

## BOOK REVIEW

**Title** : Basic Statistics for Business and Economics

**Authors** : Douglas A Lind, William G. Marchal

**Publisher** : McGraw Hill, Irwin (8th Edition 2013)

*There comes a time when you have to choose between turning the page and closing the book. - Josh Jameson*

We know that Statistics is a Science of data analysis and the same is useful for researchers of other subjects and disciplines. Sometimes students and researchers find difficulties in proper applications where statistical methods are to be used. This may lead to wrong conclusions if the appropriate methods are not applied.

There are many books available for students and researchers in the subject. This book has some specialities which make it perfect in the sense of theoretical developments and applications. There are 15 chapters in the book with about 600 pages. Topics contain basic statistics, discrete and continuous probability distributions, sampling methods, estimation theory, large and small sample tests, ANOVA, Regression analysis, Non parametric methods etc.

Many numerical illustrations are given in each chapter. Presentation of the core material of the subject is in very lucid form so that the use of this book becomes very easy and inspiring. At the end of each chapter, exercises are given which are from many practical application fields. This makes the book very useful for researchers also. Practice test work and computer packages are additional features of the book. The overall presentation of the book is extremely noteworthy for anyone who wants to use it for study purpose.

In nutshell this book is very very special as teachers manual also. This is why it has become very popular among students, teachers and research workers. It is definitely unavoidable for those who want to study the subject in behavioural sciences, management and even in engineering where use of Statistical methods is inevitable.

Date : 20/12/2015

**Dr. H. M. Dixit**

Head, Statistics Dept.,

Arts and Commerce College, PILWAI (N.G.)

- Gujarat Science Congress Conference will be held at K.S.K.V. University, BHUJ, Kutch during February 6-7, 2016.
- UGC sponsored National Seminar on BETTER DATA BETTER LIVES was held at statistics Dept., M.S.Uni. Vadodara on Oct. 20, 2015, in Conjunction with WORLD STATISTICS DAY.
- ISPS journal will now be printed by Springer & Co. Dr. K. Muralidharan is secretary of ISPS.
- 7th International Conference on Quality, Reliability, Infocom technology and Business Operations will be held during 28-30 December, 2015 at University of Delhi. This conference is organised by society for Reliability, Engineering, Quality and Operations Management (SPEQOM), Dept. of O.R., Delhi University, Delhi.
- International workshop on Advanced stochastic Modeling and Data Analysis will take place during Jan. 4-9, 2016 at V. University, Tirupati, Andhrapradesh.
- 18th Annual National Conference of Society of Statistics, Computer and Applications will be held at Dept. of Statistics, Jammu Univeristy, Jammu during Feb. 18-20, 2016. This conference is organised by Dept. of Statistics, Jammu University, Jammu.
- Next ISPS conference will be held at Aligarh Muslim University, Aligarh during Dec. 17-19, 2016.
- 103rd Annual conference of Indian Science Congress Association will be held at Uni. of Mysor, Mysore during Jan. 3-7, 2016. P. M. will inaugurate this conference.
- 52th Annual conference of the Indian Econometric Society will be held at I.I.M., Kozhikode during Jan. 4-6, 2016. Pl. contact nrbmurthy@gmail.com
- Annual convention of Indian Agricultural Statistics Society will be held during Jan. 29-31 at Indian Agricultural Research Institute at New Delhi.

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\* Head, Statistics Dept, M. S. Uni, Vadodara 380 002

### [A] FEEDBACK

• **Dr. Rakesh Srivastav**

SV has continuity in publication over the years. It is good for students, teachers and researchers. It gives sufficiently good reference materials. Idea of Biography on last coverpage is welcome.

• **Dr. A. C. Brahmbhatt**

SV issues reveal a happy blend of theory and application. They exhibit how finer interpretations and refinable solutions could be arrived at for the real world problems.

• **Dr. H. M. Dixit**

This journal provides sufficient ideas for new innovations in statistics. Quality is good and praiseworthy. If possible, include quizzes, classroom notes, specialised articles by invitation etc.

• **Prof. J. D. Patel**

SV team is doing excellent work. Some invited talks may be included. Also there is a need to train our teachers in the fast developing world. SV can be instrumental for such activities. My best wishes.

• **Dr. U. B. Gothi**

Indeed good work by the team. Make it digital for its widespread. Evaluation work is good and impartial. If possible, publish it after every quarter (4 issues p.a.)

• **Dr. P. H. Thakar**

SV journal satisfies the needs of theoreticians and practitioners as well. There is an unbiased scheme for evaluation of the articles. At each stage response is quick and it is good to accelerate fast growth in the developing world today. SV team is very actively supporting different ideas placed. You will find variety in each issues which is a speciality of this journal.

\* Rtd. Principal, H. K. Commerce College, Ahmedabad and Ex. Secretary, Gujarat Vidyasabha and Brahmarshiwadi Trust, Ahmedabad.

- **Dr. S. G. Raval**

It provides good platform for research workers. Evaluation process for articles is quick and unbiased. We all appreciate the creditable efforts of SV team. My heartiest congrats and best wishes for this creditable work.

- **Kirtan Parmar**

Even with little experience, I could judge about the creditable work, unbiasedness and efficiency of this system. Idea of Erriata in the next issue is praiseworthy. Make it an online journal and also published it frequently in an year.

**[B] CORRIGENDUM**

- June 2015 Issue

Cover Page 2 ---> EDITORIAL BOARD

---> MEMBERS LIST --> DR. B. H. PRAJAPATI

PAGE 56 (Last Page ---> Last Line --> email id is drjayesh.purohit@gmail.com

- December 2014 Issue

Page 14-21 Expression for V(x) is to be corrected.

**[C] Do you know this ?**

**TERMS : MEANING**

**GOOGLE** : Global Organization Of Oriented Group Language of Earth

**YAHOO** : Yet Another Hierarchical Official Oracle

**ECS** : Elitegroup Computer System

**LCD** : Liquid Crystal Display

**HTML** : Hyper Text Market Language

**MMS** : Multimedia Messaging Service

**IBM** : International Business Machines

**Wi-Fi** : Wireless Fidelity

**GSM** : Global System for Mobile Communication

**CDMA** : Code Division Multiple Access.

**SIM** : Subscriber Identity Module

**JPEG** : Joint Photographic Expert Group

**HTTP** : Hyper Text Transfer Protocol

**PNG** : Portable Network Graphics

**PDF** : Portable Document Format

- **Shailesh Teredesai**



## INSTRUCTIONS TO AUTHORS

Editorial board invites research articles, brief summary of research project reports, review articles, informative articles, research and project reports, review articles, informative articles, research notes, class room notes, statistical queries and other problems of interest as well as any relevant informations that can significantly highlight the applications part pertaining to statistics subject.

Accordingly the editorial board welcomes articles in the fields of agricultural and industrial statistics, operations research and operations research management, economicS and econometrics, theoretical statistics, SQC, Information and coding theory, statistical planning, Biometrics, computer programming applications, envirmetal statistics, demography etc.

TWO copies of the manuscript should be sent to **Dr. B. B. Jani, Editor, Sankhya Vignan**, at B/14, Bansidhar Apartment, Mirambica School Road, Naranpura, Ahmedabad-380013. India.

Please also send your article by email of Sankhya Vignan **svgsa2015@gmail.com** for quick action and response.

The manuscript should be typed on bond paper in double space with sufficient margins on all the sides of the paper. The title of the article should include name(s) and address of the authors. All references should be listed at the end of the article and should be numbered in alphabetical order of authors. It should be produced in an appropriate manner indicating the years of publication, details of publications should be as concise and compact as possible.

**Please note that all submitted articles are reviewed before its publication.**

- Inquiries regarding subscription and other particulars regarding **Sankhya Vignan** should be addressed to the **editorial secretary**. You may also mail to **drjayesh.purohit@gmail.com**
- GSA membership fees and other queried should be address to Secretary, GSA, C/o. Statistics Dept., Gujarat University, Ahmedabad 380009 (INDIA)
- Each author will receive a copy of the published journal. The first named author will receive 10 copies of offprints.
- All payment can be made either by cash or M.O. or Bank draft in favour of Gujarat Statistical Association, payable at Ahmedabad.

- In order to meet with the administrative, printing and postal expences, it is decided by the editorial board to accept advertisements from interested resources. Normal rates of advertisements are as under.

<b>Last full cover page</b>	<b>Rs. 20,000/-</b>
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- These rates will be effective from Jan. 2016. A discount from 20% to 30% canbe given if it is sufficiently in advance.
- Please note that we want to make this journal digital and also publish it on Quarterly Basis.

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**Note :** Members of editorial board are in no way concerned with the views, opinions or ideas expressed in this issue. Authenticity responsibility lies solely with the persons presenting them.

# Gujarat Statistical Association

Established : 1969

[Registered under Public Trust Act of 1950 (Bombay)]

R. No. E2502 A'bad-1974

The objective of the association is primarily to promote statistical ideas in pure and applied fields in the form of study, teaching and research in statistics.

The membership of GSA consists of Life / Institutional / ordinary members.

Membership	Fees	
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Institutional Member (for 3 years)	₹ 2,000/-	US \$ 500
Life Member	₹ 1,500/-	US \$ 300
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## P. V. SUKHATME\*



**Professor Pandurang Vasudeo Sukhatme** (27 July 1911 - 28 Jan 1997) was a very eminent Indian Statistician. He graduated from Ferguson College, Pune. During 1933-36 he studied at the university of London and was awarded Ph.D. degree in 1936 and D.Sc. degree in 1939 for his work on bipartitional functions. He was under the influence of eminent persons like **R. A. Fisher, J. Neyman** and **E.S. Pearson**. He has done pioneering work in agricultural statistics and in Biometry.

He was influential in the establishment of **Indian Agricultural Statistics Research Institute**. Later while working at FAO in Rome, he developed statistical models for assessing the dimensions of hunger and future food supply for size and nature of the protein gap.

Another of his notable contributions was the applications of statistical techniques for studying human nutrition. **Sukhatme-Morgen Hypothesis** suggested that at low calorie intake levels, stored energy in the body is used with greater metabolic efficiency and that the metabolic efficiency decreases as the intake increases above the homostatic range.

### Awards Received

- ☛ The Guy Medal for RSS, London, (1963)
- ☛ The D. C. Guha Memorial Lecturership at Indian Science Congress Association (1973)
- ☛ PADMA BHUSHAN award by Govt. of India (1971)
- ☛ Bal Gangadhar Tilak Lecturership of Indian Science National Academy (1982)

(\* Brief Biographical sketch is given in the Journal)

This page is specially donated by **Prof. Shailesh Teredesai** (Ex. Head) Statistics Dept., S. M. Patel Institute of Commerce, GLS, Ahmedabad-380009.

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(Journal of GSA, Ahmedabad)

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